Resistors in Series and Parallel

ECE 211 Circuits I

Lecture #3

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Objective

- Simplify a resistor network
- Find the equivalent resistance between two terminals

Approach

- Combine resistors that are in series
- Combine resistors that are in parallel
- Keep going until you are left with a single resistance

Note:

- Not always possible

Resistors in Series:

- Series: All the current through R1 also goes through R2
- I(R1) = I(R2) = I(R3)

Problem: Find the net resistance of this circuit:

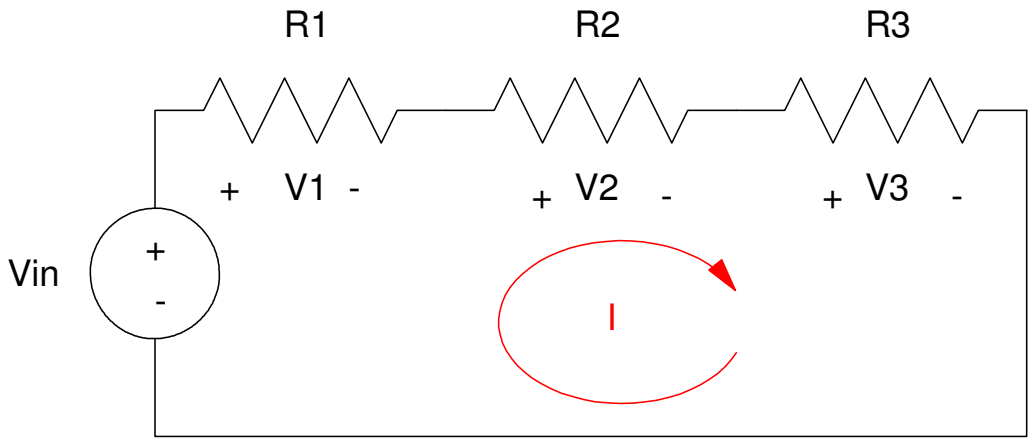
Using Kirchoff's voltage law:

$$\text{Vin} = \text{V1} + \text{V2} + \text{V3}$$

$$V_{in} = I \cdot R_1 + I \cdot R_2 + I \cdot R_3$$

$$V_{in} = I \cdot (R_1 + R_2 + R_3)$$

$$V_{in} = I \cdot R$$



Resistors in series add

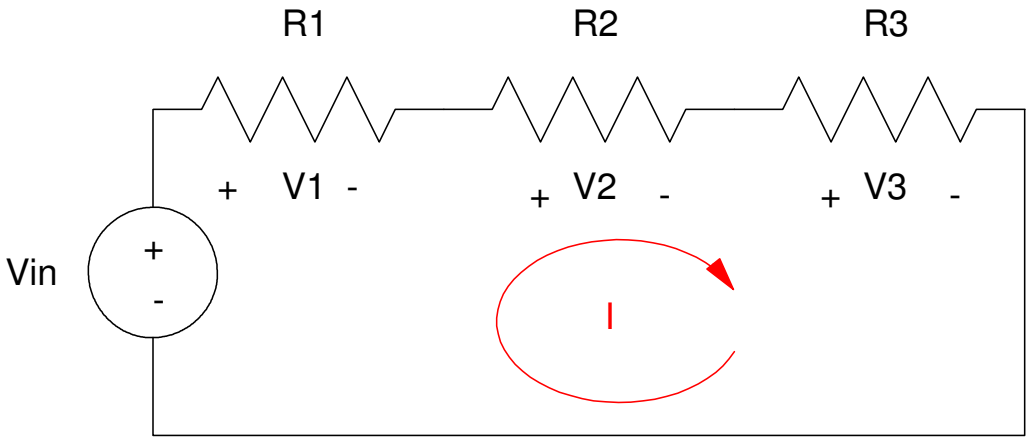
Examples:

Let R_1 = 100 Ohms, R_2 = 200 Ohms, R_3 = 300 Ohms. Find the total resistance.

$$R_{net} = R_1 + R_2 + R_3$$

$$R_{net} = 100\Omega + 200\Omega + 300\Omega$$

$$\boxed{R_{net} = 600\Omega}$$



Let R_1 = 100 Ohms, R_2 = 200 Ohms, R_{net} = 10001000 Ohms. Find R_3.

Answer: Resistors in series add:

$$R_{net} = R_1 + R_2 + R_3$$

$$1000\Omega = 100\Omega + 200\Omega + R_3$$

$$\boxed{R_3 = 700\Omega}$$

Resistors in Parallel:

- Parallel: The voltages are the same on both sides of each resistor
- Va1 = Vb1 = Vc1
- Va2 = Vb2 = Vc2

Find the net resistance:

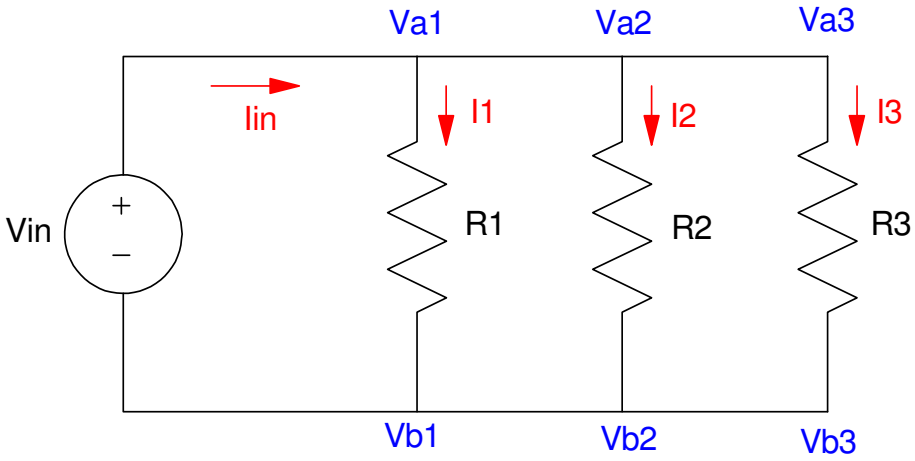
$$I_{in} = I_1 + I_2 + I_3$$

$$I_{in} = \left(\frac{V_{in}}{R_1}\right) + \left(\frac{V_{in}}{R_2}\right) + \left(\frac{V_{in}}{R_3}\right)$$

$$I_{in} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_{in}$$

$$V_{in} = I_{in} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$V_{in} = I_{in} \cdot R$$



In general:

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)^{-1}$$

Examples:

Let R1 = 100 Ohms, R2 = 200 Ohms, R3 = 300 Ohms. Find the total resistance.

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

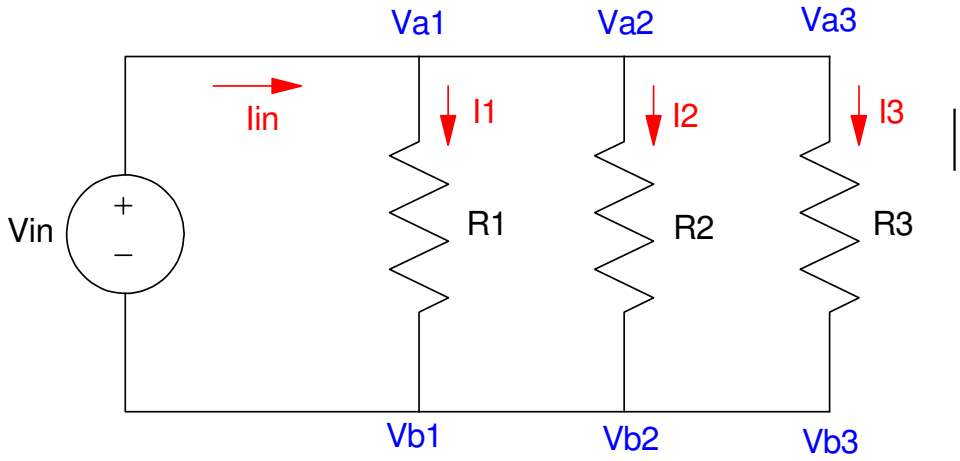
$$R_{net} = \left(\frac{1}{100} + \frac{1}{200} + \frac{1}{300}\right)^{-1}$$

$$\boxed{R_{net} = 54.54\Omega}$$

Let R2 = 200 Ohms, R3 = 300 Ohms, Rnet = 100 Ohms. Find R1.

$$R_{net} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$100\Omega = \left(\frac{1}{R_1} + \frac{1}{200} + \frac{1}{300}\right)^{-1}$$



Find the resistance from a to b

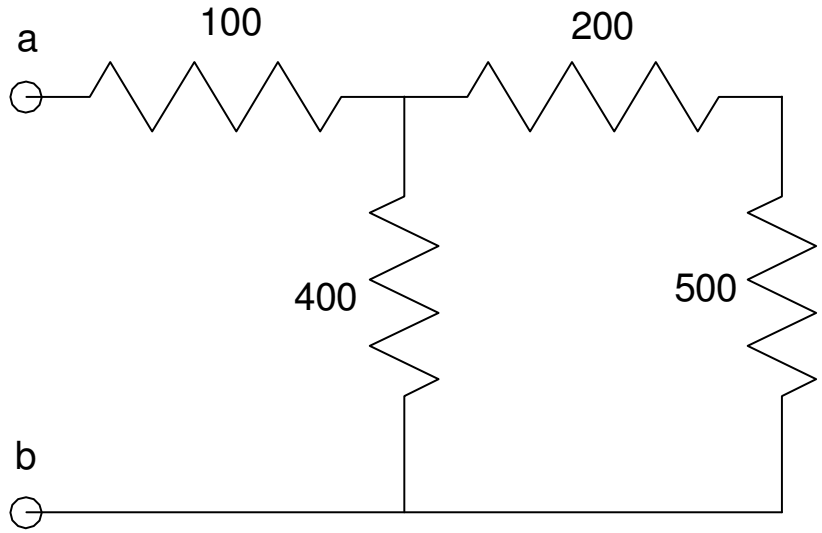
Combine resistors that are in series and parallel

Series:

- All current through R1 also goes through R2

Parallel

- Voltages on the same on both sides



Solution:

The 200 and 500 Ohm resistor are in series

$$200 + 500 = 700$$

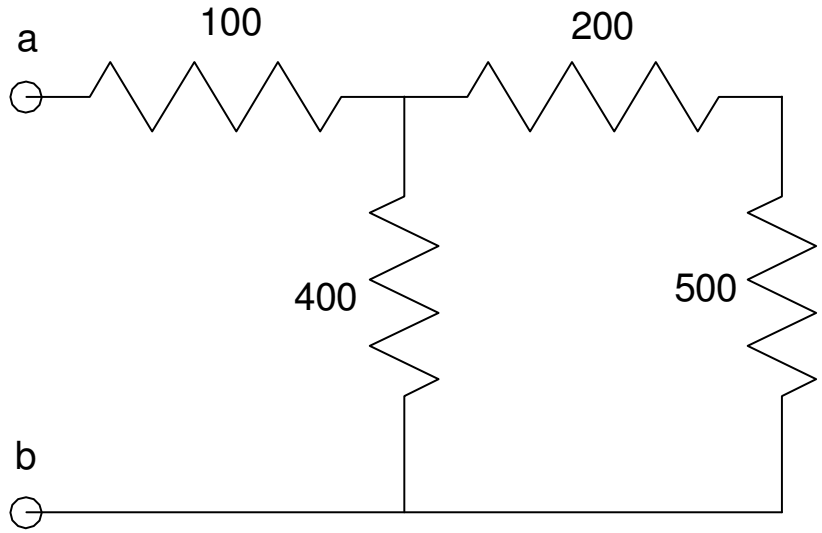
This is in parallel with 400 Ohms

$$700 \parallel 400 = \left(\frac{1}{700} + \frac{1}{400}\right)^{-1} = 254.54$$

This is in series with 100 Ohms

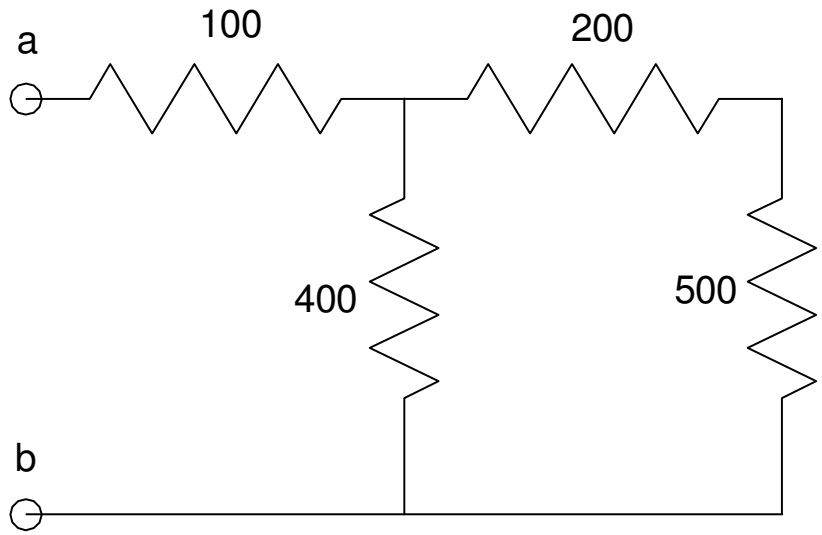
$$254.54 + 100 = 354.54$$

The net resistance is 354.54 Ohms.



Matlab Calculations

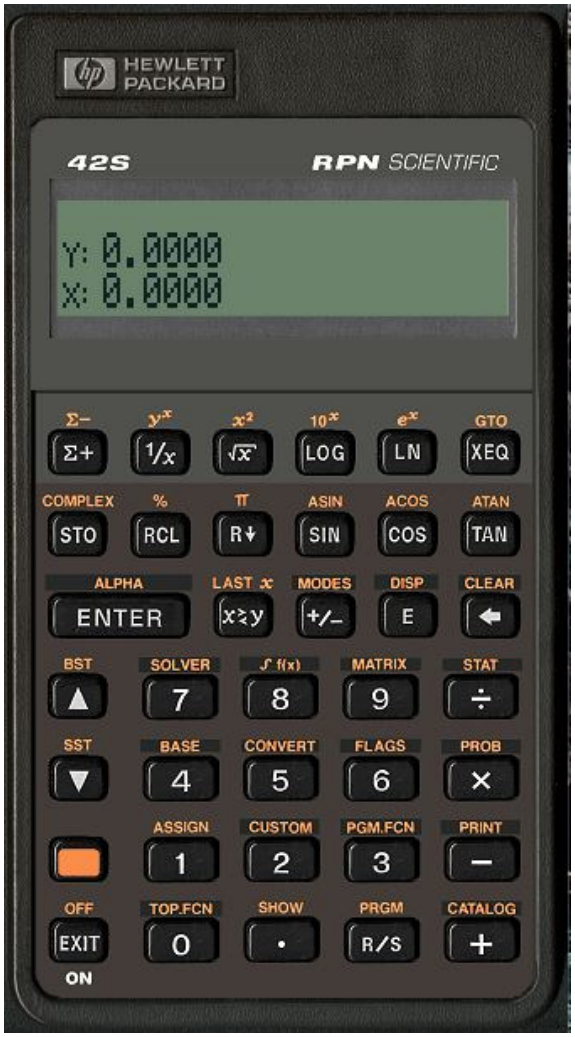
```
>> R1 = 200 + 500

R1 =   700

>> R2 = 1 / (1/R1 + 1/400)

R2 =  254.5455

>> R3 = R2 + 100

R3 =  354.5455
```



HP42 Calculations (Free42 app)

RPN Calculators are really efficient at these types of calculations

- HP is essentially a company of electrical and computer engineers
- They likewise build really good calculators for ECE majors
- Free app on your cell phone

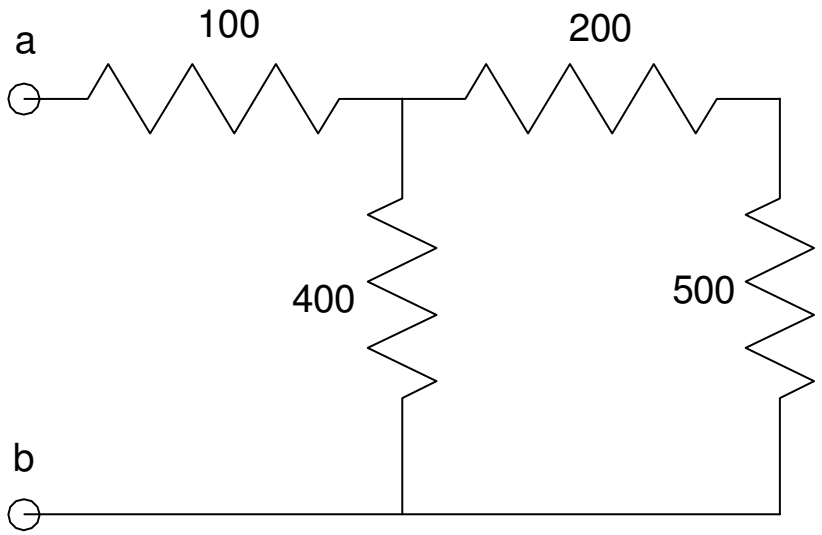


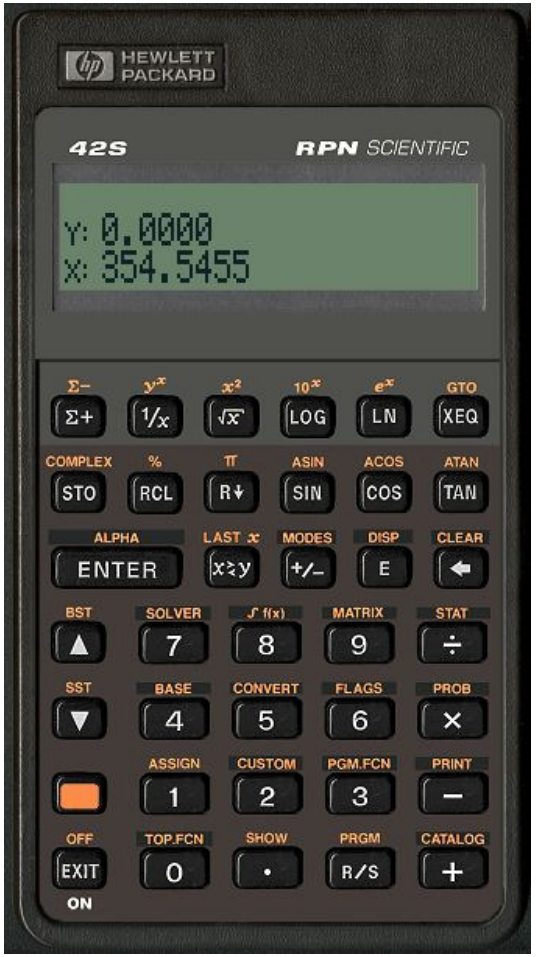
Note: These use Reverse Polish Notation (RPN)

- No equals sign
- Operation operate on the stack
  - \+ Y + X → X
  - \- Y - X → X
  - * Y * X→ X
  - / Y / X → X

RPN Calculations

```
200
enter
500
+

1/x
400
1/x
+
1/x

100
+
```





Checking in CircuitLab

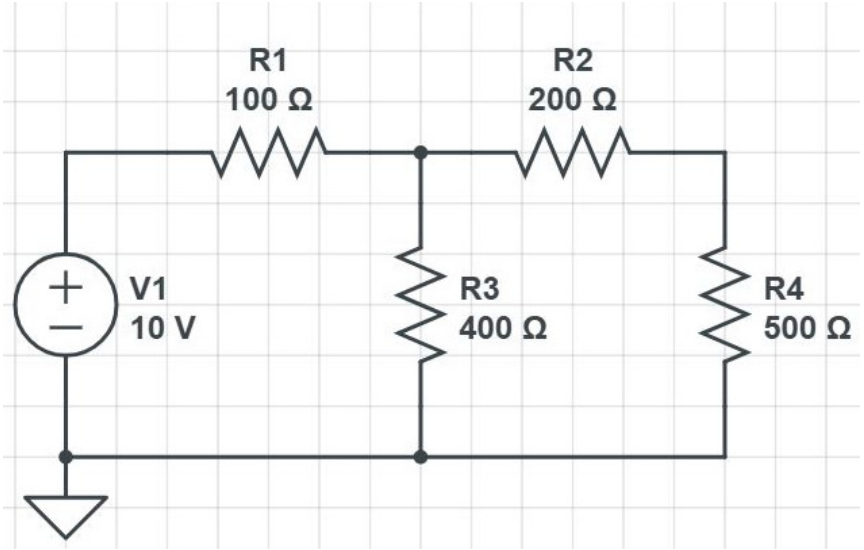
Build the circuit in CircuitLab

- Apply a test voltage (10V)
- Measure the net current draw
  - I = 28.21mA
- Compute the net resistance

$$R = \frac{V}{I}$$

$$R = \frac{10V}{28.21mA}$$

$$R = 354.48\Omega$$



Check in Lab

Build the circuit on a breadboard

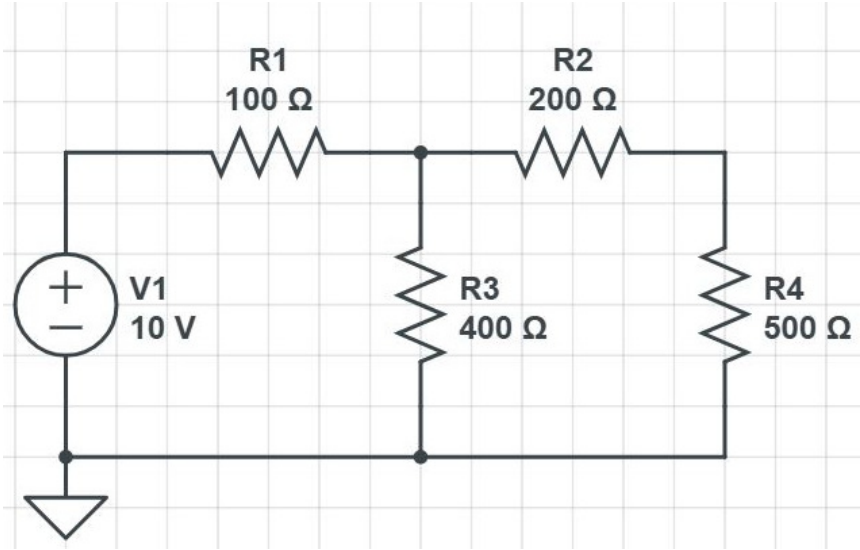
Apply a test voltage

- Whatever is convenient
- 5.0V is usually available

Measure the current draw

- Measure the voltage across the 100 Ohm resistor
- Compute the current

$$I = \frac{V_1}{R_1}$$



CircuitLab vs Chegg vs ChatGPT

CircuitLab is really useful

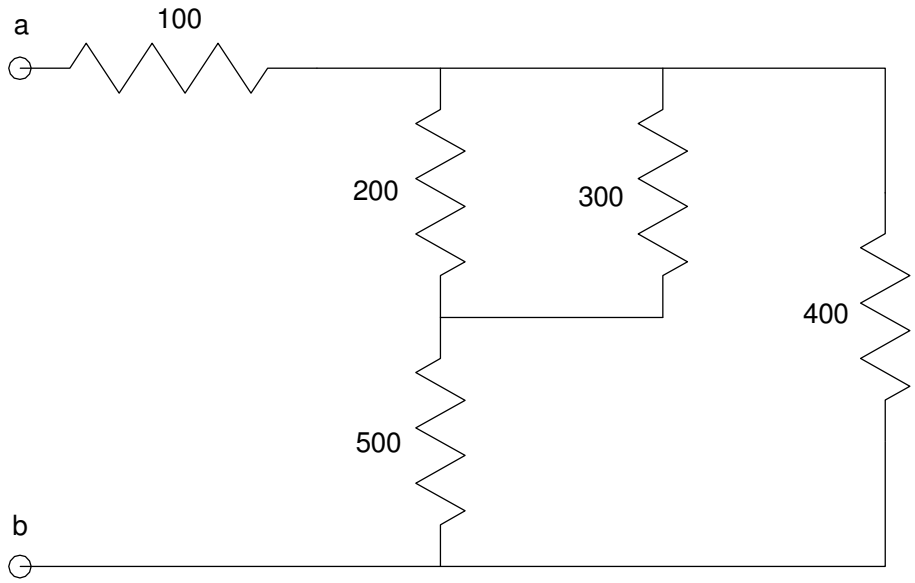
- You can check your calculations using CircuitLab
- Your results should match
- Chegg isn't needed
- ChatGPT isn't needed

If you want to know if your calculations are correct

- Check using CircuitLab, or
- Build the circuit and take measurements in lab

Example 2: Find Rab

- Combine resistors that are in series
- Combine resistors that are in parallel



Calculations

The 200 and 300 are in parallel

200 || 300 = 120 Ohms

This is in series with 500

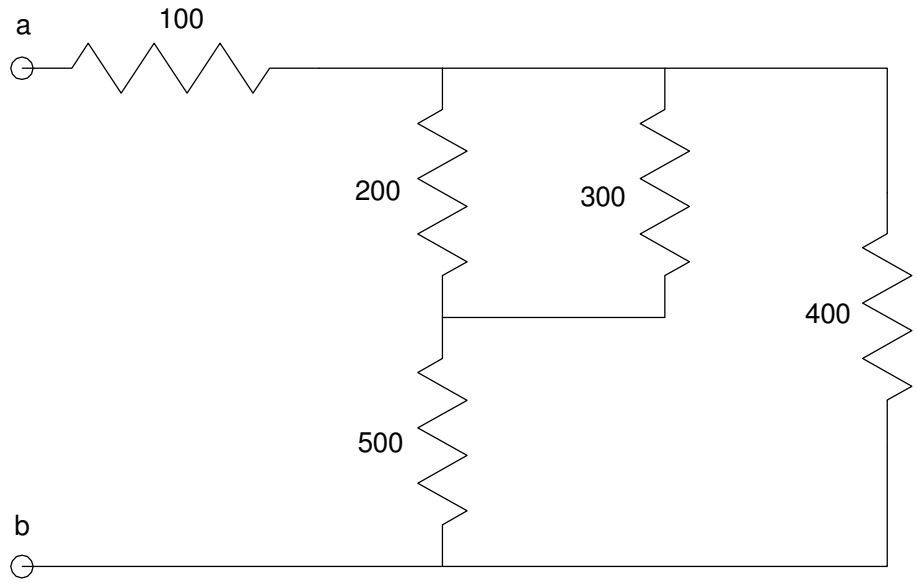
120 + 500 = 620

This is in parallel with 400

620 || 400 = 243.137

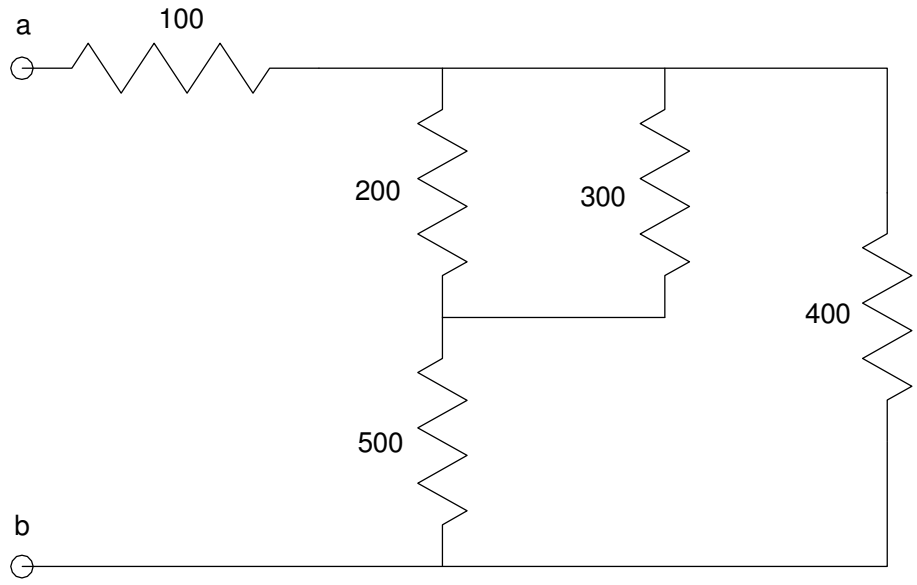
This is in series with 100

243.137 + 100 = **343.137**



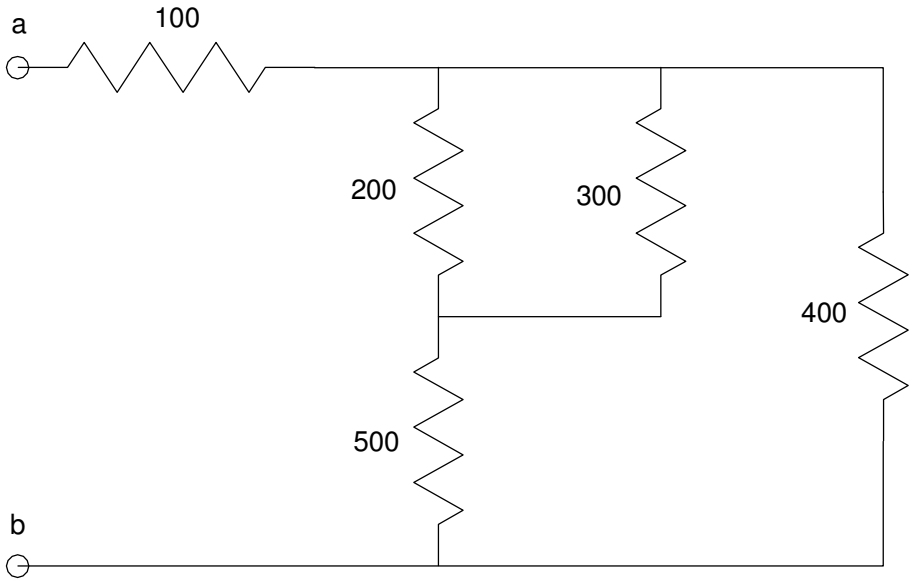
Calculations in Matlab

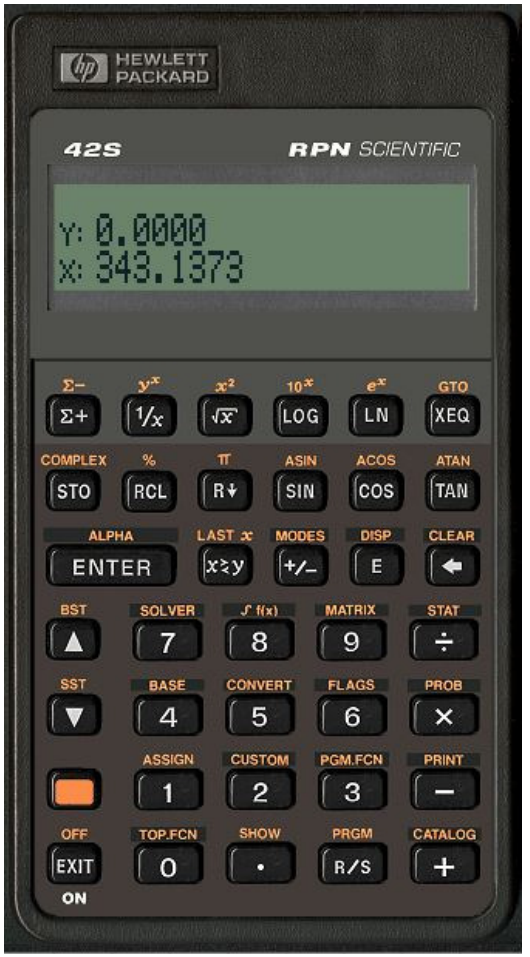
```
>> R1 = 1 / (1/200 + 1/300)

R1 =   120

>> R2 = R1 + 500

R2 =   620

>> R3 = 1 / ( 1/R2 + 1/400)

R3 =  243.1373

>> Rab = R3 + 100

Rab =  343.1373
```



Calculations with an HP42

```
200
1/x
300
1/x
+
1/x

500
+

1/x
400
1/x
+
1/x

100
+
```





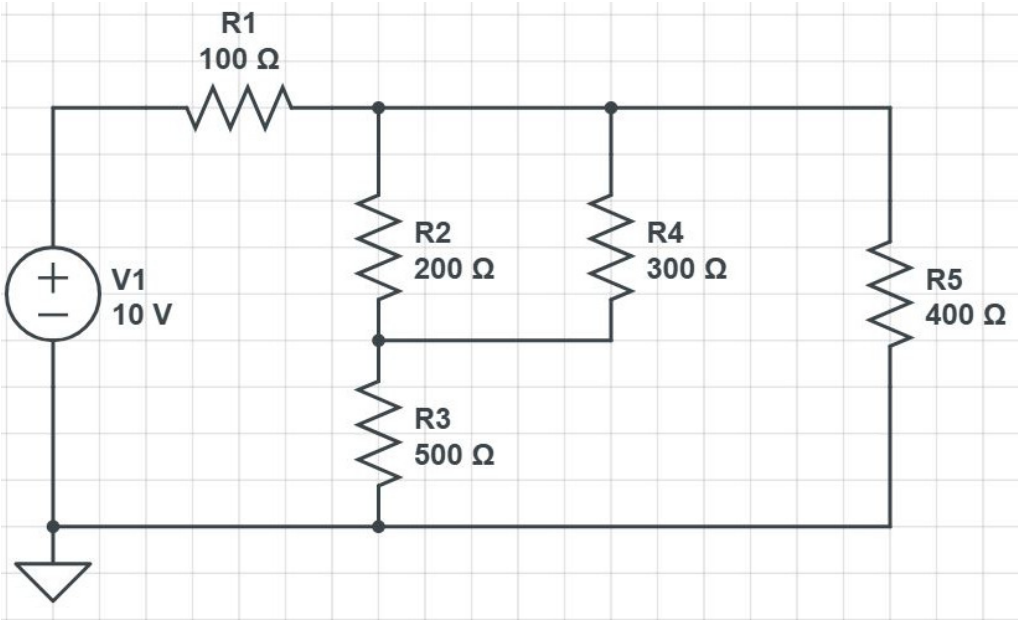
CircuitLab Calculations

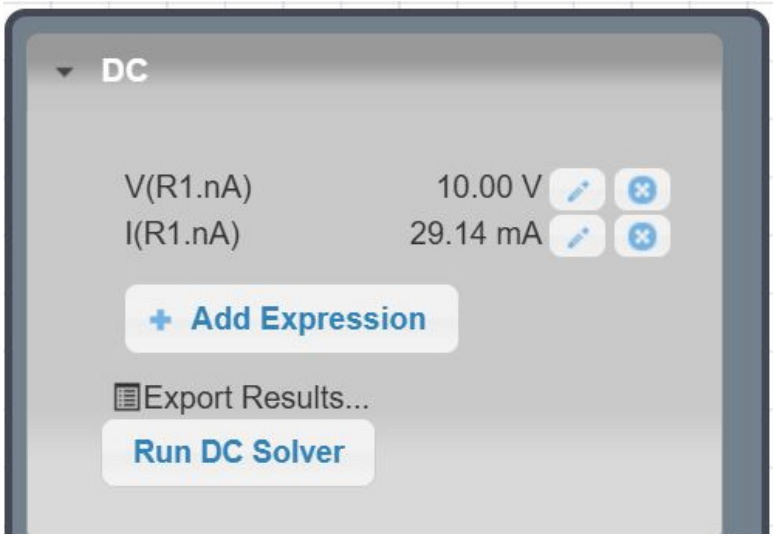
- Build the circuit
- Apply a test voltage (10V)
- Measure the net current draw
- Compute R

$$R = \frac{V}{I}$$

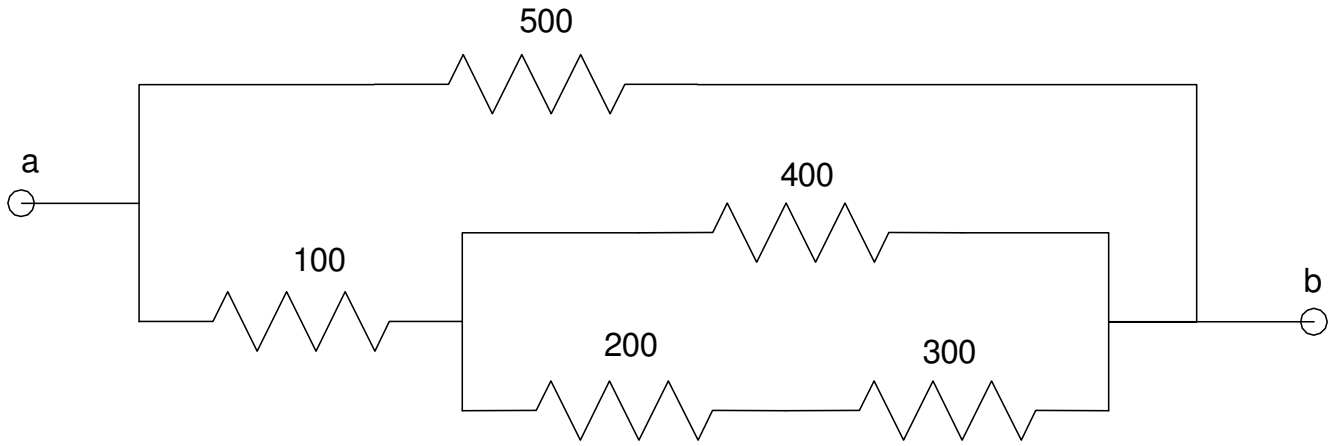
$$R = \frac{10V}{29.14mA}$$

$$R = 343.17\Omega$$





Handout: Find Rab

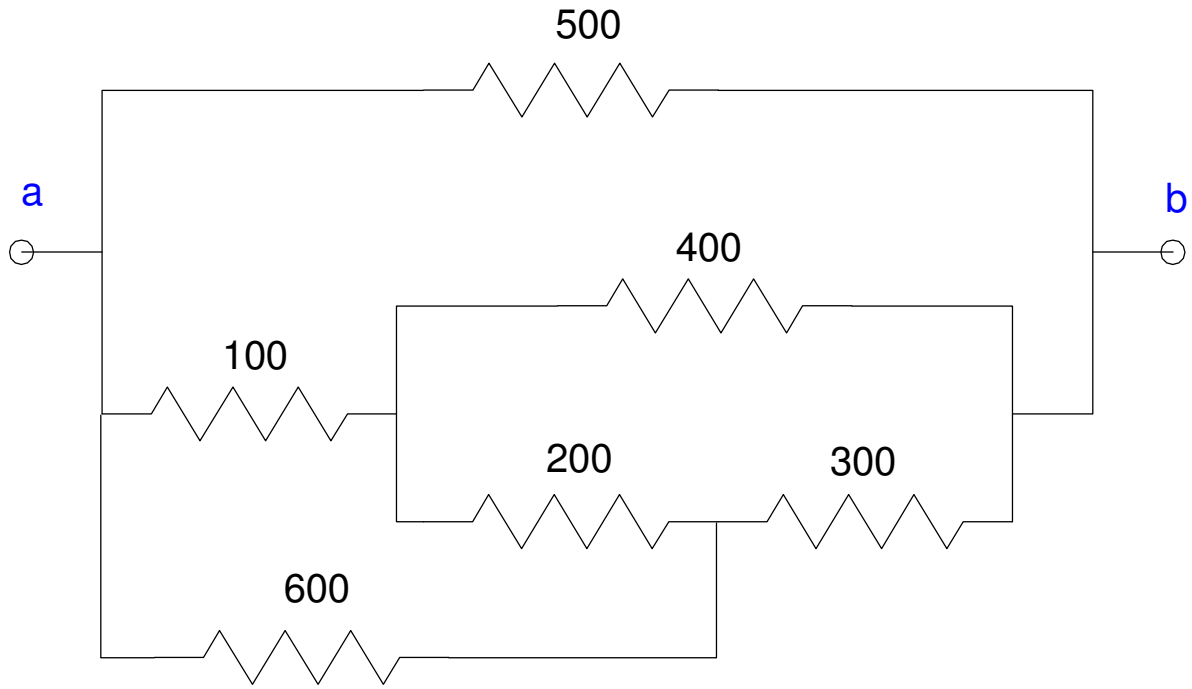


Comment

Series & parallel simplification doesn't always work

- CircuitLab always works though

Methods to compute the net resistance in these cases will be covered later in this course



Summary

Resistors in series add

$$R_{net} = R_1 + R_2 + R_3$$

Resistors in parallel add as

$$\frac{1}{R_{net}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

CircuitLab can be used to check your answers

You can also build the circuit in lab to check your answers