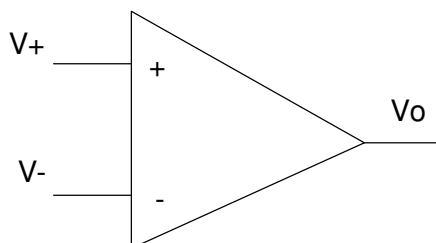


Operational Amplifiers

An operational amplifier is a 2-input device with

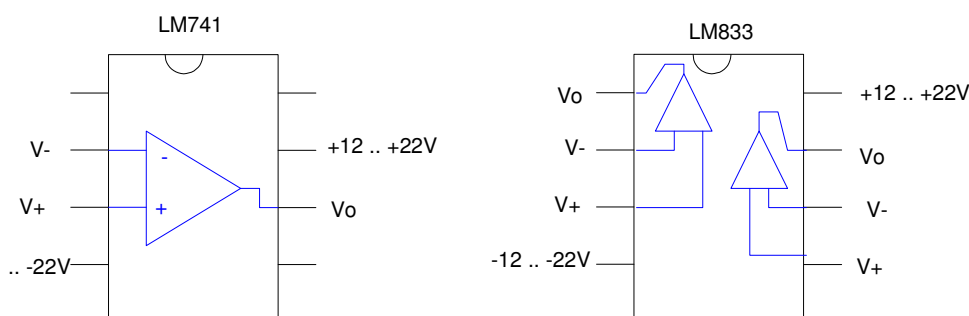
$$V_o \approx k(V^+ - V^-)$$

where k is a large number. For short, the following symbol is used for an operational amplifier:



Symbol for an operational amplifier (op-amp)

Operational Amplifier Characteristics



Pin Layout for two common op-amps: LM741 and LM833

| | LM741 | LM833 | Ideal |
|-------------------------------|--------------------|---------------------|----------------------|
| Input Resistance | 2M Ohms | - | infinite |
| Input Offset Current | 20nA | 25nA | 0 |
| Output Resistance | 75 Ohms | - | |
| Output Short Circuit Current: | 25mA | - | 0 |
| Input Offset Voltage | 1.0mV | 0.3mV | 0 |
| Operating Voltage | +/- 12V .. +/- 22V | +/- 2.5V .. +/- 15V | any |
| Differential Mode Gain | 200,000 | 100dB | infinite |
| Common Mode Rejection Ratio | 90dB | 100dB | common mode gain = 0 |
| Slew Rate | 0.5V/us | 7V/us | infinite |
| Gain Bandwidth Product | 1.5MHz | 15MHz | infinite |
| Price (qty 100) | \$0.35 | \$0.52 | - |

Input resistance / Input Offset Current: The input of the op-amp does draw some current. If you keep the currents involved much larger (meaning at 1V, resistors are less than 50M Ohm), you can ignore the current into V+ and V-.

Input Offset Voltage: If you have a lot of gain, there may be a slight DC offset in the output. You can model this as a 1mV (or 0.3mV) offset at the input, V+ or V-.

Operating Voltage: A LM741 needs at least +/-12V to power it.

Differential Mode Gain: The gain from (V+ - V-) to the output

Common Mode Rejection Ratio: The gain from (V+ + V-) is this much less than the differential mode gain. Note that

$$dB = 20 \cdot \log_{10}(gain)$$

Slew Rate: The output can't change from -10V to +10V in zero time. It can only ramp up this fast.

Gain Bandwidth Product = 1.5MHz:

- If you want a gain of one, the bandwidth is 1.5MHz
- If you want a gain of 10, the bandwidth is 150kHz.
- etc.

For a 741, for example, the output of an op-amp is

$$V_o = k_1(V^+ - V^-) + k_2(V^+ + V^-)$$

where $k_1 = 200,000$ (the differential gain) and $k_2 = 6.325$ (90dB smaller than the differential gain)

Operational Amplifier Circuit Analysis

Problem: Write the voltage node equations for the following circuit. Assume (a) a LM741 op amp. (b) an ideal op-amp.

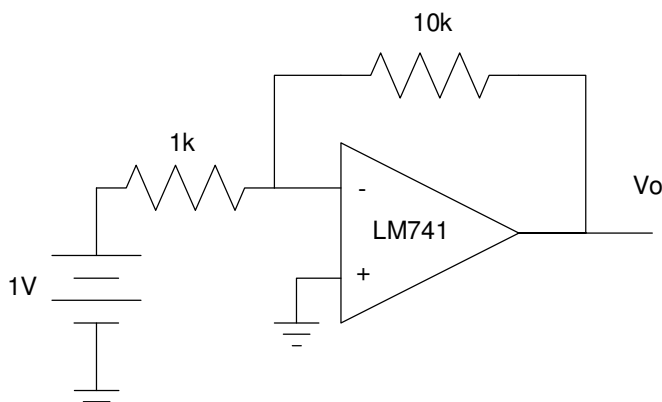
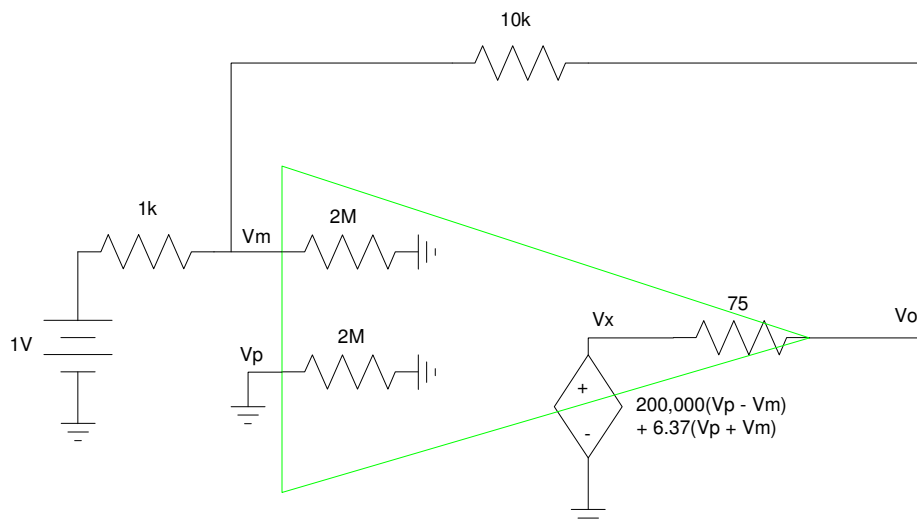


Figure 2: Find Vo for this op-amp circuit

(a) 741 Op Amp Analysis:

First, replace the op-amp with a model taking into account the input, output resistance and gains:



Solution 1: Replace the op-amp with its circuit model (LM741 used here)

Since $V_p = V_+ = 0V$, this simplifies a little: the gain of the op-amp works out to $-199,994 V_m$.

Now, write the voltage node equations:

$$@V_m: \left(\frac{V_m - 1V}{1k} \right) + \left(\frac{V_m}{2M} \right) + \left(\frac{V_m - V_x}{10k + 75} \right) = 0$$

$$@V_x: V_x = -199,994 V_m$$

Solving

$$\left(\frac{1}{1k} + \frac{1}{2M} + \frac{1 + 199,994}{10,075} \right) V_m = \left(\frac{1V}{1k} \right)$$

$$V_m = 50.4 \mu V$$

$$V_x = -199,994 V_m = -10.07V$$

$$V_o = \left(\frac{75}{10,000 + 75} \right) V_m + \left(\frac{10,000}{10,000 + 75} \right) V_x$$

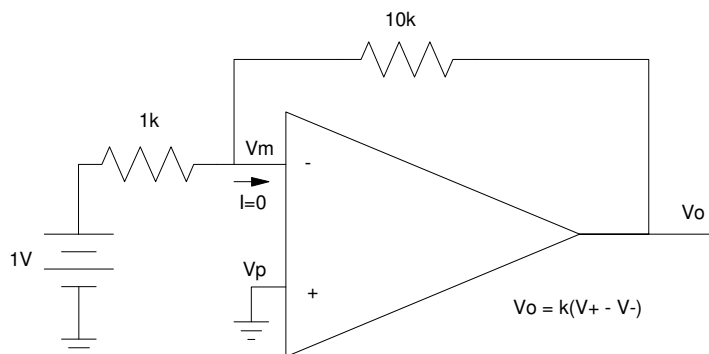
$$V_o = -9.9994V$$

(b) Ideal Op Amp:

Note that many of the terms don't affect the output all that much:

- 2M Ohms in parallel with 1k is about 1k
- $1 + 199,994$ is about 199,994.
- 50.4uV is about zero.

If you approximate these terms, you're essentially using an ideal-op amp. The circuit simplifies to:



Solution 2: Replace the op-amp with an ideal op-amp

Now the voltage node equations are:

$$@V_m: \left(\frac{V_m - 1V}{1k} \right) + \left(\frac{V_m - V_o}{10k} \right) = 0$$

$$@V_o: V_o = k(V_p - V_m)$$

Note that:

- You can't write a voltage node equation at V_o : the op-amp supplies whatever current it takes to hold the output voltage. Since you don't know what that current is, you can't sum the currents to zero.
- For an ideal op-amp, if the gain, k , is infinity and the output is finite, then $V_p = V_m$.

Solving then results in

$$V_o = -10.00V$$

which is very close to what you get for a 741 op-amp.

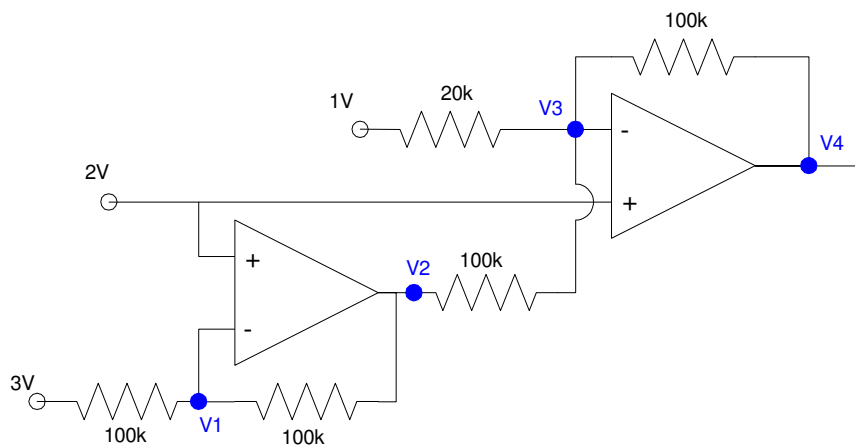
Notes:

- When analyzing an op-amp circuit, you almost have to use voltage nodes.
- If assuming an ideal op-amp, the voltage node equation at V_o is

$$V_p = V_m$$

Example 2: Assume ideal op-amps

- Write the voltage node equations for the following op-amp circuit
- Find the voltages



Example 2: Find the voltages

There are four unknown voltage nodes. We need to write 4 equations to solve for 4 unknowns. Start with the easy ones. For ideal op-amps with negative feedback

$$V_p = V_m$$

meaning

$$V_1 = 2V \quad (1)$$

$$V_3 = 2V \quad (2)$$

Now write two more equations. It's tempting, but you can't write the node equations at V2 or V4

- Equation (1) and (2) *are* the node equations at the outputs - you've already done that.
- You don't know the current from the op-amp - meaning you can't sum the currents to zero.

Instead, find two node nodes where you *can* sum the currents to zero: nodes V1 and V3.

$$\left(\frac{V_1 - 3}{100k} \right) + \left(\frac{V_1 - V_2}{100k} \right) = 0 \quad (3) \quad * 100k \text{ to clear the denominator}$$

$$\left(\frac{V_3 - V_2}{100k} \right) + \left(\frac{V_3 - 1}{20k} \right) + \left(\frac{V_3 - V_4}{100k} \right) = 0 \quad (4) \quad * 100k \text{ to clear the denominator}$$

Solving

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & -1 & 7 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

In Matlab:

```
>> A = [1,0,0,0 ; 0,0,1,0 ; 2,-1,0,0 ; 0,-1,7,-1]
```

```

1      0      0      0
0      0      1      0
2     -1      0      0
0     -1      7     -1

```

```
>> B = [2;2;3;5]
```

```

2
2
3
5

```

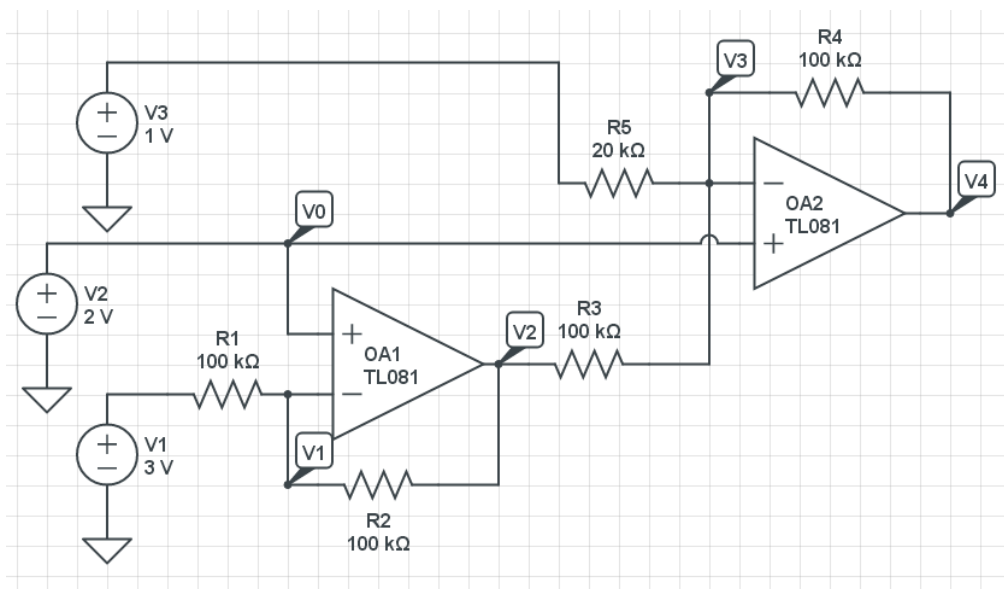
```
>> V = inv(A)*B
```

```

V1      2
V2      1
V3      2
V4      8

```

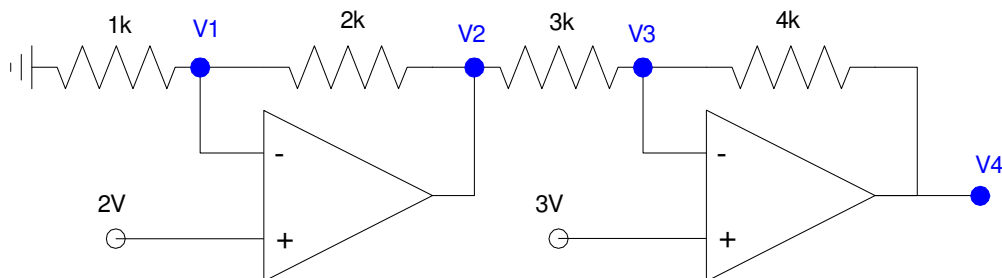
This checks with the Circuitlab solution



| | | | |
|-------|---------|--|--|
| V(V1) | 2.000 V | | |
| V(V2) | 1.000 V | | |
| V(V3) | 2.000 V | | |
| V(V4) | 8.000 V | | |

Circuitlab results for example 2: The voltages match our computations.

Example 3: Assume ideal op-amps. Find the node voltages.



There are four unknown voltages, so we need to write 4 equations to solve for 4 unknowns.

Start with the easy ones: at the output of each op-amp, $V_+ = V_-$

$$V_1 = 2 \quad (1)$$

$$V_3 = 3 \quad (2)$$

Sum the currents to zero at nodes 1 and 3 for the remaining two equations

$$\left(\frac{V_1}{1k}\right) + \left(\frac{V_1 - V_2}{2k}\right) = 0 \quad (3)$$

$$\left(\frac{V_3 - V_2}{3k}\right) + \left(\frac{V_3 - V_4}{4k}\right) = 0 \quad (4)$$

In matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \left(\frac{1}{1k} + \frac{1}{2k}\right) & \left(\frac{-1}{2k}\right) & 0 & 0 \\ 0 & \left(\frac{-1}{3k}\right) & \left(\frac{1}{3k} + \frac{1}{4k}\right) & \left(\frac{-1}{4k}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Solving:

```
>> A = [1,0,0,0 ; 0,0,1,0 ; 1/1000+1/2000, -1/2000, 0, 0 ; 0,-1/3000, 1/3000+1/4000,-1/4000]
```

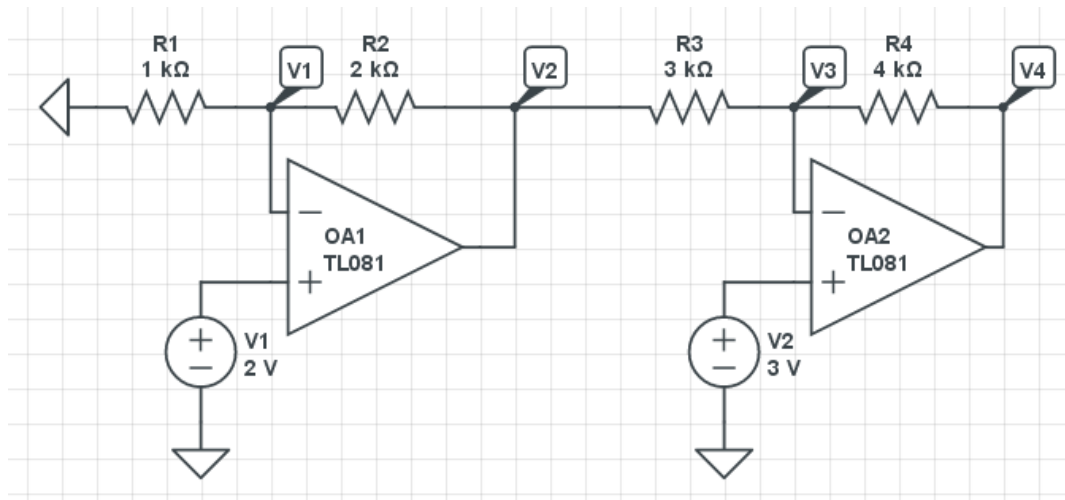
```
1.0000      0      0      0
      0      0      1.0000      0
0.0015 -0.0005      0      0
      0 -0.0003      0.0006 -0.0003
```

```
>> B = [2;3;0;0];
```

```
>> V = inv(A)*B
```

```
V1      2.0000
V2      6.0000
V3      3.0000
V4     -1.0000
```

Again, this matches with what Circuitlab gives



| | | | |
|-------|-----------|--|--|
| V(V1) | 2.000 V | | |
| V(V2) | 6.000 V | | |
| V(V3) | 3.000 V | | |
| V(V4) | -999.9 mV | | |

Circuitlab Solution: The node voltages match our calculations.