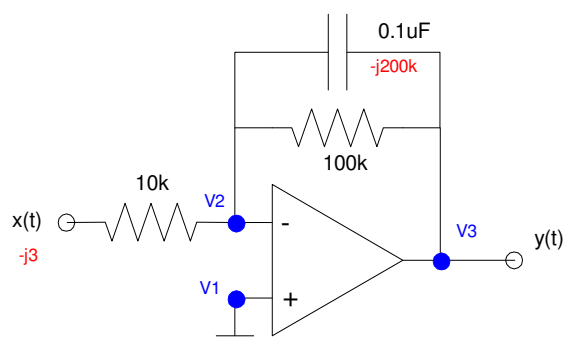


Op-Amp Circuits with Phasors

Single-Pole Low-Pass Filter

Find the voltage, $y(t)$, for

$$x(t) = 3 \sin(50t)$$



Solution: Convert to phasor notation

$$3 \sin(50t) \rightarrow 0 - j3$$

$$0.1 \mu F \rightarrow \frac{1}{j\omega C} = -j200k$$

Write the voltage node equations. At node V3, $V_+ = V_-$

$$V_2 = V_1 \quad (1)$$

$$V_1 = 0 \quad (2)$$

Sum the current to zero at node V2

$$\left(\frac{V_2 - (-j3)}{10k} \right) + \left(\frac{V_2 - V_3}{100k} \right) + \left(\frac{V_2 - V_3}{-j200k} \right) = 0 \quad (3)$$

Solve. Plug in $V_1 = V_2 = 0$ into (3)

$$\left(\frac{0 - (-j3)}{10k} \right) + \left(\frac{0 - V_3}{100k} \right) + \left(\frac{0 - V_3}{-j200k} \right) = 0$$

$$\left(\frac{-1}{100k} + \frac{-1}{-j200k} \right) V_3 = \left(\frac{-j3}{10k} \right)$$

$$V_3 = 12 + j24 = 26.8 \angle 63^\circ$$

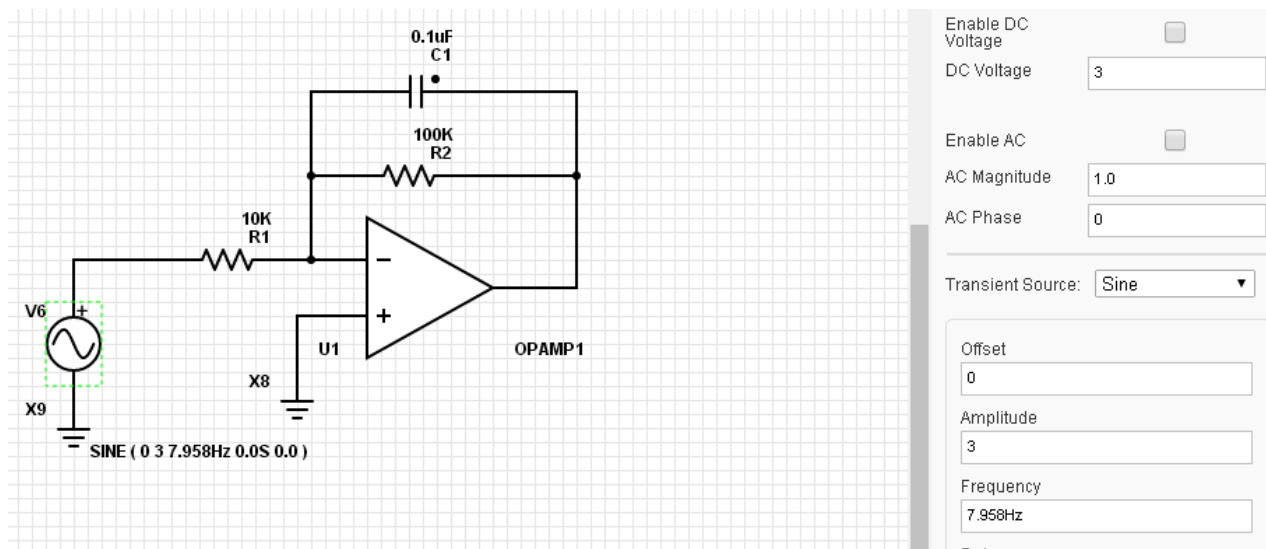
meaning

$$v_3(t) = 12 \cos(50t) - 24 \sin(50t)$$

$$v_3(t) = 26.8 \cos(50t + 63^\circ)$$

Checking in PartSim: The input is

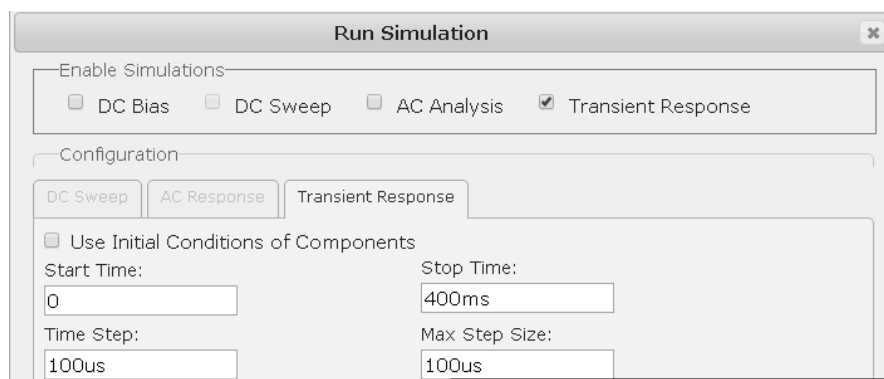
$$Hz = \frac{\omega}{2\pi} = \frac{50 \text{ rad/sec}}{2\pi} = 7.958Hz$$



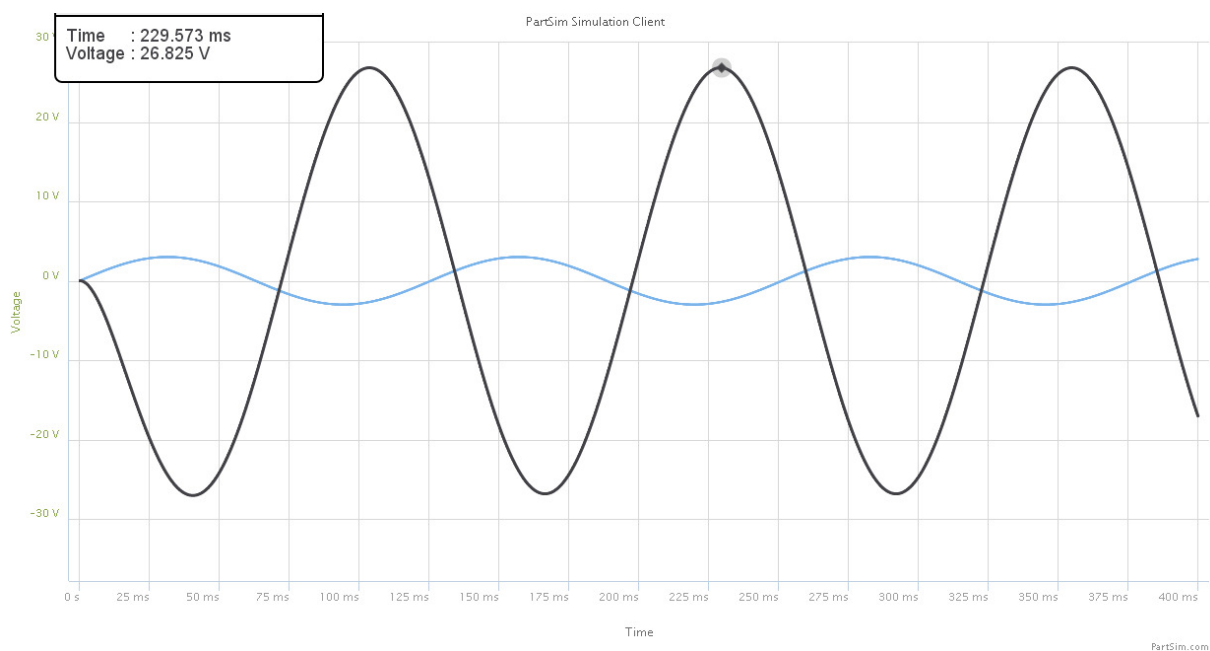
To see the output, select the probes to be the input and the output.

Run a transient simulation for 3 cycles

$$t_{\max} = 3T = \frac{3}{f} = \frac{3}{7.958Hz} = 377ms$$



This results in



Note:

- The period is 126ms ($7.958\text{Hz} = 50\text{ rad/sec}$)
- The peak is 26.825V (vs. 26.8V calculated)
- The peak for the output ($\cos(0)$) is 22ms ahead of the zero crossing for the input ($\sin(0)$). This works out to

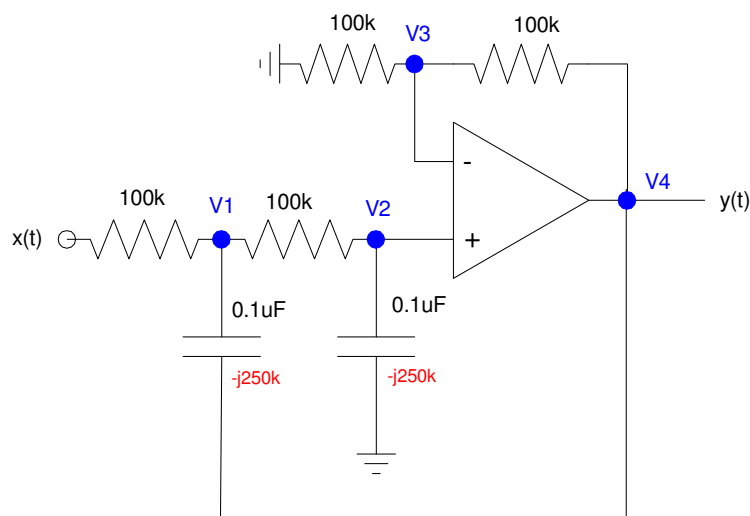
$$\phi = \left(\frac{22\text{ms time lead}}{126\text{ms period}} \right) \cdot 360^\circ = 62.8^\circ \text{ phase shift (vs. 63 degrees computed)}$$

Also note that if you change the frequency of $x(t)$, you have to resolve the entire problem.

Example 2: Two-Pole Op-Amp Circuit

Find $y(t)$ for

$$x(t) = 3 \cos(40t)$$



Step 1: Convert to phasors

$$3 \cos(40t) \rightarrow 3 + j0$$

$$0.1 \mu F \rightarrow \frac{1}{j\omega C} = -j250k$$

Step 2: Write the voltage node equations. With 4 nodes, we need 4 equations. Start with the easy one: for negative feedback, $V_+ = V_-$

$$V_2 = V_3 \quad (1)$$

Now write three more

$$\left(\frac{V_1 - 3}{100k} \right) + \left(\frac{V_1 - V_4}{-j250k} \right) + \left(\frac{V_1 - V_2}{100k} \right) = 0 \quad (2)$$

$$\left(\frac{V_2 - V_1}{100k} \right) + \left(\frac{V_2 - 0}{-j250k} \right) = 0 \quad (3)$$

$$\left(\frac{V_3 - 0}{100k} \right) + \left(\frac{V_3 - V_4}{100k} \right) = 0 \quad (4)$$

Step 3: Solve. Group terms

$$V_2 - V_3 = 0$$

$$\left(\frac{1}{100k} + \frac{1}{-j250k} + \frac{1}{100k}\right)V_1 - \left(\frac{1}{100k}\right)V_2 - \left(\frac{1}{-j250k}\right)V_4 = \left(\frac{3}{100k}\right)$$

$$\left(\frac{-1}{100k}\right)V_1 + \left(\frac{1}{100k} + \frac{1}{-j250k}\right)V_2 = 0$$

$$\left(\frac{1}{100k} + \frac{1}{100k}\right)V_3 + \left(\frac{-1}{100k}\right)V_4 = 0$$

Place in matrix form

$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ \left(\frac{1}{100k} + \frac{1}{-j250k} + \frac{1}{100k}\right) & \left(\frac{-1}{100k}\right) & 0 & \left(\frac{-1}{-j250k}\right) \\ \left(\frac{-1}{100k}\right) & \left(\frac{1}{100k} + \frac{1}{-j250k}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{1}{100k} + \frac{1}{100k}\right) & \left(\frac{-1}{100k}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \left(\frac{3}{100k}\right) \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
a1 = [0,1,-1,0];
a2 = [1/100000 + 1/(-j*250000) + 1/100000, -1/100000, 0, 1/(j*250000)]
a3 = [-1/100000, 1/100000 + 1/(-j*250000), 0, 0]
a4 = [0,0,2/100000, -1/100000]
```

```
A = [a1;a2;a3;a4]
```

```
0 1. -1. 0
0.00002 + 0.000004i - 0.00001 0 - 0.000004i
- 0.00001 0.00001 + 0.000004i 0 0
0 0 0.00002 - 0.00001
```

```
B = [0;3/100000;0;0]
```

```
0.
0.00003
0.
0.
```

```
V = inv(A)*B
```

```
V1 3.4658041 - 0.2218115i
V2 2.9112754 - 1.3863216i
V3 2.9112754 - 1.3863216i
V4 5.8225508 - 2.7726433i
```

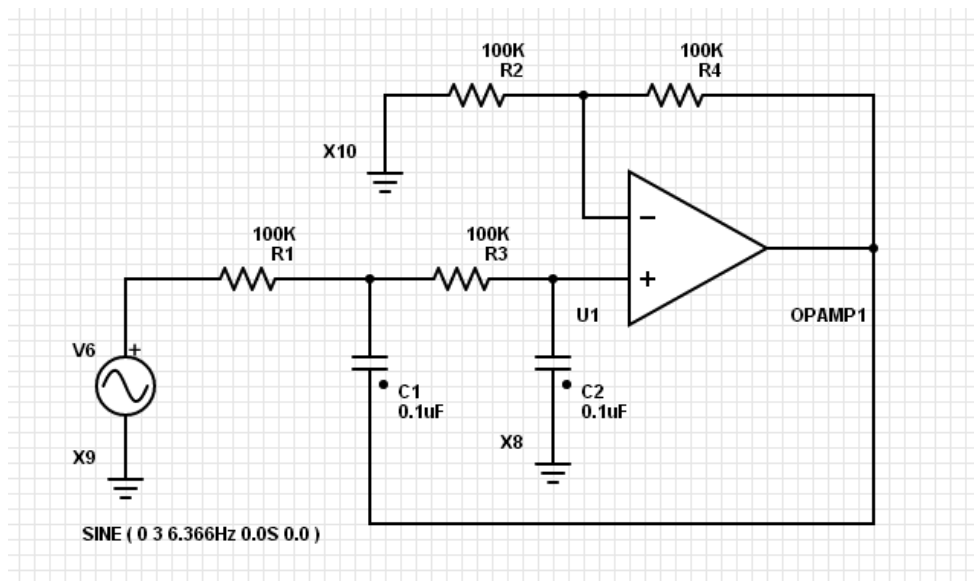
So

$$v_4(t) = 5.822 \cos(40t) + 2.77 \sin(40t)$$

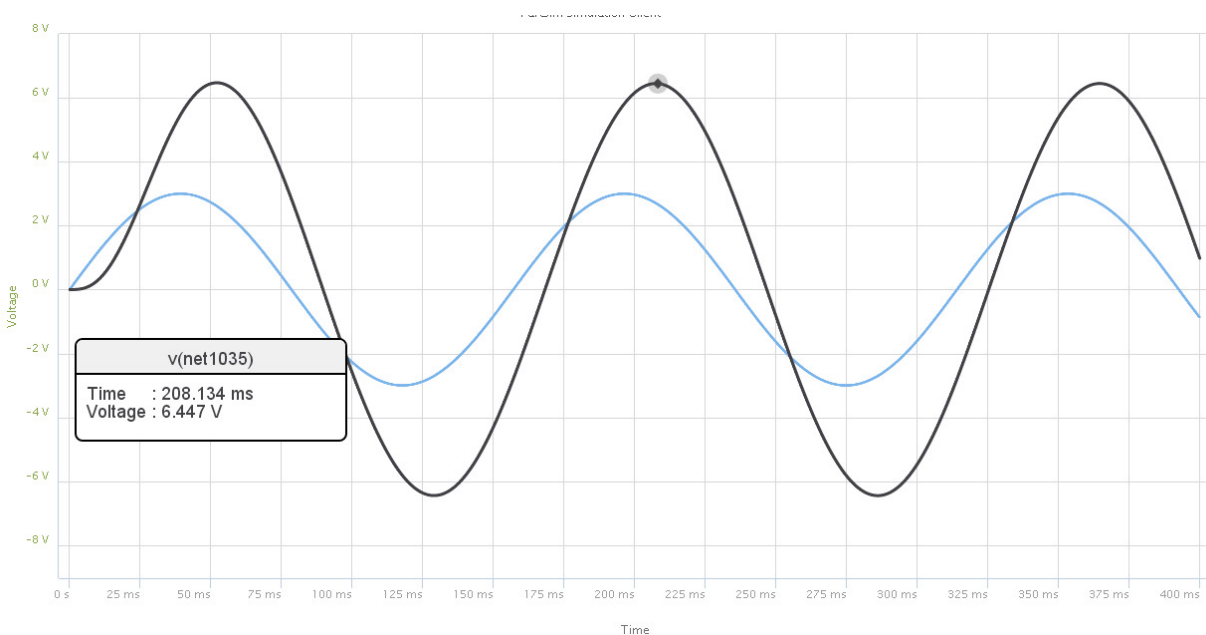
rectangular form

$$v_4(t) = 6.447 \cos(40t - 25^\circ) \quad \text{polar form}$$

Checking in PartSim: Note that 40 rad/sec = 6.366Hz



Running a transient simulation for 400ms (about 3 cycles)



The output peak is 6.447V (vs. 6.447V computed)

The output is delayed by 9.9ms from the input. The phase shift is

$$\phi = \left(\frac{9.9\text{ms delay}}{157\text{ms period}} \right) \cdot 360^\circ = 22.7^\circ \text{ delay} \quad (\text{ vs. } 25^\circ \text{ degrees computed})$$

