

Super-Loops (a.k.a. Super-Meshes)

Current Loops with Current Sources (Super Loops)

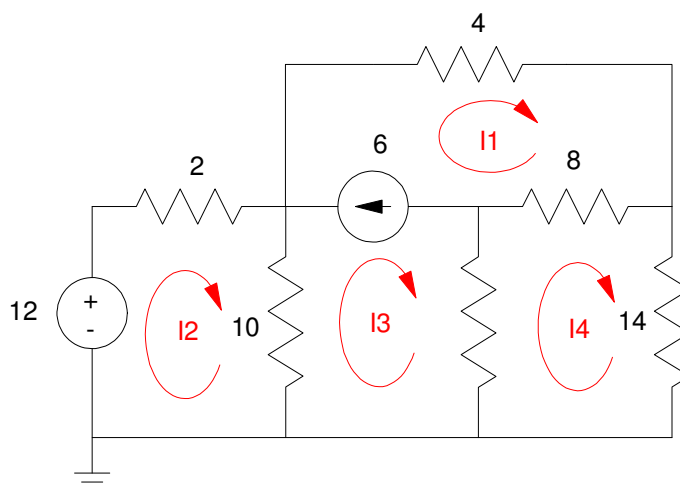
Current sources cause a little trouble with current loops. A current source supplies whatever voltage it takes to maintain current. This is a problem since we're trying to sum voltages to zero: not knowing what a voltage is prevents us from writing the loop equation.

To get around this problem, write two loop equations are:

- One equation is the current as defined by the current loop
- For the second equation, find a path around the circuit which avoids the current source but includes elements not covered by other loops.

The second step is a little tricky and often has multiple options.

For example, write the loop equations for the following circuit:



There are four windows. Hence, we need to write 4 equations to solve for 4 unknowns. Start with the easy one

$$1) \quad I_1 - I_3 = 6$$

Now write the loop equations around I2 and I4:

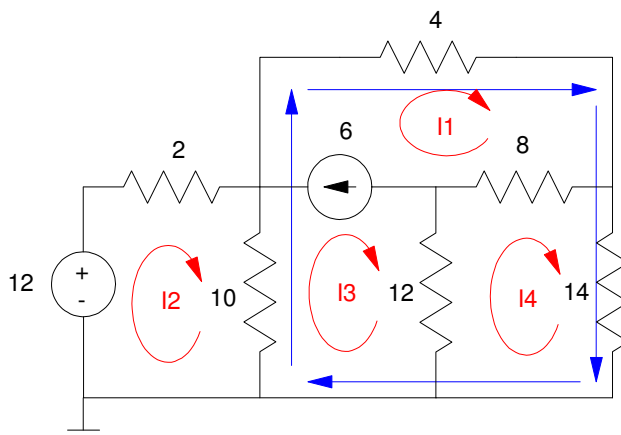
$$2) \quad -12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$3) \quad 12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

We now need a fourth equation. Loop I1 and I3 don't work since the current source has an unknown voltage across it. We still need one more equation.

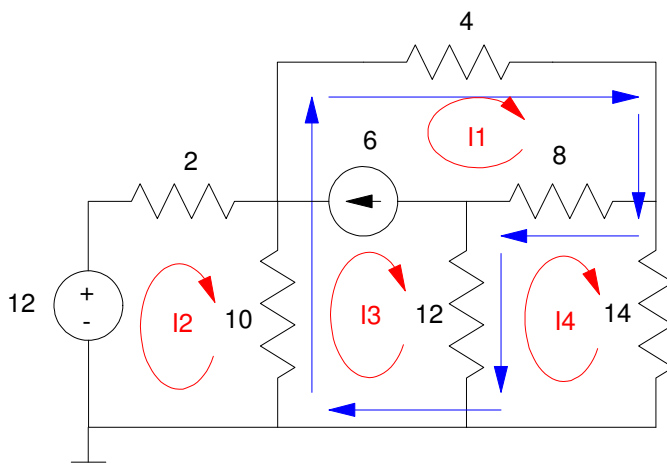
Note that the 4 Ohm resistor does not appear anywhere yet. For the 4th equation, define a loop which includes the 4 Ohm resistor and does not include the current source.

One option is to use the loop in blue shown below:



$$4) 10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$

A second option is to use a slightly different path:



$$4) 10(I_3 - I_2) + 4(I_1) + 8(I_1 - I_4) + 12(I_3 - I_4) = 0$$

Both equations are valid and can be used as the 4th current loop equation.

Solving in MATLAB: First, write 4 equations for 4 unknowns. Repeating these here for convenience:

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

$$I_1 - I_3 = 6$$

$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$

Group like terms gives 4 equations to solve for 4 unknowns:

$$(1) \quad 12I_2 - 10I_3 = 12$$

$$(2) \quad -8I_1 - 12I_3 + 34I_4 = 0$$

$$(3) \quad I_1 - I_3 = 6$$

$$(4) \quad 4I_1 - 10I_2 + 10I_3 + 14I_4 = 0$$

Put in matrix form:

$$\begin{bmatrix} 0 & 12 & -10 & 0 \\ -8 & 0 & -12 & 34 \\ 1 & 0 & -1 & 0 \\ 4 & -10 & 10 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 6 \\ 0 \end{bmatrix}$$

Throw into MATLAB and solve

```
-->A = [0,12,-10,0; -8,0,-12,34; 1,0,-1,0; 4,-10,10,14]
```

```
    0.    12.   -10.    0.
  - 8.     0.   -12.   34.
    1.     0.    -1.    0.
    4.   -10.    10.   14.
```

```
-->B = [12;0;6;0]
```

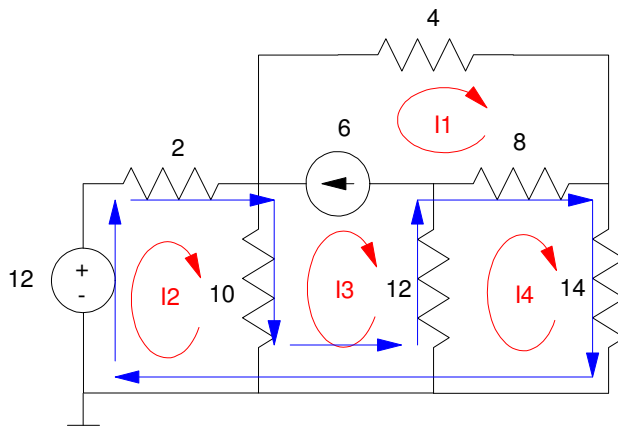
```
    12.
     0.
     6.
     0.
```

```
-->inv(A)*B
```

```
I1    3.5712271
I2   -1.0239774
I3   -2.4287729
I4   -0.0169252
```

Current can be negative. It just means the direction we assumed for I1..I4 was wrong.

Note that some paths for the 4th equation are not valid. For example, suppose you use this loop:



$$-12 + 2(I_2) + 10(I_2 - I_3) + 12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

This makes the 4 equations to solve for 4 unknowns:

- (1) $12I_2 - 10I_3 = 12$
- (2) $-8I_1 - 12I_3 + 34I_4 = 0$
- (3) $I_1 - I_3 = 6$
- (4) $-8I_1 + 12I_2 - 22I_3 + 34I_4 = 12$

Putting these in matrix form and solving in MATLAB gives the following results:

```
-->A = [0,12,-10,0; -8,0,-12,34; 1,0,-1,0; -8,12,-22,34]
```

```
    0.    12.   -10.    0.
   - 8.     0.   -12.   34.
    1.     0.    -1.    0.
   - 8.    12.   -22.   34.
```

```
-->B = [12;0;6;12]
```

```
    12.
     0.
     6.
    12.
```

```
-->inv(A)*B
      !--error 19
Problem is singular.
```

A is not invertible. MATLAB is trying to tell you that you don't have 4 independent equations - meaning there is insufficient information to determine $I_1..I_4$.

The problem is the 4 Ohm resistor never appears in any of the 4 equations. This means information is missing. This show up in MATLAB as a singular matrix.

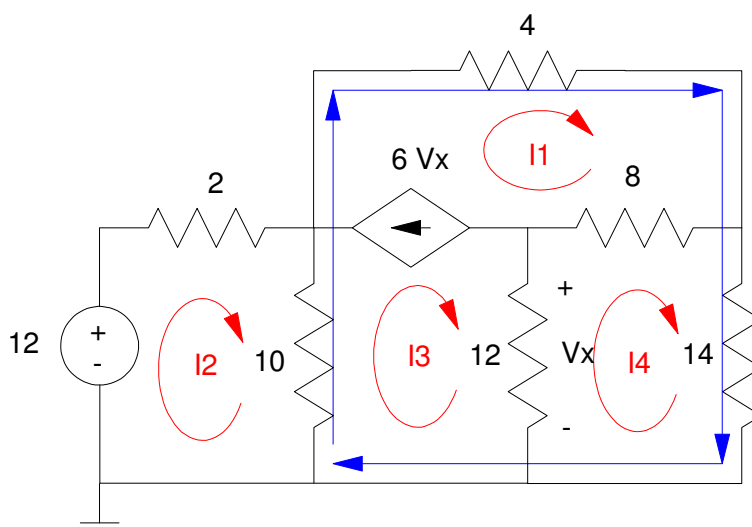
You need to use a different (valid) path for the 4th equation.

Super Loops with Dependent Sources

If you have dependent sources, it just means you need more equations:

- N equations for each of the N current loops,
- Plus one equation for each dependent source

For example, find $I_1..I_4$



Step 1: Define the currents (shown in red)

Step 2: Determine how many equations you need. There are five unknowns (I_1, I_2, I_3, I_4, V_x). Write 5 equations for 5 unknowns.

Step 3: Start writing equations. Start with the easy ones:

$$V_x = 12(I_3 - I_4)$$

$$6V_x = I_1 - I_3$$

Write the loop equations around I_2 and I_4

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$

For the last equation, use a super-loop (shown in blue)

$$10(I_3 - I_2) + 4(I_1) + 14(I_4) = 0$$

This gives 5 equations to solve for 5 unknowns. Grouping terms:

$$12I_3 - 12I_4 - V_x = 0$$

$$I_1 - I_3 - 6V_x = 0$$

$$12I_2 - 10I_3 = 12$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

$$4I_1 - 10I_2 + 10I_3 + 14I_4 = 0$$

Write these in matrix form

$$\begin{bmatrix} 0 & 0 & 12 & -12 & -1 \\ 1 & 0 & -1 & 0 & -6 \\ 0 & 12 & -10 & 0 & 0 \\ -8 & 0 & -12 & 34 & 0 \\ 4 & -10 & 10 & 14 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Throwing these equations into MATLAB and solve:

```
A = [0,0,12,-12,-1 ; 1,0,-1,0,-6 ; 0,12,-10,0,0];
A = [A ; -8,0,-12,34,0 ; 4,-10,10,14,0]
```

```
0.    0.    12.   -12.   -1.
1.    0.    -1.    0.    -6.
0.    12.   -10.    0.    0.
-8.    0.   -12.   34.    0.
4.   -10.   10.   14.    0.
```

```
B = [0;0;12;0;0]
```

```
0.
0.
12.
0.
0.
```

```
I = inv(A)*B
```

```
I1    1.0219378
I2    1.3210662
I3    0.3852794
I4    0.3764369
Vx    0.1061097
```