Circuit Elements and Kirchoff's Laws

ECE 211 Circuits I

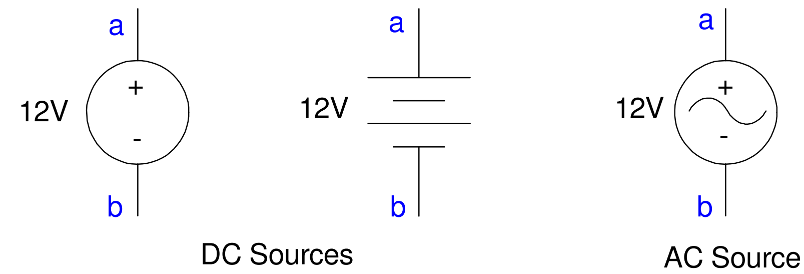
Lecture #2

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Circuit Elements: Voltage Sources

Voltage sources provide a fixed voltage across two points

- Think of a battery

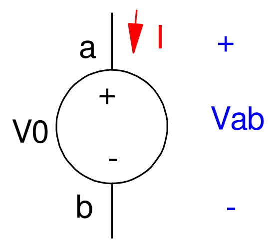
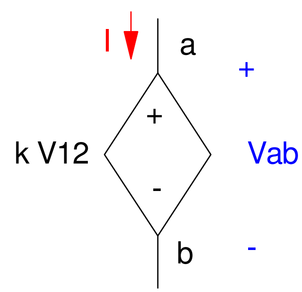


Voltage	Example	Enough voltage to...
1.5V	AA battery	Turn on a single LED
5.0V	USB Battery	Power a computer
13.2V	Car battery	Turn a motor (starter)
120V	Wall Outlet	Power a computer, hair dryer
13,200V	Residential transmission lines	Power a residential subdivision Kill a person
345,000V	Long distance transmission lines	Power a small city Create an arc 11cm long

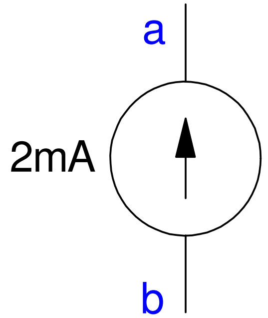
Voltage Sources

This class uses

- Independent sources (voltage is fixed)
- Dependent sources (voltage depends upon something else)

Element	Symbol	VI Relationship
Voltage Source (battery)		$V_{ab} = V_0$ $I = \text{any}$
Voltage Controlled Voltage Source (amplifier)		$V_{ab} = kV_{12}$ $I = any$

Circuit Elements: Current Sources



Current sources provide a fixed current through a wire

- For LEDs, current = brighness
- For DC motors, current = torque

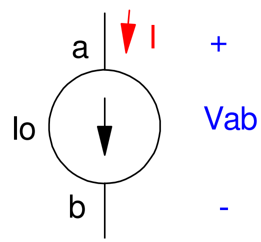
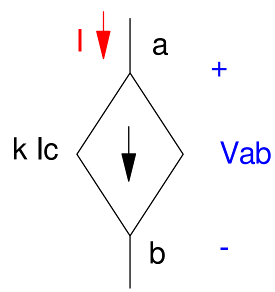
The current determines the thickness of a wire needed

Current	Diameter of wire to carry.	Example
1uA	0.000 7mm	Sleep mode for a TV remote
1mA	0.021 mm	Power a single LED (dim)
1A	0.72mm	Charging a cell phone
10A	2.3mm	Hair dryer
100A	7.3mm	Welding

Current Sources

This class uses

- Independent sources (current is fixed)
- Dependent sources (current is a funciton of something else)

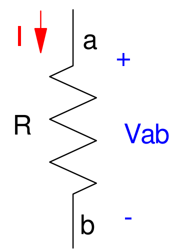
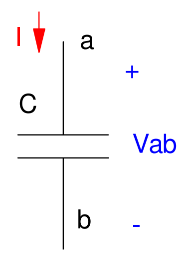
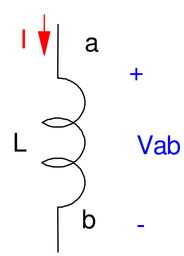
Element	Symbol	VI Relationship
Current Source (LED driver)		$I = I_0$ $V_{ab} = any$
Current Controlled Current Source (transistor)		$I = kI_c$ $V_{ab} = any$

Passive Circuit Elements

Passive elements do not need a battery

- They are set by the properties of the materials they're made from and their construction

This class uses three passive circuit elements

Element	Symbol	VI Relationship
Resistor (basic circuit element)		$V_{ab} = IR$
Capacitor (basis circuit element)		$I = C\,\frac{dV_{ab}}{dt}$
Inductor (basic cicuit element)		$V_{ab} = L\frac{dI}{dt}$

Resistors

Like it or not, all wires have resistance

$$R = \frac{\rho L}{A}$$

Example: Find the resistance of a transmission line

- L = 1000km (Dickenson to Minneapolis)
- A = 1cm2
- $\rho = 1.68 \cdot 10^{-8} \Omega m$ (copper)

Solution

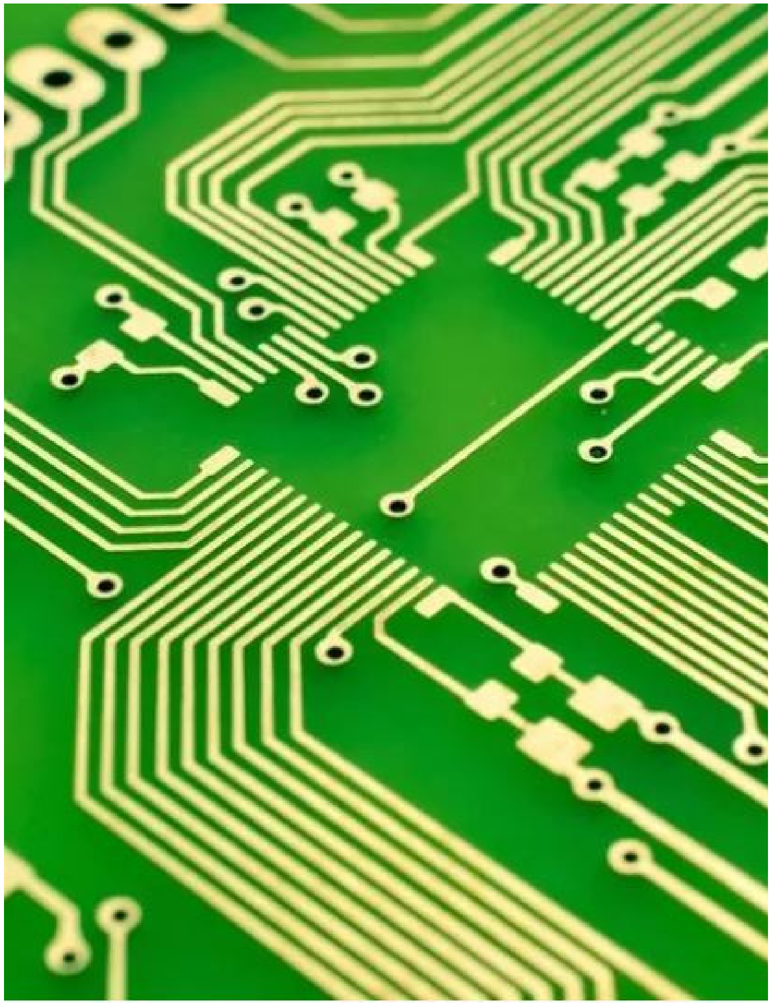
$$R = \frac{(1.68e{-}8\Omega m)(1e6m)}{(0.01m)^2}$$

$$R = 167.8\Omega$$

The transmission line has a resistance of 167.8 Ohms

Find the resistance of a copper trace on a printed circuit board (PCB)

- Length = 4cm
- Width = 1mm
- Thickess = 35um
- Copper (1oz copper)



Solution

$$R = \frac{\rho L}{A}$$

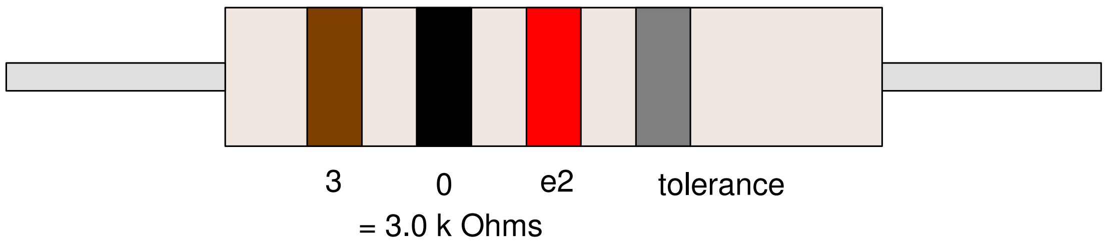
$$R = \frac{(1.68e{-}8\Omega m)(0.04m)}{(0.001m)(0.000035m)}$$

$$R = 0.0192\Omega$$

The resistance of copper traces are *almost* zero, but not quite

Resistor Color Codes

Resistors are labeled with colored bands



Value

0	1	2	3	4	5	6	7	8	9
black	brown	red	orange	yellow	green	blue	violet	grey	white

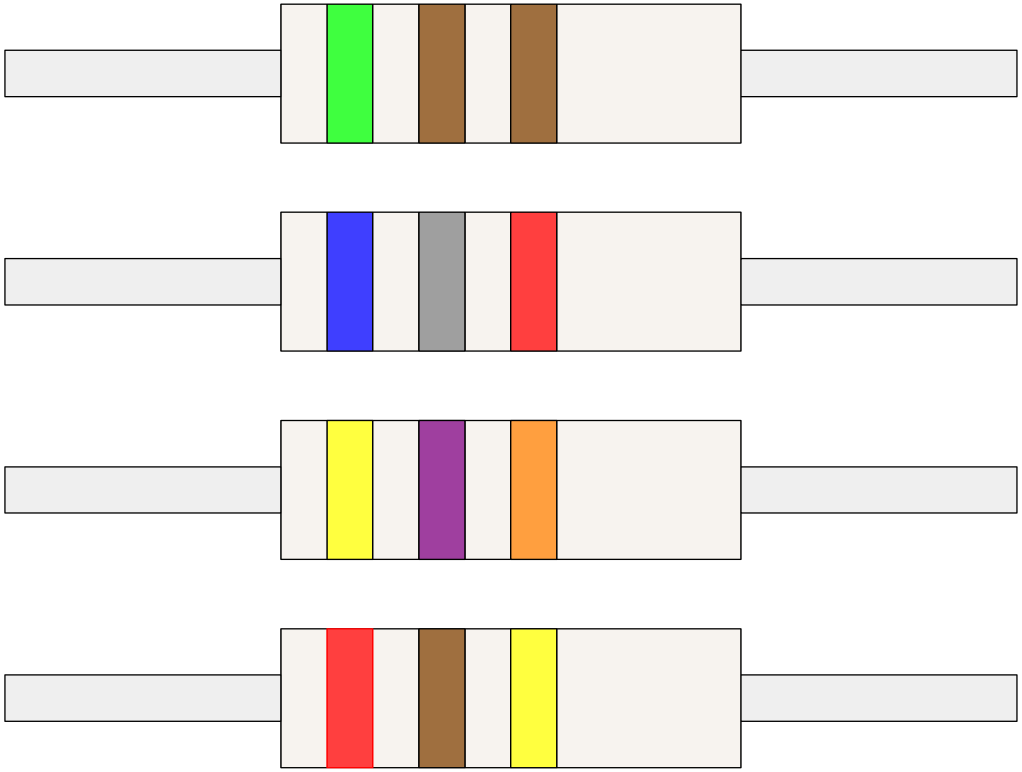
Tolerance

Silver	Gold	Red	Brown	Green
+/- 10%	+/- 5%	+/- 2%	+/- 1%	+/- 0.5%

Handout

Determine the value of each resistor

0	1	2	3	4	5	6	7	8	9
black	brown	red	orange	yellow	green	blue	violet	grey	white



Capacitors

Some capacitors have their value printed on them

Others use a three-digit code

- XXY

This is the value in pF

$$102 \rightarrow 10 \cdot 10^2 pF$$

$$103 \rightarrow 10 \cdot 10^3 pF$$

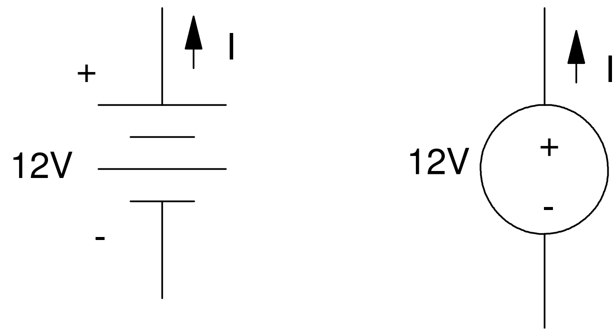
$$104 \rightarrow 10 \cdot 10^4 pF$$

Problem: Determine the value of these capacitors

Independent Sources

Voltage Source: Like a battery: the voltage is fixed

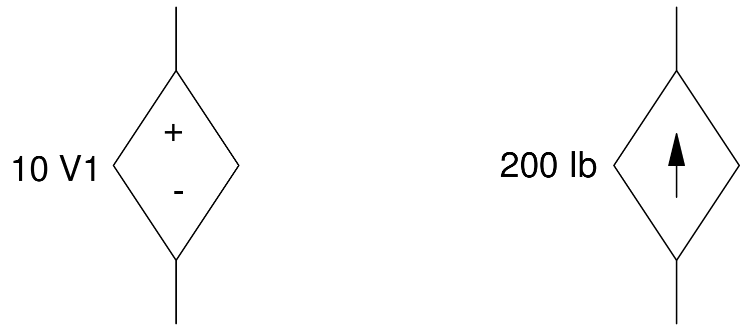
- Current depends upon the load (can be anything, positive or negative)



Current Source: LED driver: the current is fixed

Dependent Sources

Controlled Current and Voltage Sources: A diamond indicates a controlled voltage source or a controlled current source.



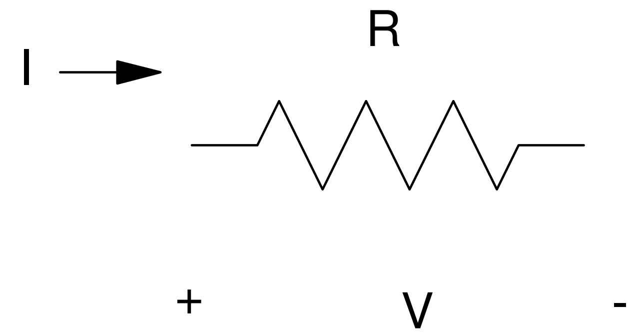
Controlled sources arise from various components covered in ECE 320 Electronics

- Operational Amplifiers (voltage controlled voltage source)
- Transistors (current controlled current source)
- MOSFET (voltage controlled current source)

For this class, just treat them as a device.

Ohm's Law

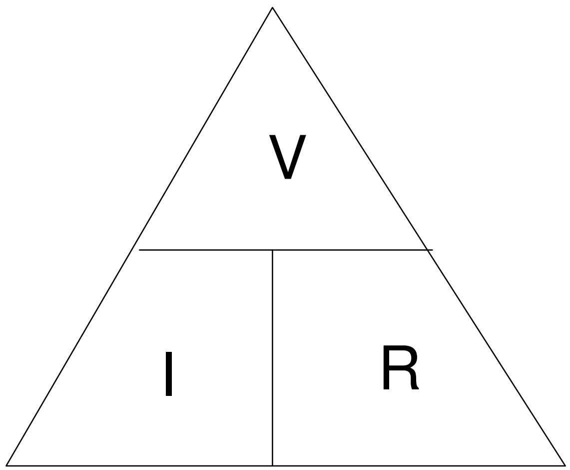
- V = I R
- Current goes into the + terminal



Other Forms:

$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$



Power:

$$P = V\,I$$

$$P = \frac{V^2}{R}$$

$$P = I^2\,R$$

Problems

- A 1000km transmission line has a resistance of 167.8 Ohms
- A PCB trace has a resisance of 0.0192 Ohms

Find the implied voltage drop, current, and power

Equations

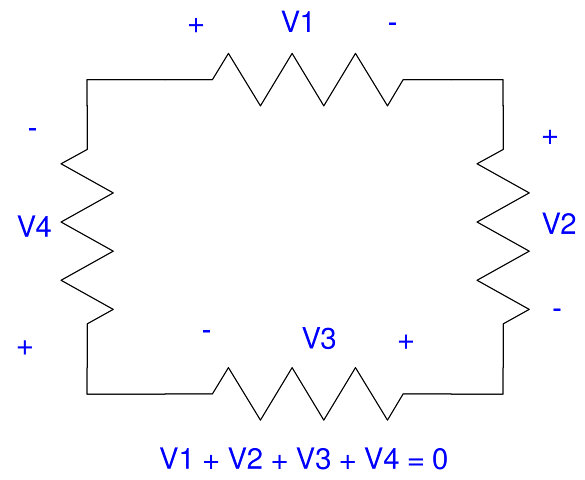
- V = I R
- P = V I

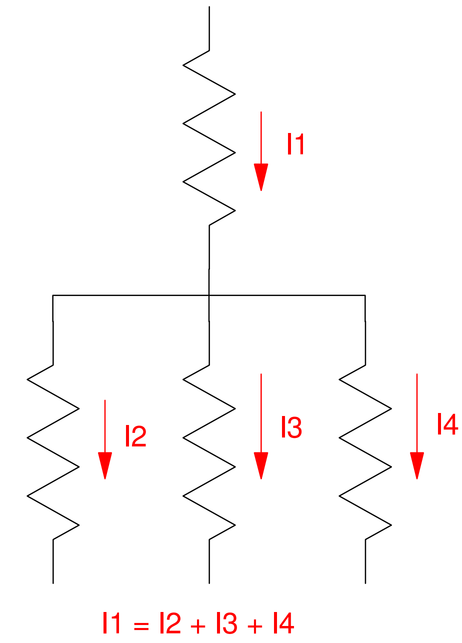
	Volts (V)	Amps (I)	Ohms (R)	Power (W)
a)			167.8 Ohms	10kW
b)		100 A	167.8 Ohms	
c)			0.0192 Ohms	2.0 W
d)		2.0A	0.0192 Ohms	

Kirchoff's Laws

Kirchoff's laws simply restate the conservation of voltage and current:

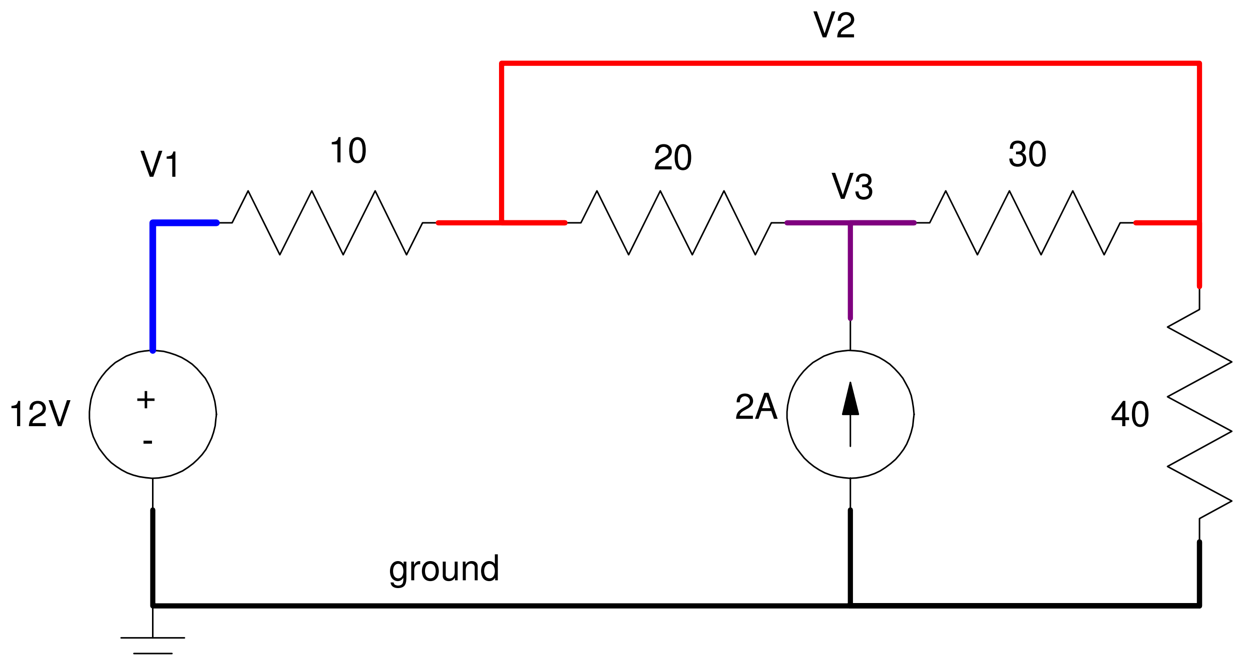
- Conservation of Voltage
  - The sum of the voltages around any closed path must be zero
- Conservation of Current
  - The sum of all currents from a node must be zero
  - Current In = Current Out





Voltage Nodes

- Indicated by a solid line.
- All points are the same voltage
- The following circuit has four voltage nodes including ground.



Ground

- The voltage node which all other voltages are referenced to.
- Usually, ground is the same as earth ground - it doesn't have to be, however.
- Birds sitting on a high-voltage line treat the 13kV line as their ground reference.

Symbols:

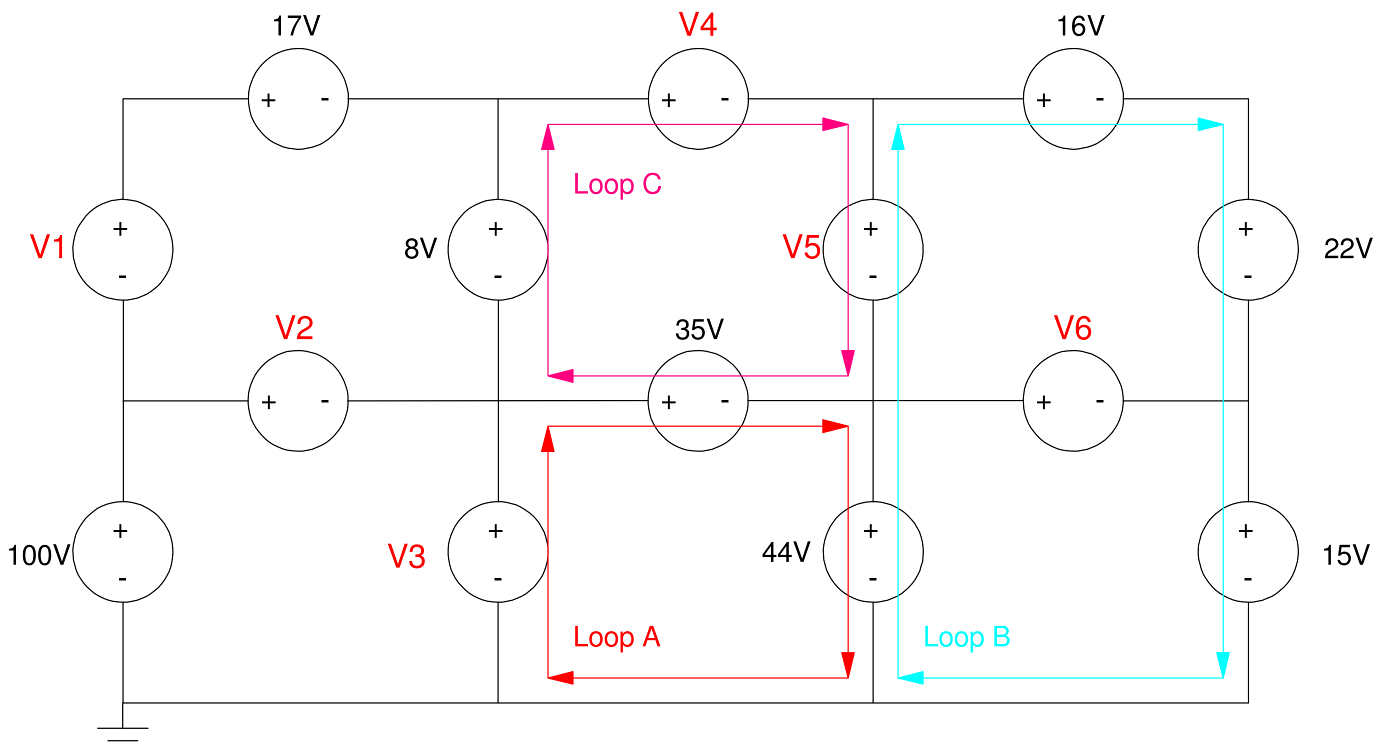
- If both appear in a circuit it means two different ground references
- Don't short the two together

Conservation of Voltage: Kirchoff's Voltage Law

The sum of the voltages around any closed path must be zero

- i.e. If you short the leads of a volt meter together, you'll measure 0V.

Example: Determine V1..V6



Using Kirchoff's Voltage Law,

- Sum the voltages around a closed path
- To be consistent, add if you hit the + sign first, subtract if you hit the - sign first.
- Look for paths with one unknown.

Loop A:

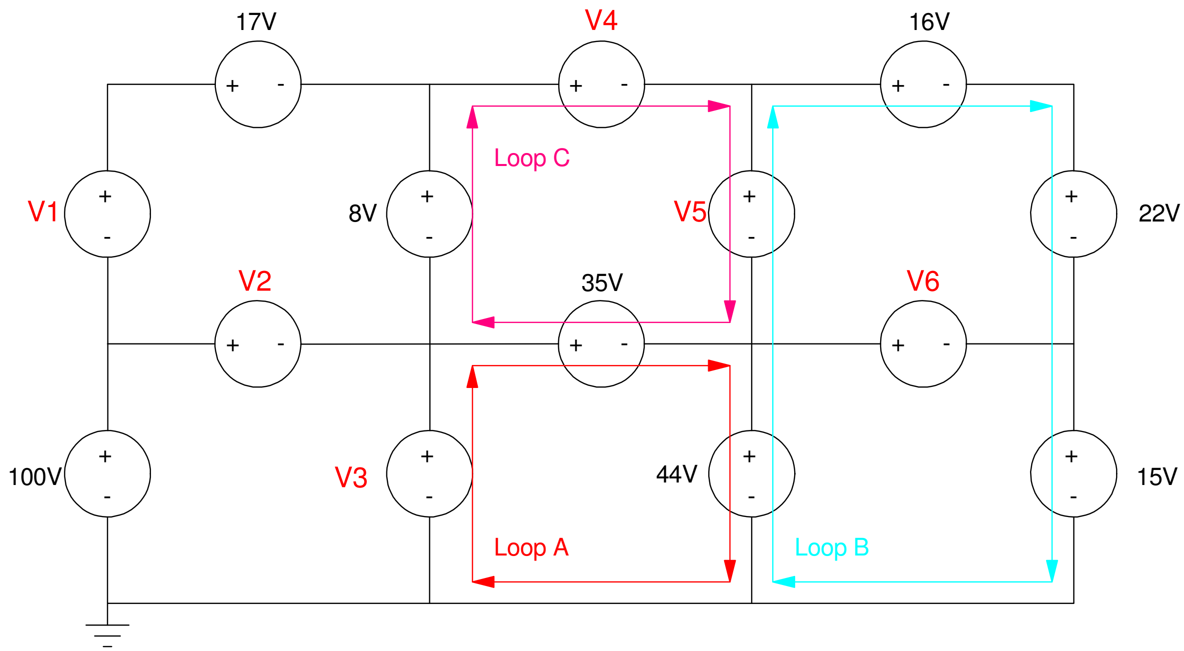
- -V3 + 35 + 44 = 0
- V3 = 79V

Loop B:

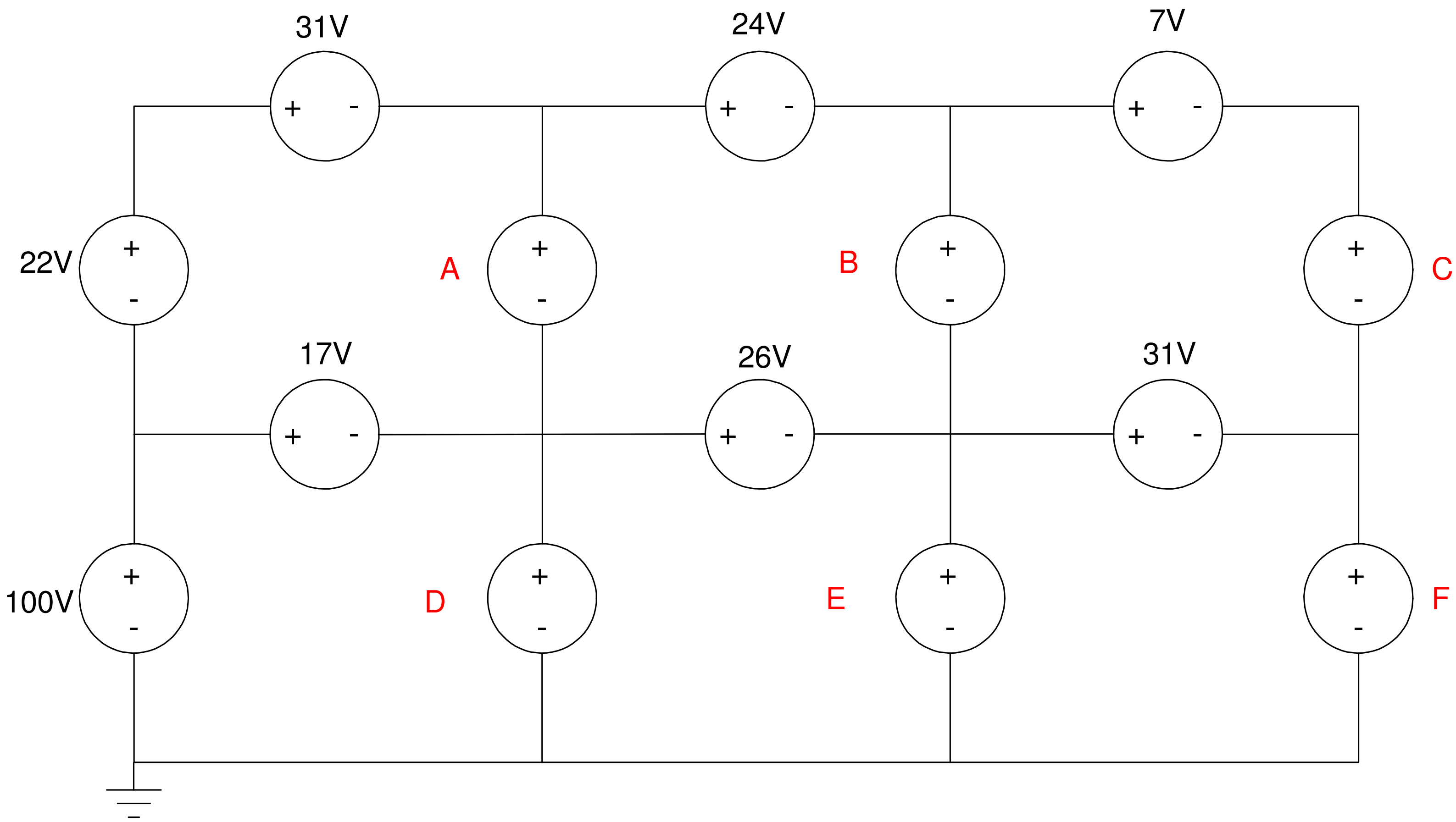
- -44 - V5 + 16 + 22 + 15 = 0
- V5 = 9V

Loop C:

- -8 + V4 + V5 - 35 = 0
- -8 + V4 + 9 - 35 = 0
- V4 = 34V



Problem: Determine the unknown voltages



Conservation of Current: Kirchoff's Current Law

The sum of the currents leaving a node must be zero

- The sum of the currents going into a node must be zero
- Current in = Current Out

Kirchoff's current law simply states you cannot create matter.

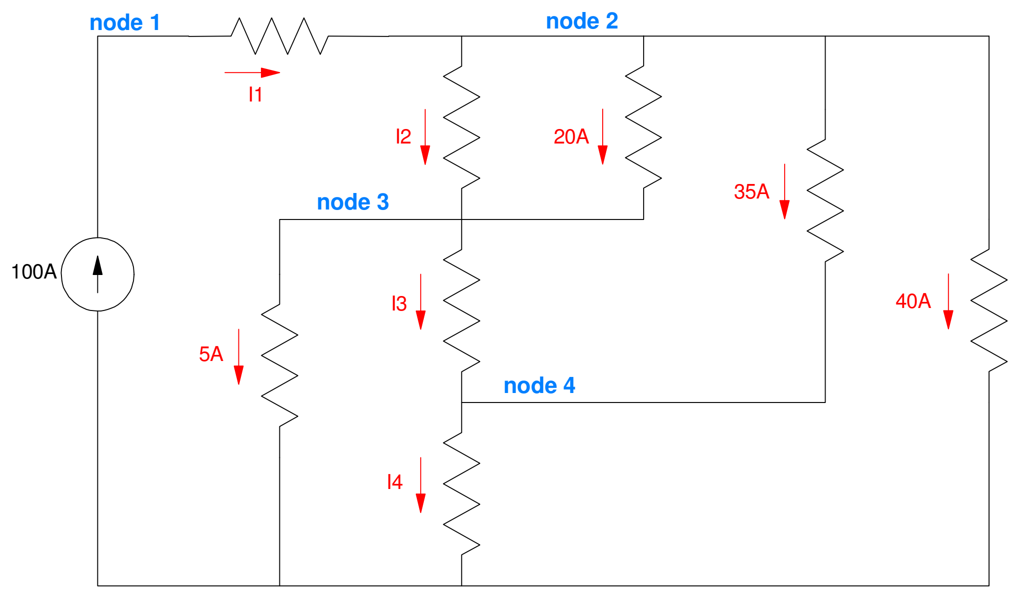
- Current is the flow of electrons, a physical entity.
- If you cannot create matter, you cannot create electrons
- (i.e. current in must match with the current out of a node).

Conservation of Current

- Determine the unknown currents for the following circuit:

At Node #1, current in = current out

$$100 = I1$$



At Node #2, current in = current out

$$I1 = I2 + 20 + 35 + 40$$

$$I2 = 5A$$

At Node #3, current in = current out

$$I2 + 20 = 5 + I3$$

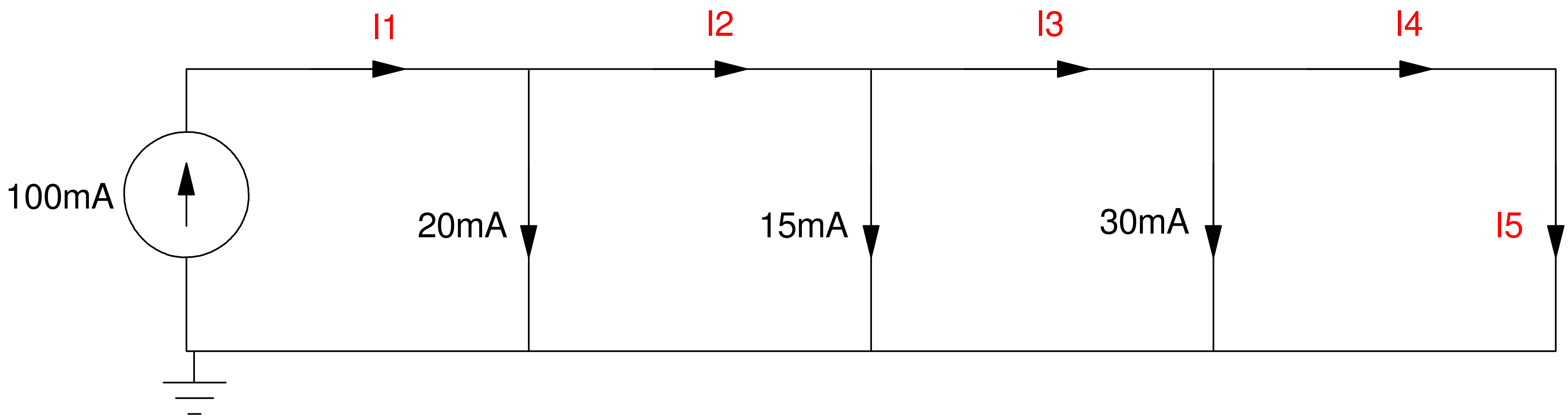
$$I3 = 20A$$

At Node #4, current in = current out

$$I3 + 35 = I4$$

$$I4 = 55A$$

Problem: Determine the unknown currents



Summary

Kirchoff's laws are conservation of voltage and current

Conservation of Voltage

- The sum of the voltages around any closed path must be zero

Conservation of Current

- The sum of all currents from a node must be zero
- Current in = Current out

These will be the bases for many techniques used in this class to analyze circuits