Superposition (take 3)

Previously, we looked at how to analyze an AC to DC converter and a Buck converter. To solve these circuits, we changed the problem so that the input contained

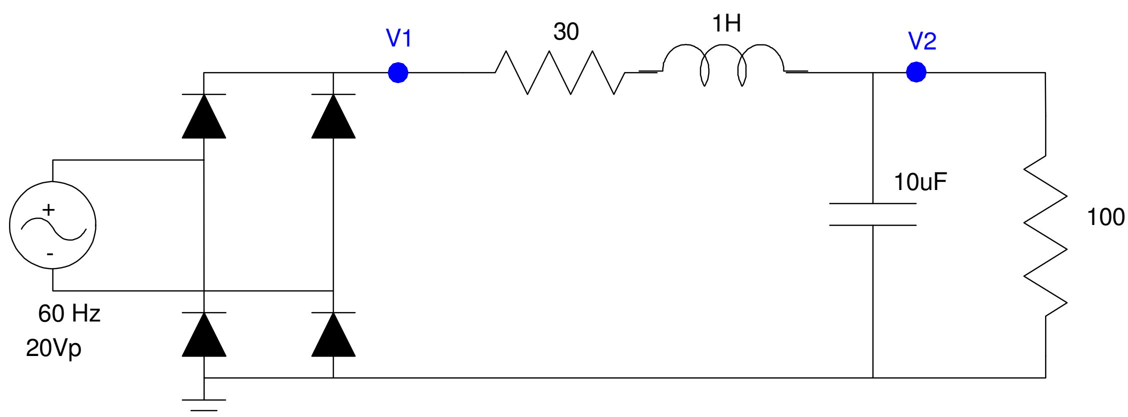
- A DC signal, and
- An AC signal

The resulting output that we calculated was close but slightly off from the actual output voltage.

A more accurate way to analyze these circuits is to express the input in terms of its Fourier Series. Then, superposition can be used to determine the output at each frequency.

Example 1: AC to DC Converter

The following circuit is an AC to DC converter that we'll cover in ECE 320 Electronics I. Determine the voltage at V2:



AC to DC Converter covered in ECE 320 Electronics I

From PartSim, the signal at V1 is

$$v_1(t) = |20\sin(377t)| - 1.4$$

Previously, we approximated this as

- A DC term which matched the DC term of v1, and
- An AC term which was the same frequency as v1(t) (120Hz) and same peak-to-peak voltage

$$v_1(t) \approx 11.33 + 10\cos(754t)$$

A more accurate approximation would be the Fourier Series approximation.

Step 1: Find the Fourier Series Approximation for x(t):

```
T = pi;
t = [0:0.0001:1]' * T;
Wo = 2*pi/T;

x = 20*sin(t) - 1.4;

a0 = mean(x)

a1 = 2*mean(x .* cos(Wo*t))
a2 = 2*mean(x .* cos(2*Wo*t))
a3 = 2*mean(x .* cos(3*Wo*t))
a4 = 2*mean(x .* cos(4*Wo*t))
a5 = 2*mean(x .* cos(5*Wo*t))

b1 = 2*mean(x .* sin(Wo*t))
b2 = 2*mean(x .* sin(2*Wo*t))
b3 = 2*mean(x .* sin(3*Wo*t))
b4 = 2*mean(x .* sin(4*Wo*t))
b5 = 2*mean(x .* sin(5*Wo*t))
```

resulting in

n	0	1	2	3	4	5
w (rad/sec)	0	754	1,508	2,262	3,016	3,770
a_n	11.331	-8.488	-1.698	-0.728	-0.404	-0.257
b_n	0	0	0	0	0	0

meaning

$$v_1(t) = 11.331 - 8.488\cos(754t) - 1.698\cos(1508t) - 0.728\cos(2262t)$$
$$-0.404\cos(3016t) - 0.257\cos(3770t)$$

Not that this is a little different from what we previously approximated the waveform at V1 as:

$$v_1(t) \approx 11.33 + 10\cos(754t)$$

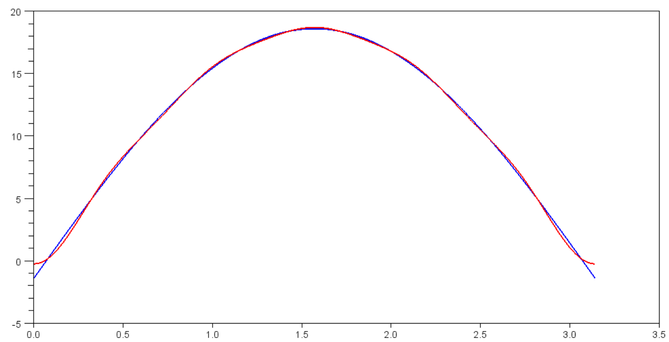
The Fourier series approximation is

- More accurate, but
- Much harder to determine.

Proceeding with the Fourier Series approximation, the actual waveform at V1 and it's approximation taken out to the 5th harmonic are:

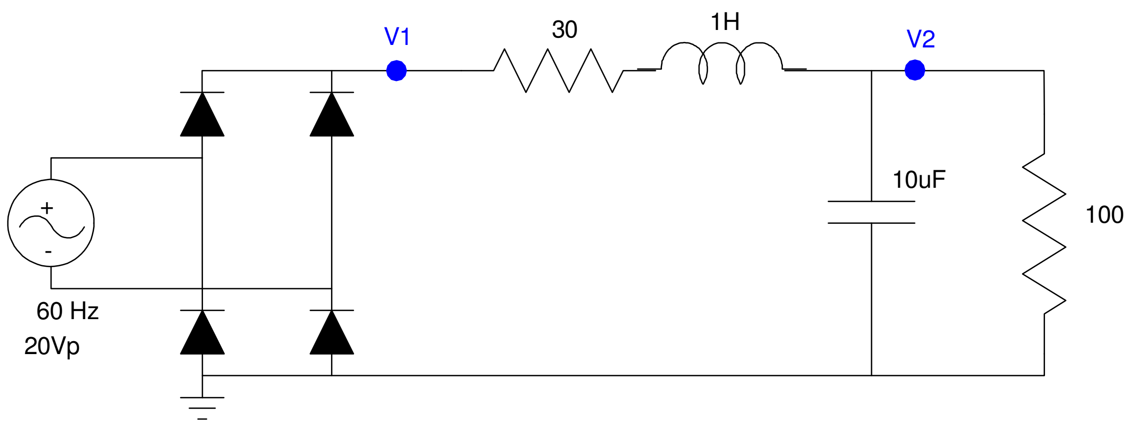
```
y = a0 + a1*cos(Wo*t) + a2*cos(2*Wo*t) + a3*cos(3*Wo*t) + a4*cos(4*Wo*t) + a5*cos(5*Wo*t);

y = y + b1*sin(Wo*t) + b2*sin(2*Wo*t) + b3*sin(3*Wo*t) + b4*sin(4*Wo*t) + b5*sin(5*Wo*t);
```

```
plot(t,x,'b',t,y,'r');
```



Signal at V1 (blue) and Fourier Series Approximation (red)

Step 2: Use Superposition at Each Frequency



$$v_1(t) = 11.331 - 8.488\cos(754t) - 1.698\cos(1508t) - 0.728\cos(2262t)$$
$$-0.404\cos(3016t) - 0.257\cos(3770t)$$

DC:

$$V1 = 11.331$$

$$V_2 = \left(\frac{100}{100+30}\right) \cdot 11.331$$

$$V_2 = 8.716$$

754 rad/sec (120Hz: fundamental)

$$v_1(t) = -8.488 \cos(754t)$$

$$V_1 = -8.488 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j132.6\Omega$$

$$L \rightarrow j\omega L = j754\Omega$$

$$-j132.6 || 100 = 63.76 - j48.07$$

$$V_2 = \left(\frac{(63.76 - j48.07)}{(63.76 - j48.07) + (30 + j754)} \right) \cdot (8.488 + j0)$$

$$V_2 = -0.468 - j0.829$$

$$v_2(t) = -0.468 \cos(754t) + 0.829 \sin(754t)$$

At 1508 rad/sec (240Hz: 2nd harmonic)

$$v_1 = -1.698 \cos(1508t)$$

$$V_1 = -1.698 + j0$$

$$\omega = 1508$$

$$C \rightarrow \frac{1}{j\omega C} = -j66.31\Omega$$

$$L \rightarrow j\omega L = j1508\Omega$$

$$100 \; || \; -j66.31 = 30.54 - j46.05$$

$$V_2 = \left(\frac{30.54 - j46.05}{(30.54 - j46.05) + (30 + j1508)} \right) \cdot (-1.698 + j0)$$

$$V_2 = 0.052 + j0.038$$

$$v_2(t) = 0.052 \cos(1508t) - 0.038 \sin(1508t)$$

At 2262 rad/sec: (360Hz: 3rd harmonic)

$$v_1(t) = -0.728\cos(2262t)$$

$$\omega = 2262$$

$$V_1 = -0.728 + j0$$

$$C \to \frac{1}{j\omega C} = -j44.21\Omega$$

$$L \to j\omega L = j2262\Omega$$

$$100 \,||\, -j44.21 = 16.35 - j36.98$$

$$V_2 = \left(\frac{(16.35-j36.98)}{(16.35-j36.98)+(30+j2262)}\right) \cdot (-0.728 + j0)$$

$$V_2 = 0.012 + j0.006$$

$$v_2(t) = 0.012\cos(2262t) - 0.006\sin(2262t)$$

At 3016 rad/sec (480Hz: 4th harmonic)

$$v_1(t) = -0.404\cos(3016t)$$

$$V_1 = -0.404 + j0$$

$$C \to \frac{1}{j\omega C} = -j33.15\Omega$$

$$L \to j\omega L = j3016\Omega$$

$$100 \,||\, -j33.15\Omega = 9.90 - j29.87$$

$$V_2 = \left(\frac{(9.90-j29.87)}{(9.90-j29.87)+(30+j3016)}\right) \cdot (-0.404 + j0)$$

$$V_2 = 0.004 + j0.001$$

$$v_2(t) = 0.004\cos(3016t) - 0.001\sin(3016t)$$

At 3770 rad/sec (5th harmonic)

$$v_1(t) = -0.257\cos(3770t)$$

$$V_1 = -0.257 + j0$$

$$C \rightarrow \frac{1}{j\omega C} = -j26.53\Omega$$

$$L \rightarrow j\omega L = j3770\Omega$$

$$100 \,||\, -j26.53\Omega = 6.58 - j24.79$$

$$V_2 = \left(\frac{(6.58-j24.79)}{(6.58-j24.79)+(30+j3770)}\right) \cdot (-0.257 + j0)$$

$$V_2 = 0.0017 + j0.0005$$

$$v_2(t) = 0.0017\cos(3370t) - 0.0005\sin(3370t)$$

The total answer will be the sum of all the terms

$$v_2(t) = 8.716$$

$$-0.468\cos(754t) + 0.829\sin(754t)$$

$$+0.052\cos(1508t) - 0.038\sin(1508t)$$

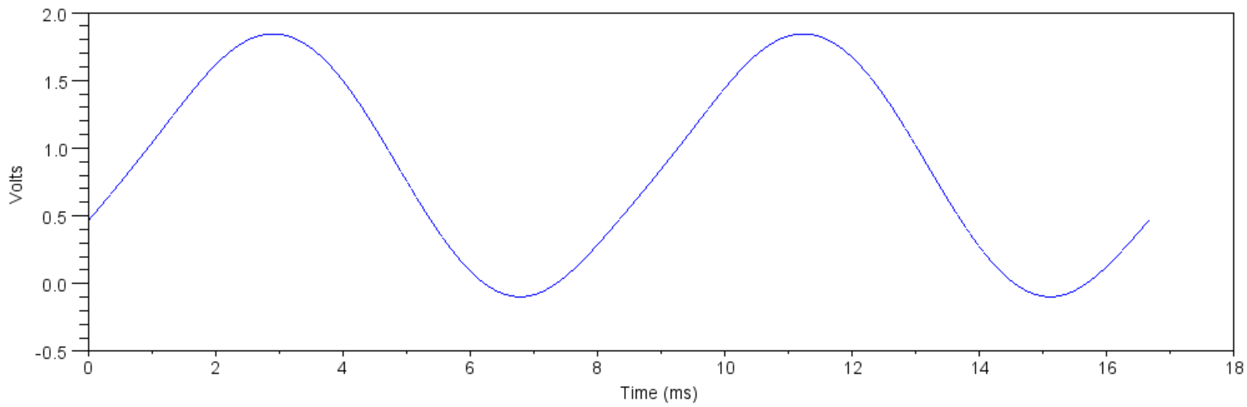
$$+0.012\cos(2262t) - 0.006\sin(2262t)$$

$$+0.004\cos(3016t) - 0.001\sin(3016t)$$

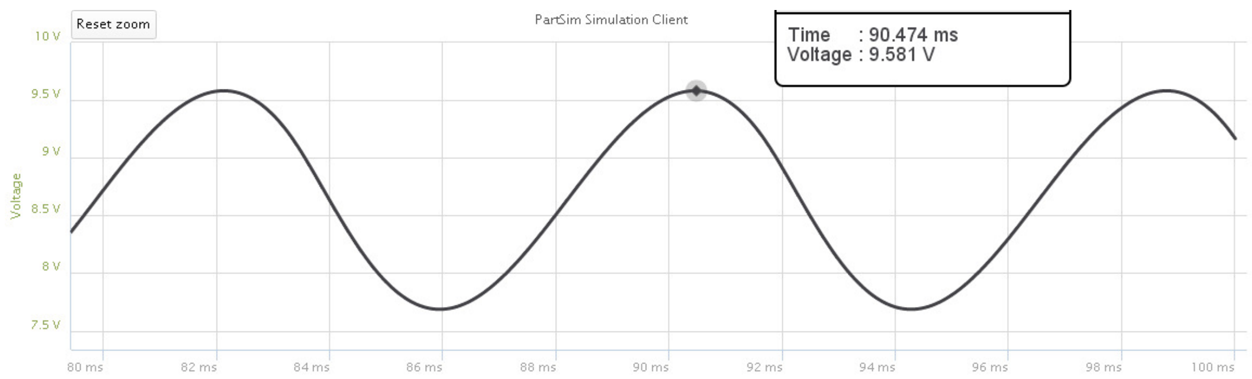
$$+0.0017\cos(3370t) - 0.0005\sin(3370t)$$

$$+\ldots$$

Comparing the calculated and PartSim signal at V2:



Calculate Signal At V2: Fourier Series Taken Out to the 5th Harmonic



Signal at V2 as computed by PartSim

	Calculated V2 (lecture #30)	PartSim V2	Calculated V2 (Fourier Series)
DC Value	8.70 V	8.644 V	8.716 V
AC Value	2.242 Vpp	1.895 Vpp	1.903Vpp

Note the following:

- By taking the Fourier Series approximation for the signal at V1, our computed answer (column #4) matches up with the simulation results very well.

- The reason our previous answer from lecture #30 (column #2) was off is we overestimated the 1st harmonic. Since V1 is 20Vpp, we assumed that the 1st harmonic was also 20Vpp. Actually, it's 16.98Vpp (2 x 8.488V)

- In theory, you have to take the Fourier Series out to infinity. Actually, you can get a pretty good approximation just using the DC term and the 1st harmonic. The signal at V2 was

$$
\begin{aligned}
v_2(t) = 8.716 & \\
&-0.468\cos(754t) + 0.829\sin(754t) \\
&+0.052\cos(1508t) - 0.038\sin(1508t) \\
&+0.012\cos(2262t) - 0.006\sin(2262t) \\
&+0.004\cos(3016t) - 0.001\sin(3016t) \\
&+0.0017\cos(3370t) - 0.0005\sin(3370t) \\
&+\ldots
\end{aligned}
$$

It isn't that bad of an approximation to only include the first two terms of V2:

$$v_2(t) \approx 8.716 - 0.468\cos(754t) + 0.829\sin(754t)$$

meaning it isn't that bad of an approximation to take V1:

$$v_1(t) = 11.331 - 8.488\cos(754t) - 1.698\cos(1508t) - 0.728\cos(2262t)$$
$$-0.404\cos(3016t) - 0.257\cos(3770t)$$

and only consider the first two terms:

$$v_1(t) \approx 11.331 - 8.488\cos(754t)$$

This is what we did back in lecture #30, only with a slightly less accurate way of determining the amplitude of the 1st harmonic. A better (and harder) way to do this is to use a Fourier Series approximation taken out to two terms (DC and AC fundamental).