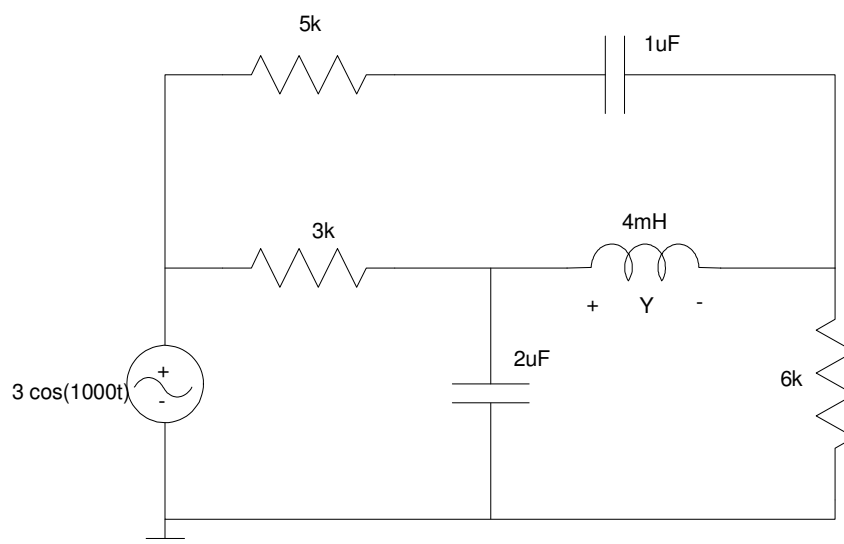


KCL with Phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a \cos(\omega t) + b \sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L \frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C \frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Current loops also works with phasors

Example 1:

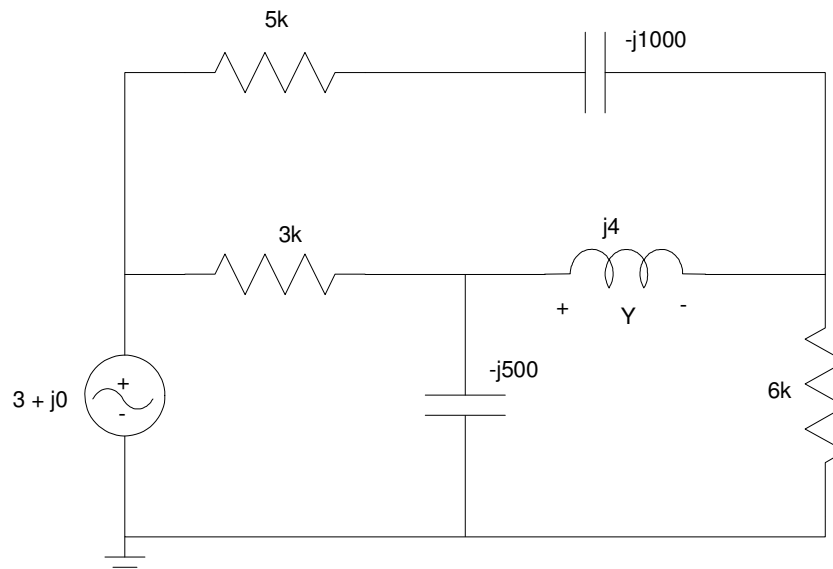


Example 1: Solve the following RLC circuits using current loops and voltage nodes.

First, convert to phaser notation. In this case,

$$\omega = 1000$$

due to the input's frequency.

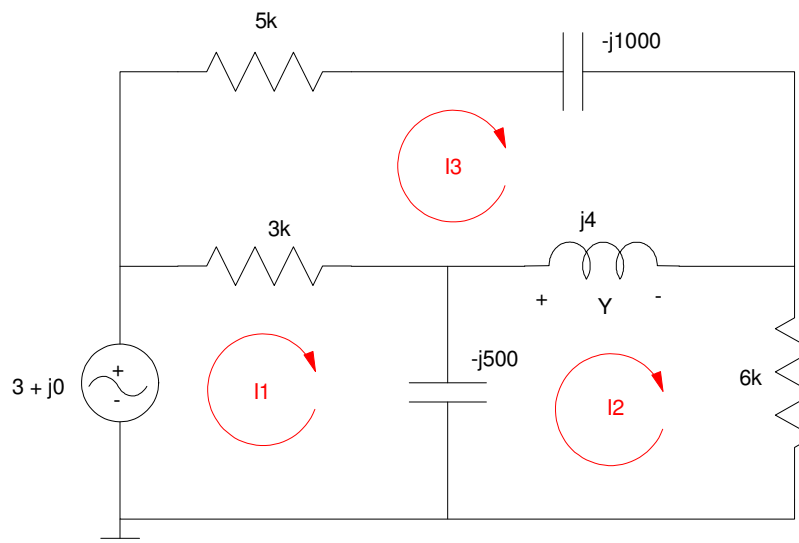


Step 1: Convert to phasor notation.

At this point, you could use current loops or voltage nodes.

Current Loops:

Using current loops, the first step is to define the loop currents (shown in blue):



Step 2 for Current Loops: Define the current loops (shown in blue)

Summing the voltage around each loop to zero gives three equations for three unknowns:

$$I1: \quad -3 + 3000(I_1 - I_3) - j500(I_1 - I_2) = 0$$

$$I2: \quad -j500(I_2 - I_1) + j4(I_2 - I_3) + 6000I_2 = 0$$

$$I3: \quad -j1000(I_3) + 5000(I_3) + j4(I_3 - I_2) + 3000(I_3 - I_1) = 0$$

Grouping terms:

$$(3000 - j500)I_1 + (j500)I_2 + (-3000)I_3 = 3$$

$$(j500)I_1 + (6000 - j496)I_2 + (-j4)I_3 = 0$$

$$(-3000)I_1 + (-j4)I_2 + (8000 - j996)I_3 = 0$$

Solving (using SciLab or MATLAB):

```
A = [3000-j*500, j*500, -3000; j*500, 6000-j*496, -j*4; -3000, -j*4, 8000-j*996]
```

```

3000. - 500.i      500.i      - 3000.
500.i      6000. - 496.i      - 4.i
- 3000.      - 4.i      8000. - 996.i

```

```
I = inv(A) * [3;0;0]
```

```
I1: 0.0014008 + 0.0004598i
```

```
I2: 0.0000475 - 0.0001125i
```

```
I3: 0.0004962 + 0.0002342i
```

meaning (converting to mA)

$$i_1(t) = 1.4008 \cos(1000t) - 0.4598 \sin(1000t) \text{ mA}$$

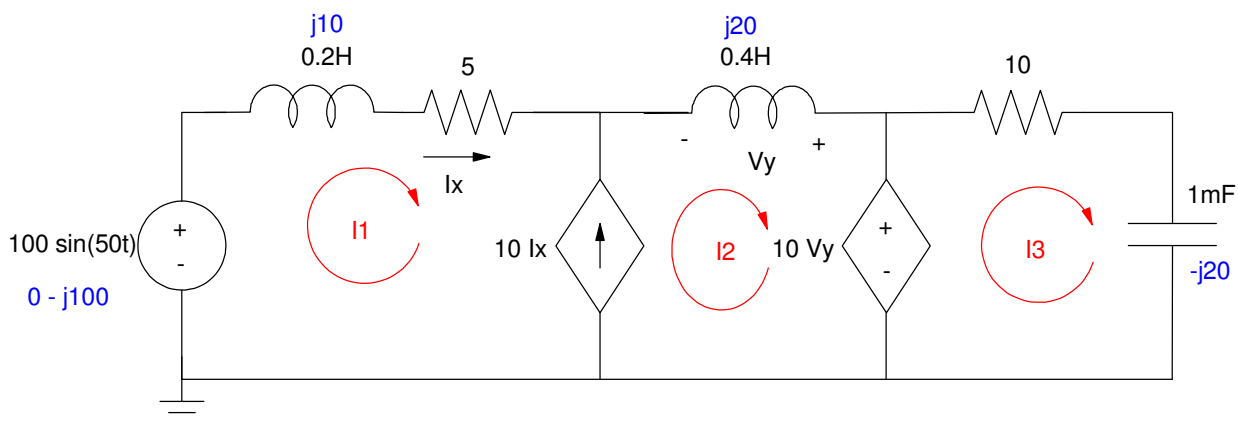
$$i_2(t) = 0.0475 \cos(1000t) + 0.1125 \sin(1000t) \text{ mA}$$

$$i_3(t) = 0.4962 \cos(1000t) - 0.2342 \sin(1000t) \text{ mA}$$

Current Loops with Dependent Sources

Not surprisingly, it also works with dependent sources

Example: Determine the currents, I_1 , I_2 , and I_3



Step 1: Convert to the phasor domain: (shown in blue)

$$\omega = 50$$

$$100 \sin(50t) \rightarrow 0 - j100$$

$$0.2H \rightarrow j\omega L = j10$$

$$0.4H \rightarrow j\omega L = j20$$

$$1mF \rightarrow \frac{1}{j\omega C} = -j20$$

Step 2: Write N equations for N unknowns

$$I_x = I_1$$

$$V_y = j20 \cdot I_2$$

$$10I_x = I_1 - I_2$$

$$-10V_y + 10I_3 - j20I_3 = 0$$

Super Loop

$$-(0 - j100) + (5 + j10)I_1 + j20 \cdot I_2 + (10 - j20)I_3 = 0$$

Group terms

$$I_x - I_1 = 0$$

$$V_y - j20 \cdot I_2 = 0$$

$$10I_x - I_1 + I_2 = 0$$

$$-10V_y + 10I_3 - j20I_3 = 0$$

$$(5 + j10)I_1 + j20 \cdot I_2 + (10 - j20)I_3 = -j100$$

Place in matrix form

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & -j20 & 0 & 0 & 1 \\ -1 & 1 & 0 & 10 & 0 \\ 0 & 0 & 10 - j20 & 0 & -10 \\ (5 + j10) & (j20) & (10 - j20) & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_x \\ V_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -j100 \end{bmatrix}$$

Solve in Matlab

```
A = [-1,0,0,1,0 ; 0,-j*20,0,0,1];
A = [A; -1,1,0,10,0 ; 0,0,10-j*20,0,-10];
A = [A ; 5+j*10,j*20,10-j*20,0,0]

- 1.          0          0          1.          0
  0          - 20.i       0          0          1.
- 1.          1.          0         10.          0
  0           0         10. - 20.i       0        - 10.
  5. + 10.i    20.i      10. - 20.i       0          0

B = [0;0;0;0;-j*100]

  0
  0
  0
  0
- 100.i
```

```
I = inv(A) * B
```

```
I1    0.0507611  -0.0001288i
I2   -0.4568498  +0.0011595i
I3    3.6501607  -1.8366755i
Ix    0.0507611  -0.0001288i
Vy   -0.0231903  -9.136997i
```

meaning

$$i_1(t) = 0.0507 \cos(50t) + 0.0001288 \sin(50t)$$

$$i_2(t) = -0.4568 \cos(50t) - 0.0011 \sin(50t)$$

$$i_3(t) = -0.0232 \cos(50t) + 1.8367 \sin(50t)$$