Voltage and Current Division

ECE 211 Circuits I

Lecture #4

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Introduction

In the previous lecture, we looked at simplifying resistor networks

- Resistors in series add
- Resistors in parallel add as the sum of the inverses, inverted

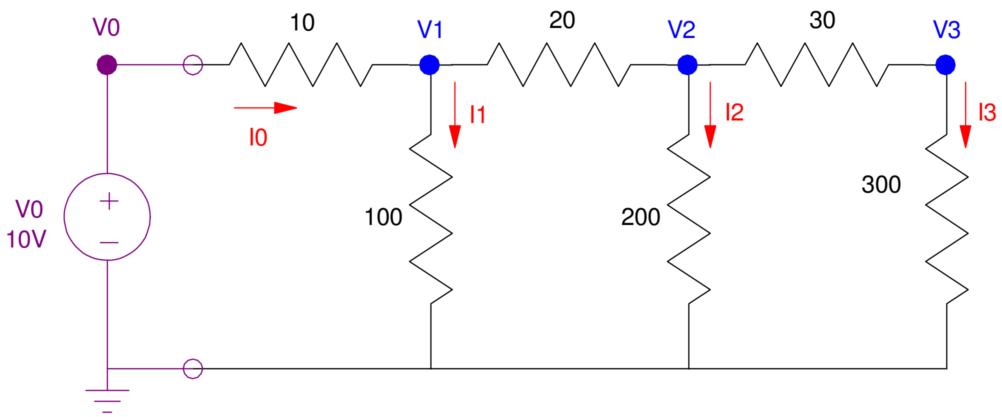
In this lecture, we add a power supply

- Voltage source or
- Current source

Problem: How do you compute the voltages and currents for this circuit?

This lecture looks at two techniques:

- Votlage Division and
- Current Division



Voltage Division

Problem:

- Given a circuit with resistors in series,
- Find the voltage across one of the resistors

Solution: Find the current:

$$I = \frac{V_{in}}{R_1 + R_2 + R_3}$$

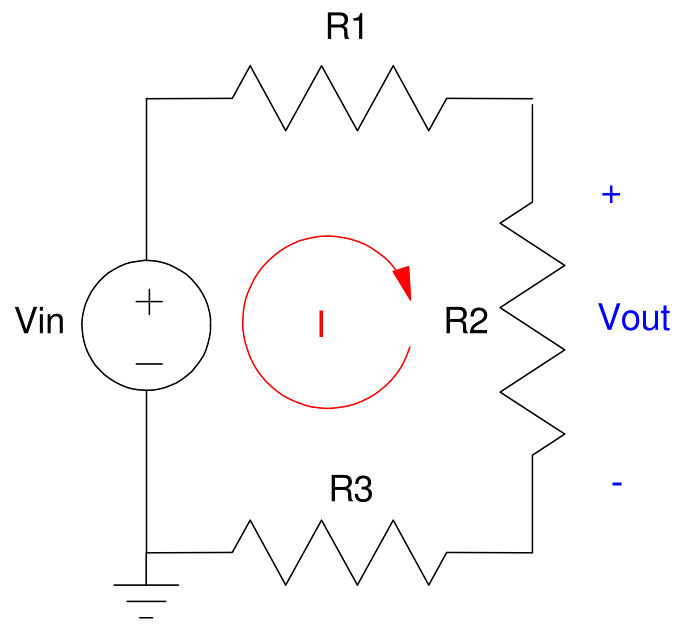
Then find Vout:

$$V_{out} = I \cdot R_2$$

$$V_{out} = \left(\frac{R_2}{R_1 + R_2 + R_3} \right) V_{in}$$

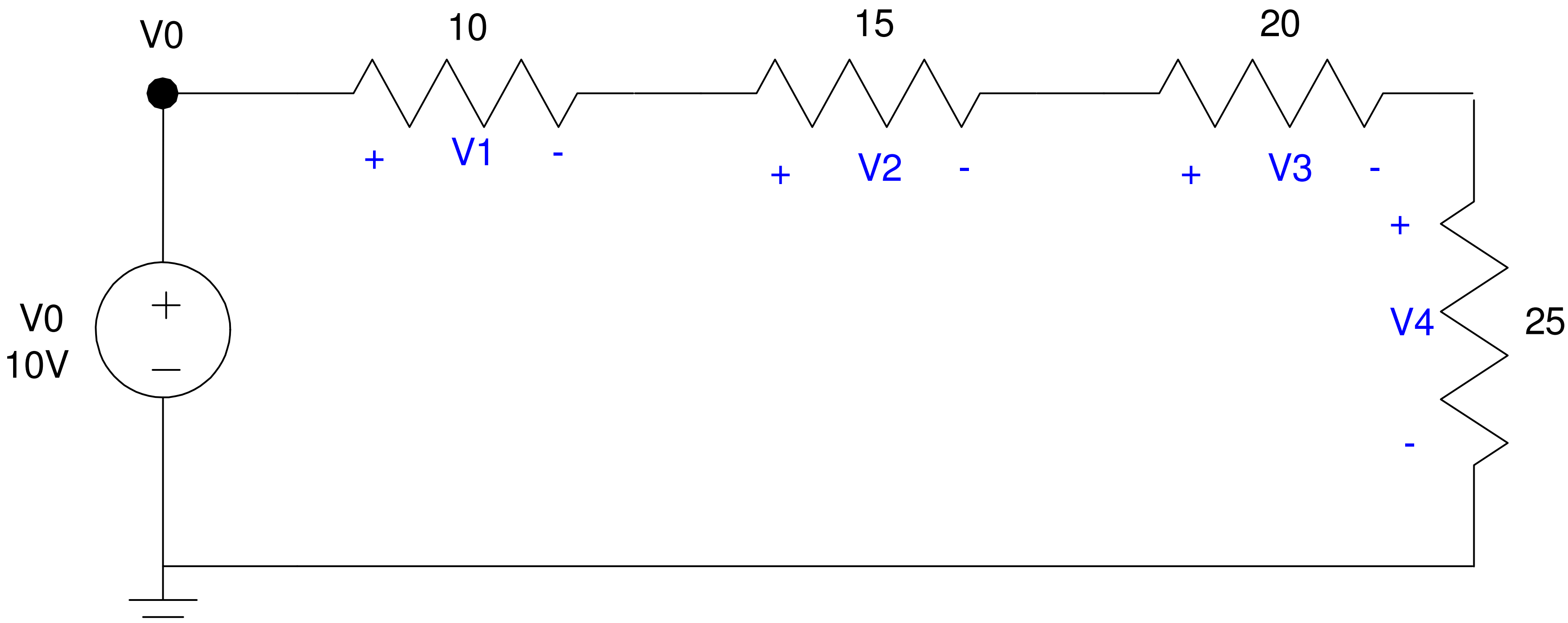
or

$$V_{out} = \left(\frac{\text{The resistance you're measuring across}}{\text{The total resistance}} \right) V_{in}$$



Handout

Find V1, V2, V3, and V4



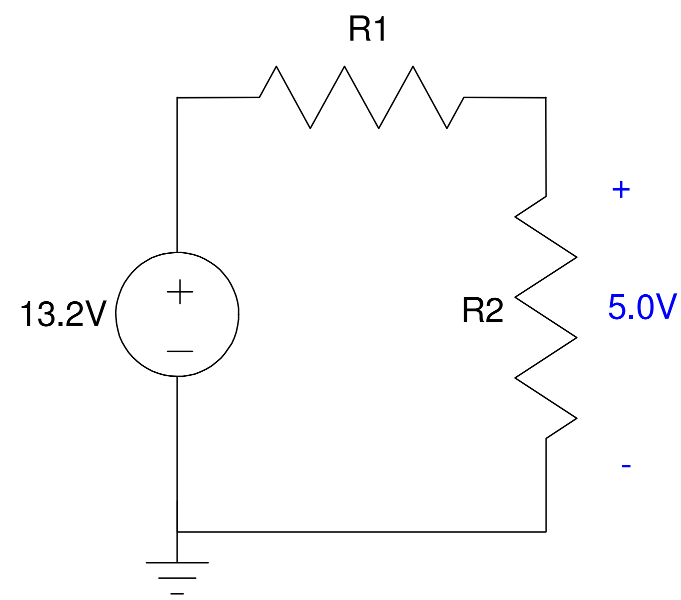
Voltage Division: Example 1

One use of voltage division is to reduce the votlage

- Example: Convert 13.2V to 5.0V

$$V_{out} = \left(\frac{R_2}{R_1+R_2}\right) V_{in}$$

$$5.0V = \left(\frac{R_2}{R_1+R_2}\right) 13.2V$$



There are an infinite number of solutions

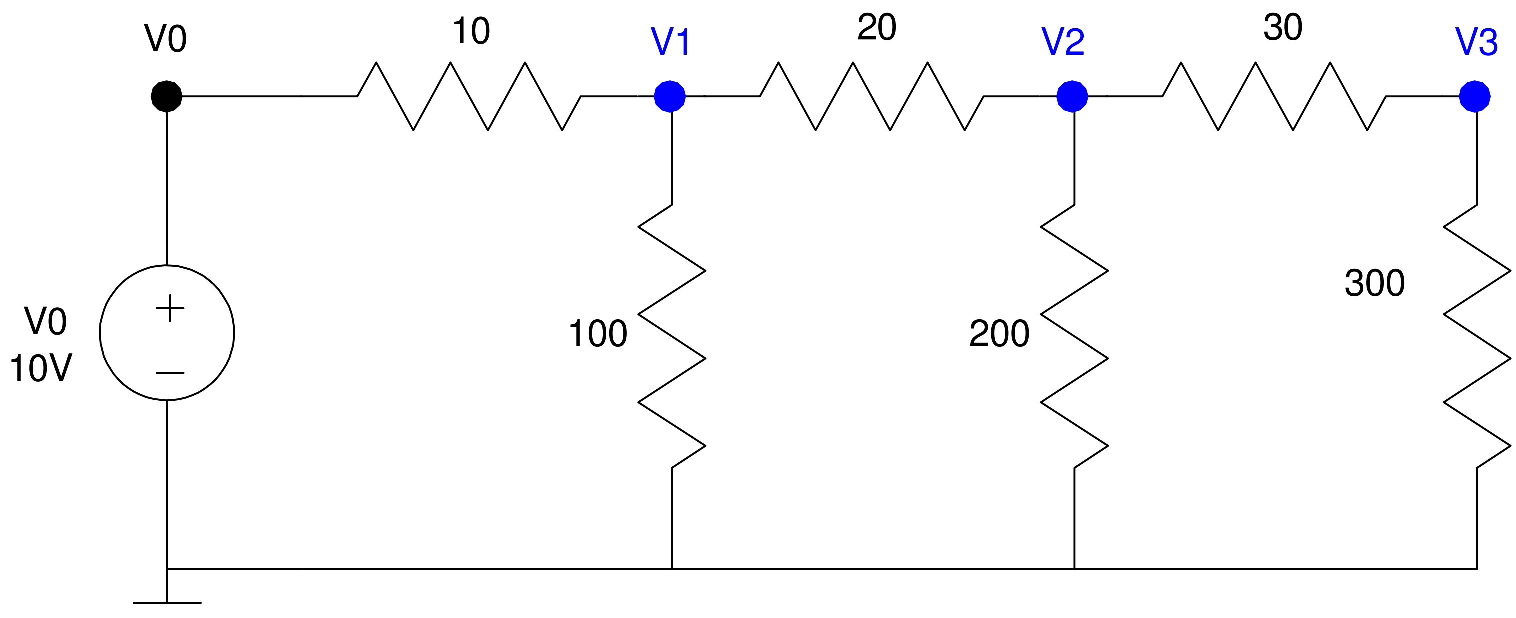
- Pick R2 *R2 = 100k*
- Solve for R1 *R1 = 164k*

You can scale the results

- Scale both R1 and R2 by X
- You get the same result

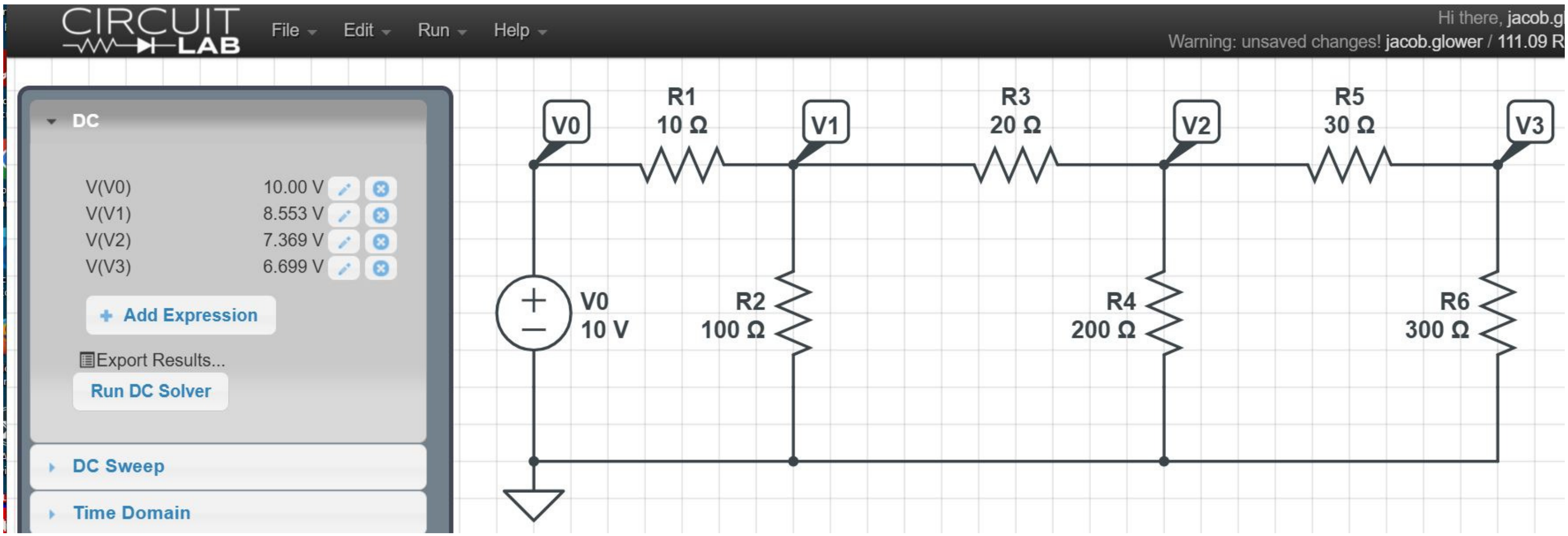
Votage Division: Example 2

Find the volages {V1, V2, V3} using voltage division



Solution #1: CircuitLab

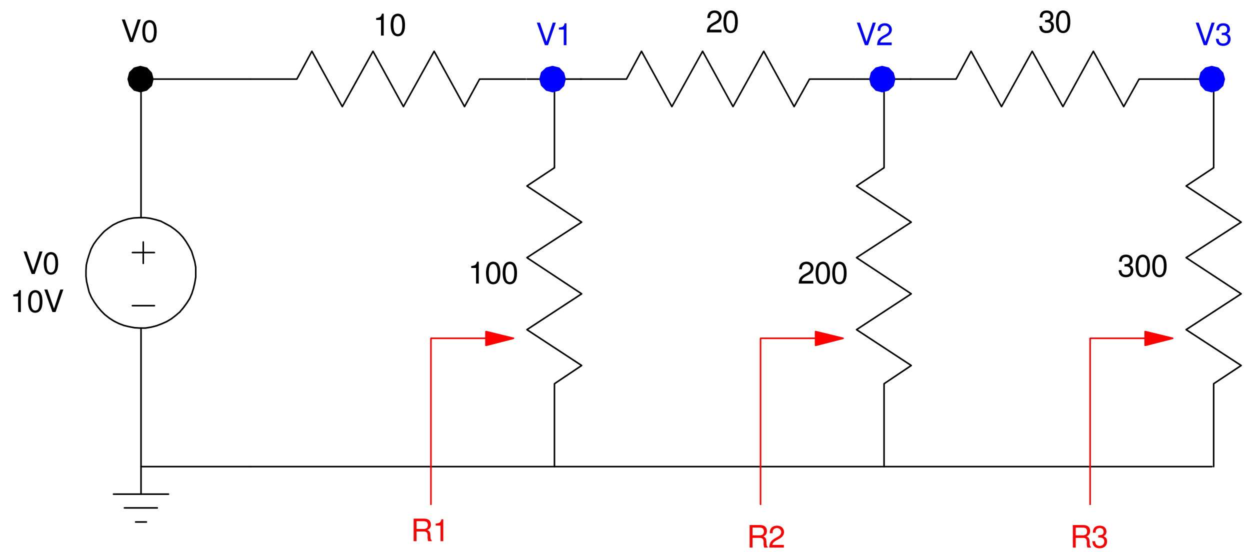
- Build the circuit in CircuitLab
- Solve using the DC Solver



Solution #2: Voltage Division

Compute the resistances looking right

- R1
- R2
- R3



By voltage division

$$V_1 = \left(\frac{R_1}{R_1+10}\right) V_0$$

$$V_2 = \left(\frac{R_2}{R_2+20}\right) V_1$$

$$V_3 = \left(\frac{R_3}{R_3+30}\right) V_2$$

Find the resistances looking right:

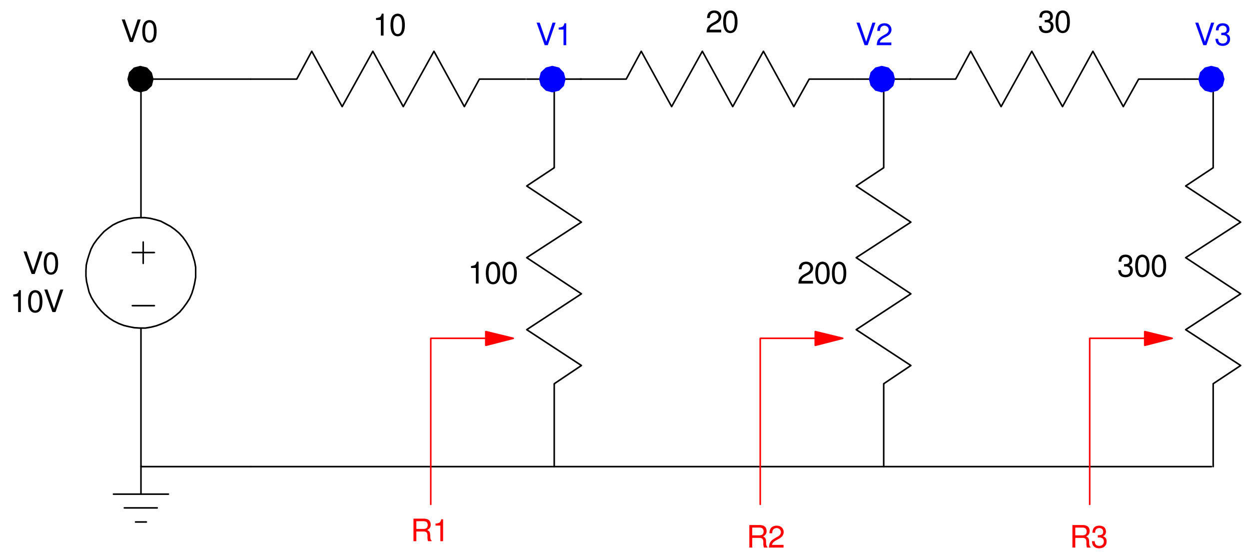
$$R_3 = 300\Omega$$

$$R_2 = \left(\frac{1}{200} + \frac{1}{330}\right)^{-1}$$

$$R_2 = 124.528\Omega$$

$$R_1 = \left(\frac{1}{100} + \frac{1}{R_2+20}\right)^{-1}$$

$$R_1 = 59.105\Omega$$



Now find the voltages

$$V_1 = \left(\frac{R_1}{R_1+10}\right) V_0$$

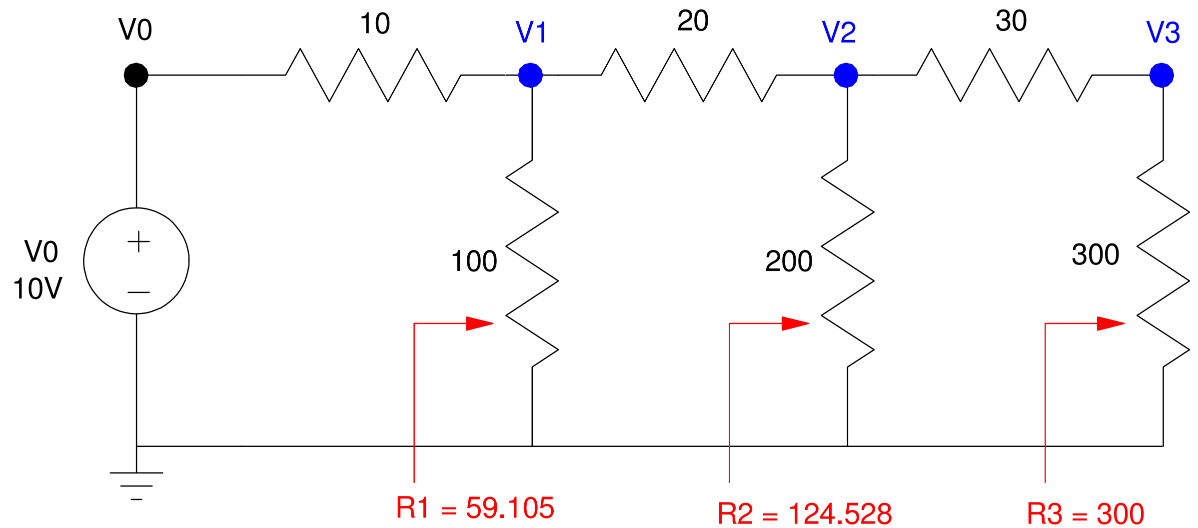
$$V_1 = 8.553V$$

$$V_2 = \left(\frac{R_2}{R_2+20}\right) V_1$$

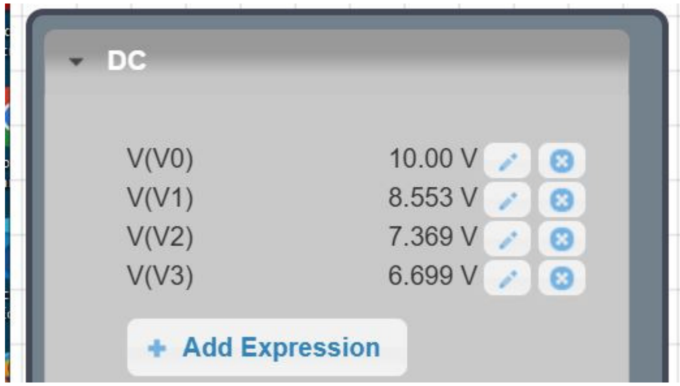
$$V_2 = 7.369V$$

$$V_3 = \left(\frac{R_3}{R_3+30}\right) V_2$$

$$V_3 = 6.699V$$



Same results as CircuitLab



Voltage Division & Ohm Meters

Voltage division is how Ohm meters work

About the only things you can measure directly are

- Distance,
- Voltage, and
- Time

Pretty much everything else has to be measured indirectly.

- Including resistance

Ohm Meter Equations

Ohm meters typically include

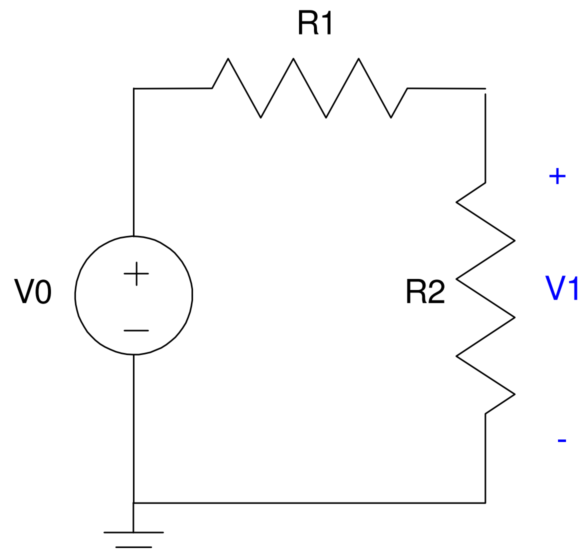
- A voltage source and
- A known resistance (R1)

By measuring V0 and V1, you can compute R2

$$V_1 = \left(\frac{R_2}{R_1+R_2}\right) V_0$$

Doing some algebra

$$R_2 = \left(\frac{V_1}{V_0-V_1}\right) R_1$$

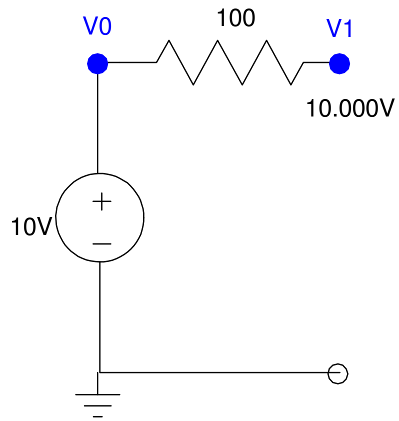


Ohm Meter Example

To find the equivalent resistance

- Measure V1 without a load
- Measure V1 with a load

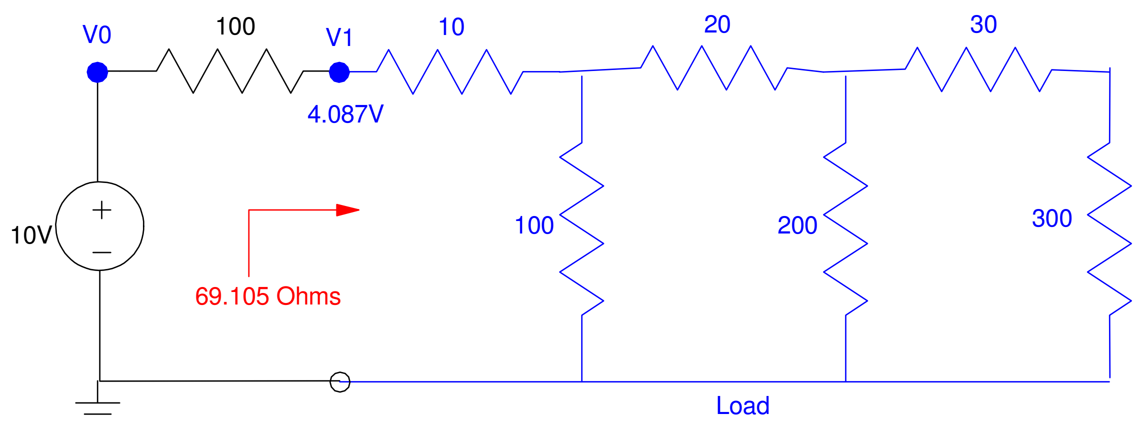
Compute the implied resistance



$$R_2 = \left(\frac{V_1}{V_0 - V_1}\right) R_1$$

$$R_2 = \left(\frac{4.087V}{10V - 4.087V}\right) 100\Omega$$

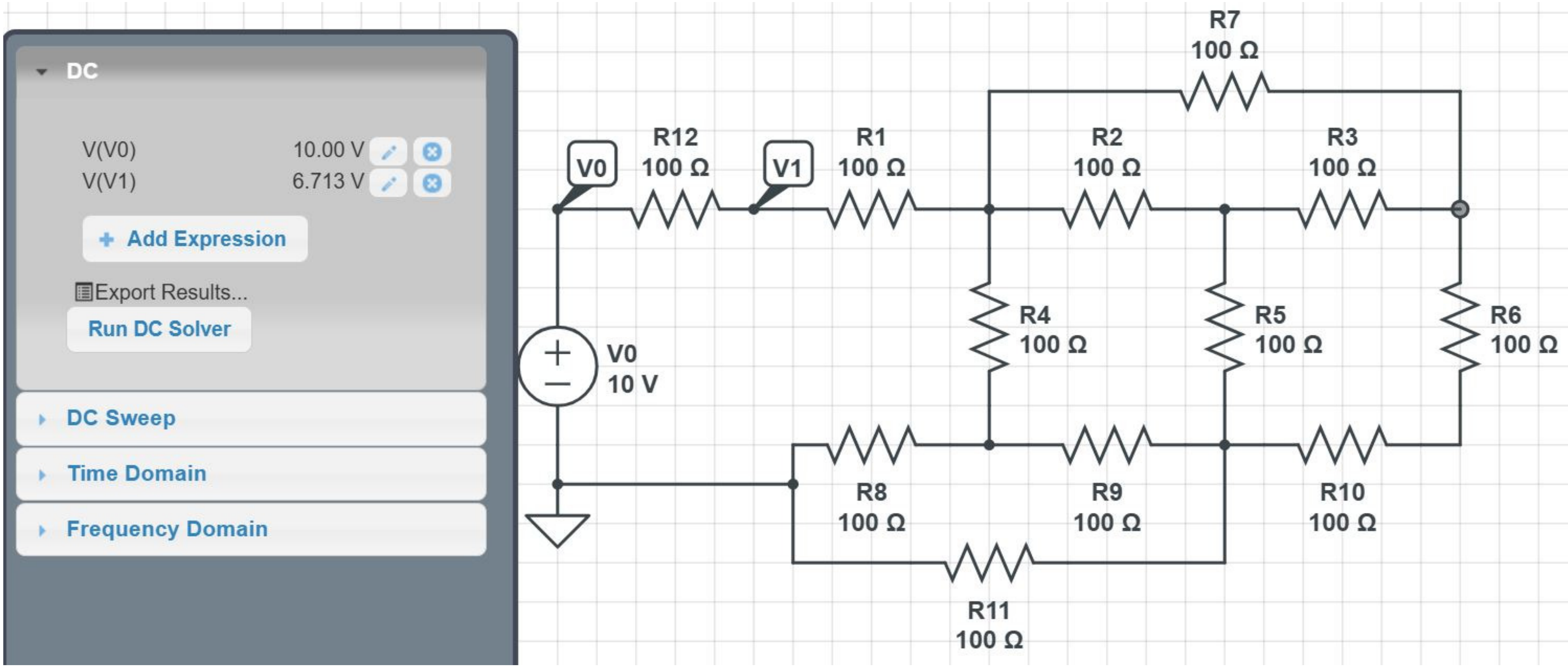
$$R_2 = 69.105\Omega$$



Same answer we got when adding resistors in series and parallel

Problem:

Compute the equivalent resistance of this circuit using V0 and V1



Current Division

Given a circuit with resistors in parallel,

- Find the current through one of the resistors

Solution: Find the voltage

$$R_{total} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

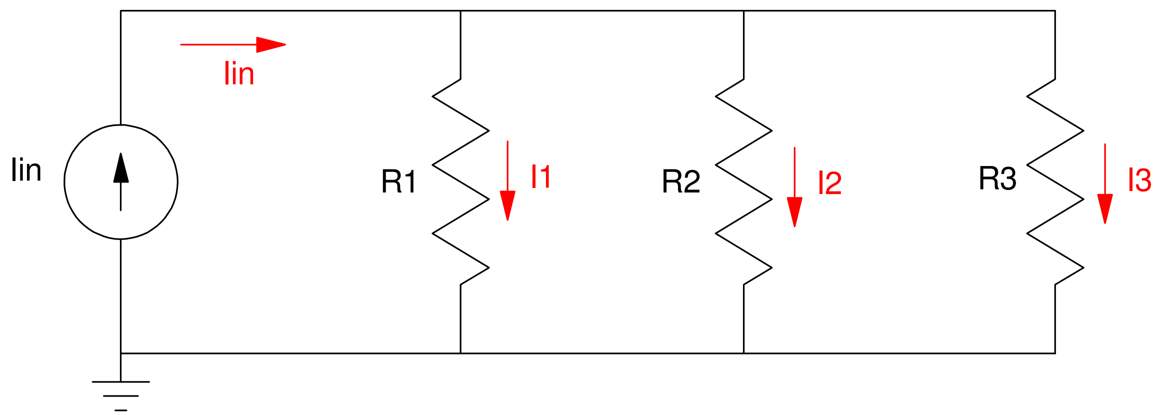
$$V = R_{total} \cdot I_{in}$$

The current I2 is then

$$I_2 = \frac{V}{R_2}$$

$$I_2 = \left(\frac{R_{total}}{R_2}\right) I_{in}$$

$$I_2 = \left(\frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right) I_{in}$$



Current Division

Find I1, I2, and I3

Solution:

$$I_1 = \left(\frac{\frac{1}{50}}{\frac{1}{50}+\frac{1}{100}+\frac{1}{150}}\right)100mA$$

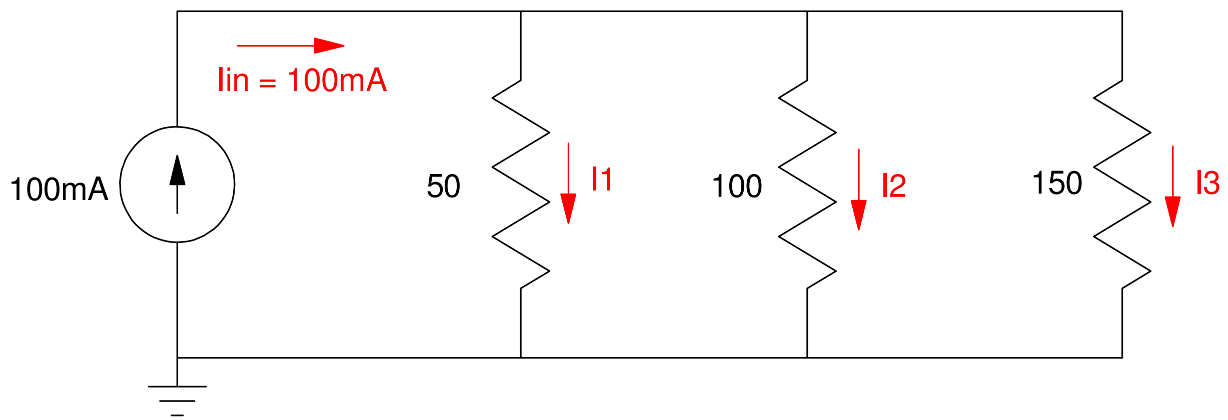
$$I_1 = 54.545mA$$

$$I_2 = \left(\frac{\frac{1}{100}}{\frac{1}{50}+\frac{1}{100}+\frac{1}{150}}\right)100mA$$

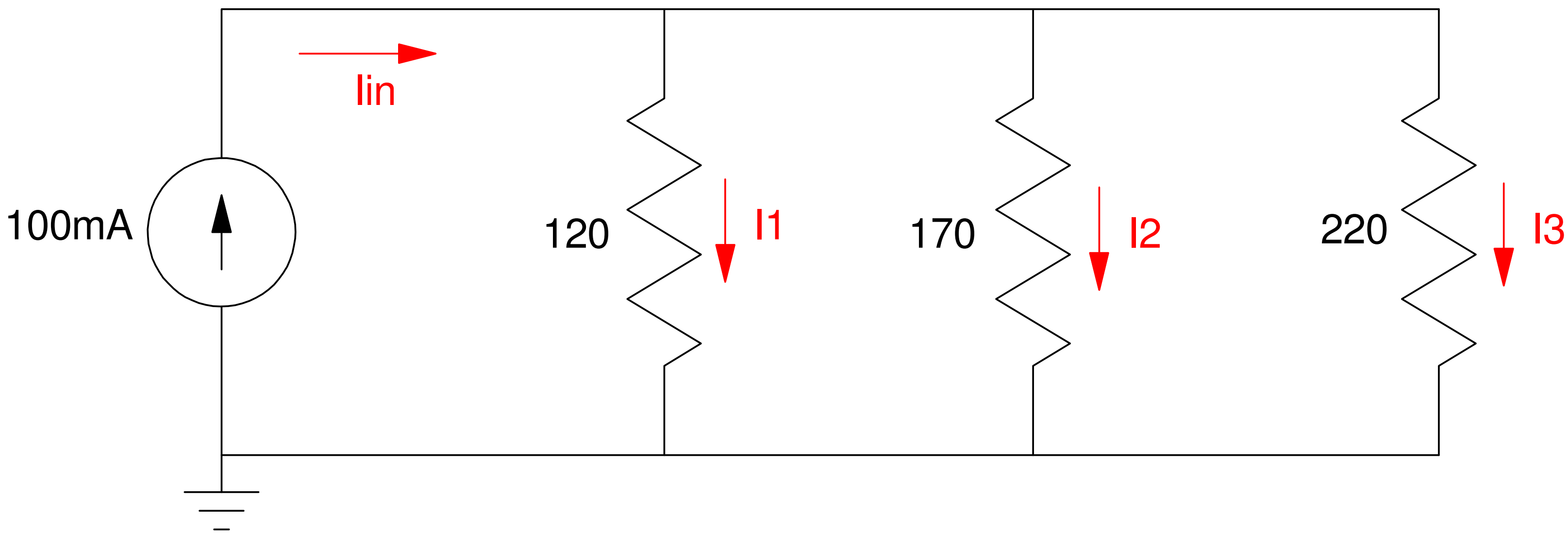
$$I_2 = 27.568mA$$

$$I_3 = \left(\frac{\frac{1}{150}}{\frac{1}{50}+\frac{1}{100}+\frac{1}{150}}\right)100mA$$

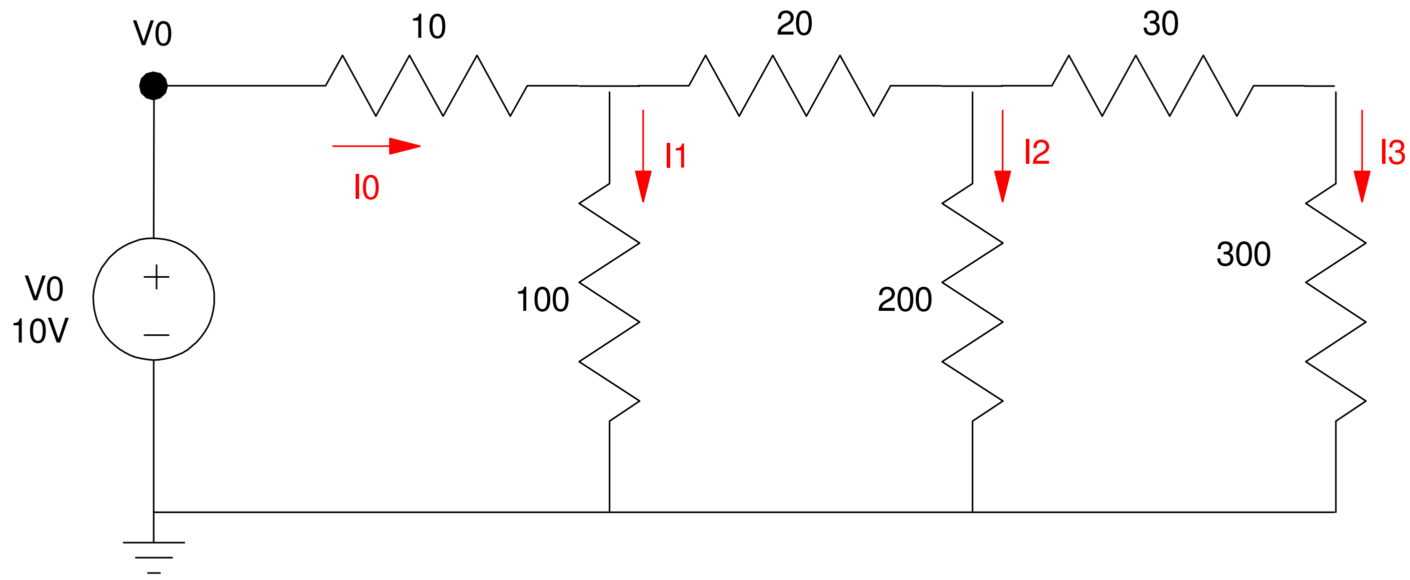
$$I_3 = 18.378mA$$



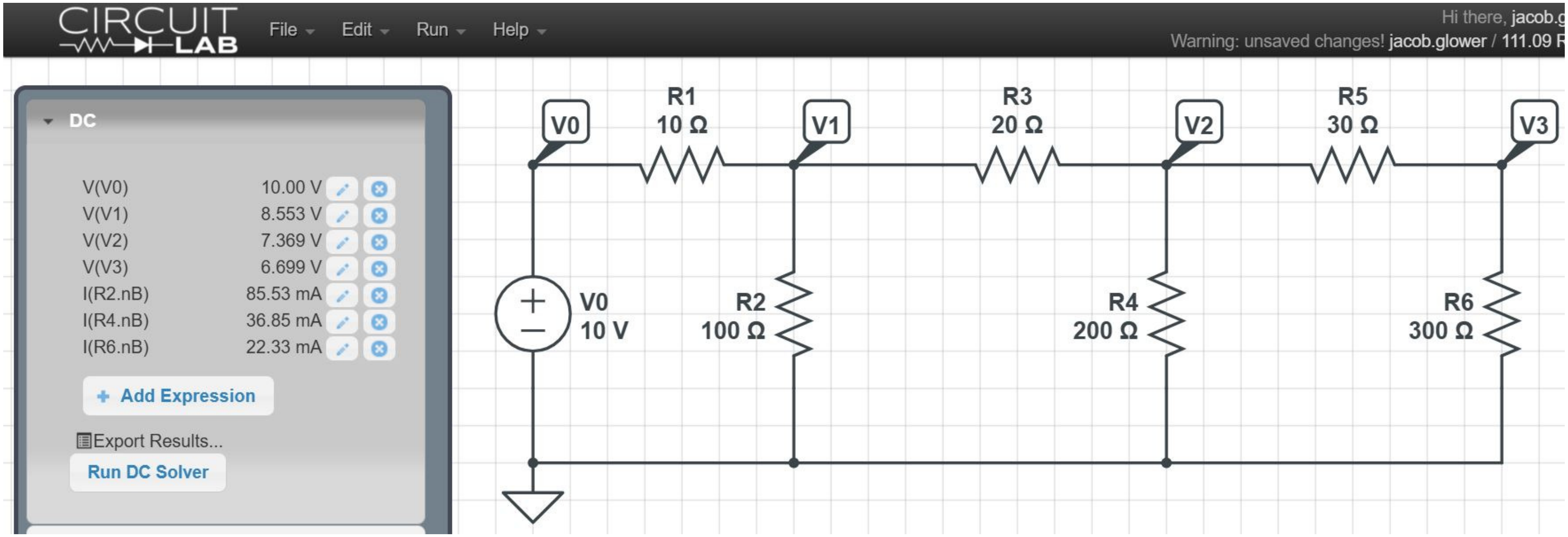
Handout: Find I1, I2, and I3



Example #3: Find the currents



CircuitLab Solution



Calculations

$$I_0 = \left(\frac{10V}{10\Omega + 59.105\Omega}\right)$$

$$I_0 = 144.707mA$$

$$I_1 = \left(\frac{\frac{1}{100}}{\frac{1}{100} + \frac{1}{20+124.528}}\right) I_0$$

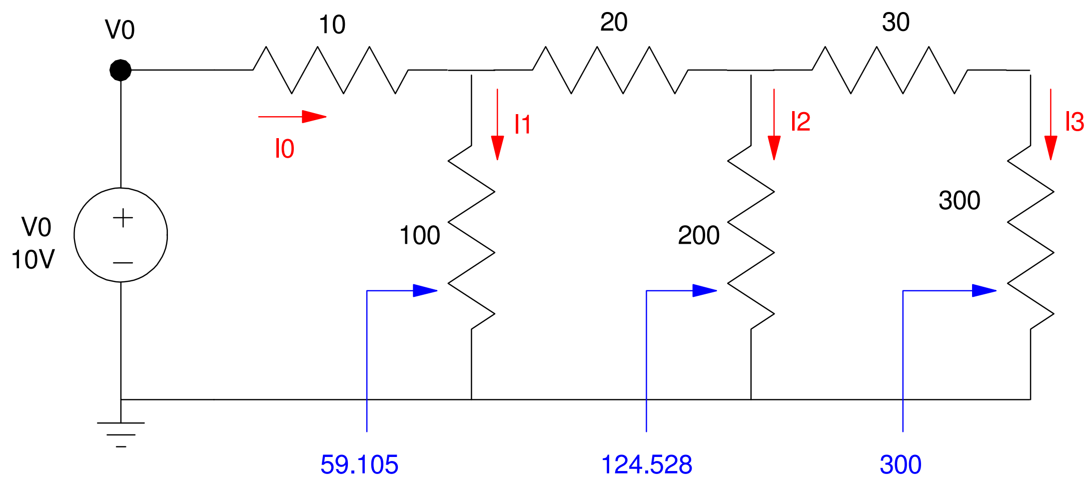
$$I_1 = 85.529mA$$

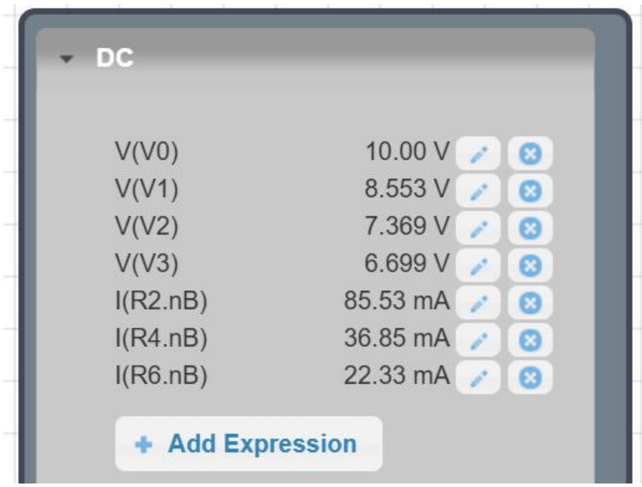
$$I_4 = I_0 - I_1 = 59.178mA$$

$$I_2 = \left(\frac{\frac{1}{200}}{\frac{1}{200} + \frac{1}{330}}\right) I_4 = 36.847mA$$

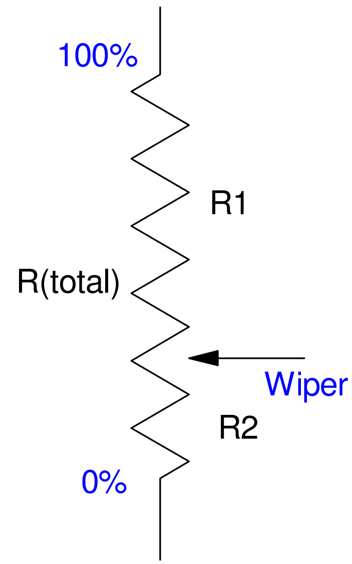
$$I_3 = \left(\frac{\frac{1}{330}}{\frac{1}{200} + \frac{1}{330}}\right) I_4 = 22.331mA$$

Same as CircuitLab





New Topic: Potentiometers



A potentiometer lets you adjust R1 and R2

- R(total) = R1 + R2 = fixed (value of pot)
- R2 varies from 0% to 100% of R(total)

Equations

$$0 \le \alpha \le 1$$

$$R_2 = \alpha \cdot R_{total}$$

$$R_1 = (1 - \alpha) \cdot R_{total}$$

Pot: Variable Resistor

One use of a potentiometer is as a variable resistor

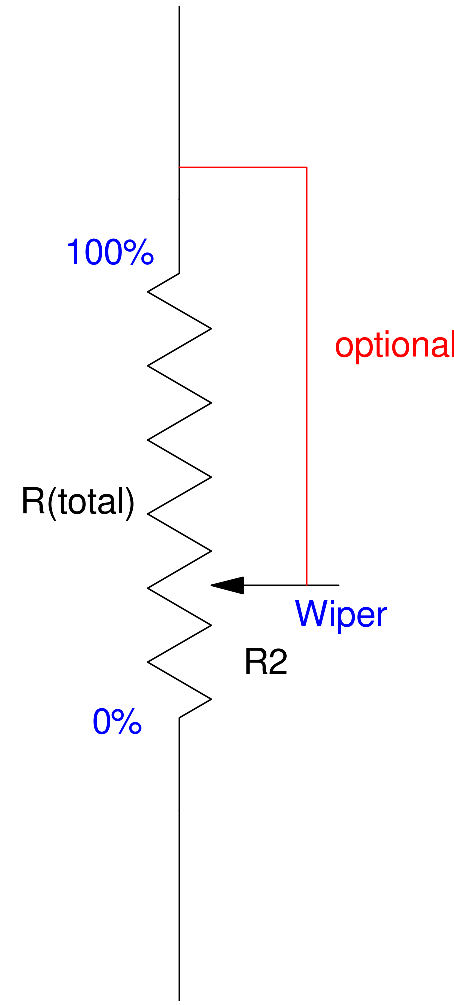
- Move the wiper
- R2 varies from 0% to 100% of R(total)

Note: The wiper can fail open

- Normal wear and tear
- You try to apply infinite current when R2 = 0

The optional wire sets the resistance when the wiper fails

- R2 = infinity (no wire)
- R2 = R(total) (with wire)



Pot: Variable Voltage

- Use as a voltage divider
- Output voltage varies from 0% to 100% of input voltage

Also serves as a variable gain

- 0% to 100% of the input signal
- Used in amplifiers

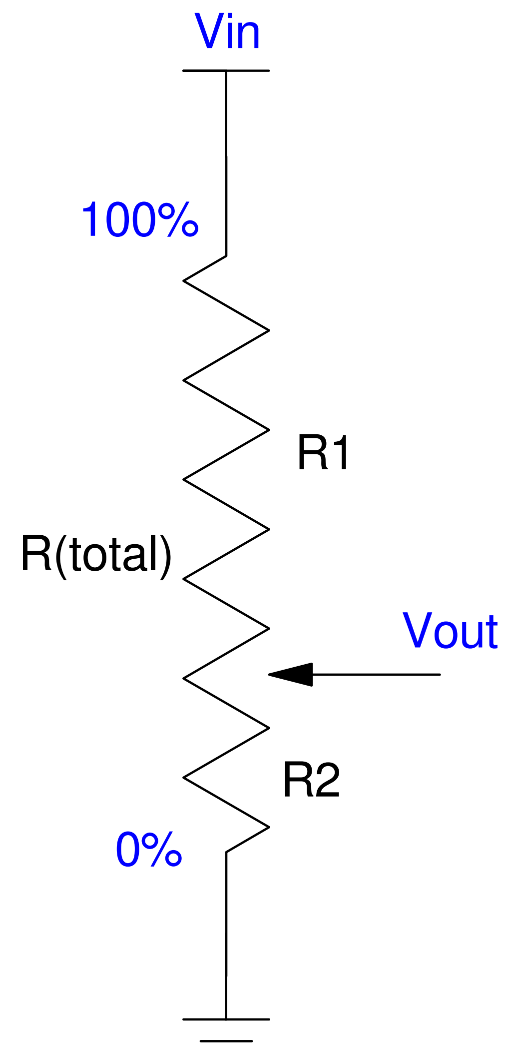
$$V_{out} = \left(\frac{R_2}{R_1+R_2}\right) V_{in}$$

$$R_2 = \alpha \cdot R_{total}$$

$$R_1 = (1-\alpha) \cdot R_{total}$$

Net

$$V_{out} = \alpha \cdot V_{in}$$

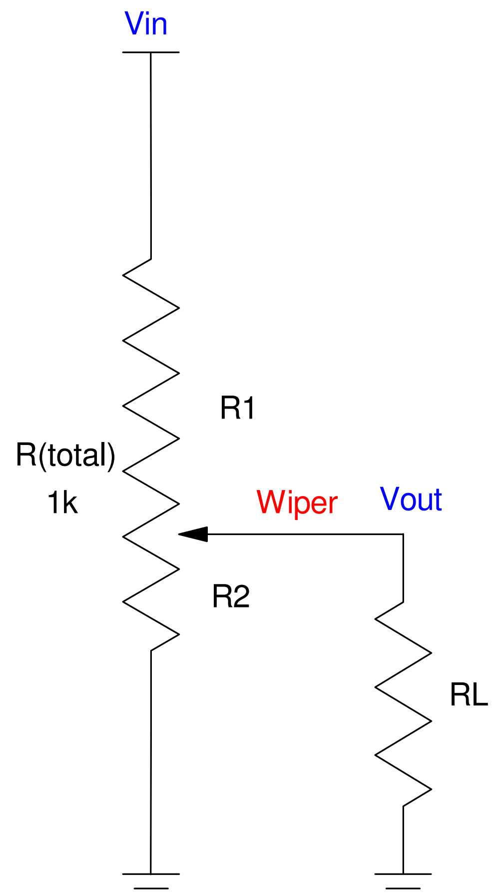


Loading

If you add a load to a potentiometer, the voltage changes

- The resistance to ground is R2 || RL
- The reduces the effective R2
- This reduces the voltage at Vout

This is called loading



Loading (cont'd)

If α is the wiper position

$$R_2 = \alpha \cdot R_{total}$$

$$R_1 = (1 - \alpha) R_{total}$$

The output voltage is

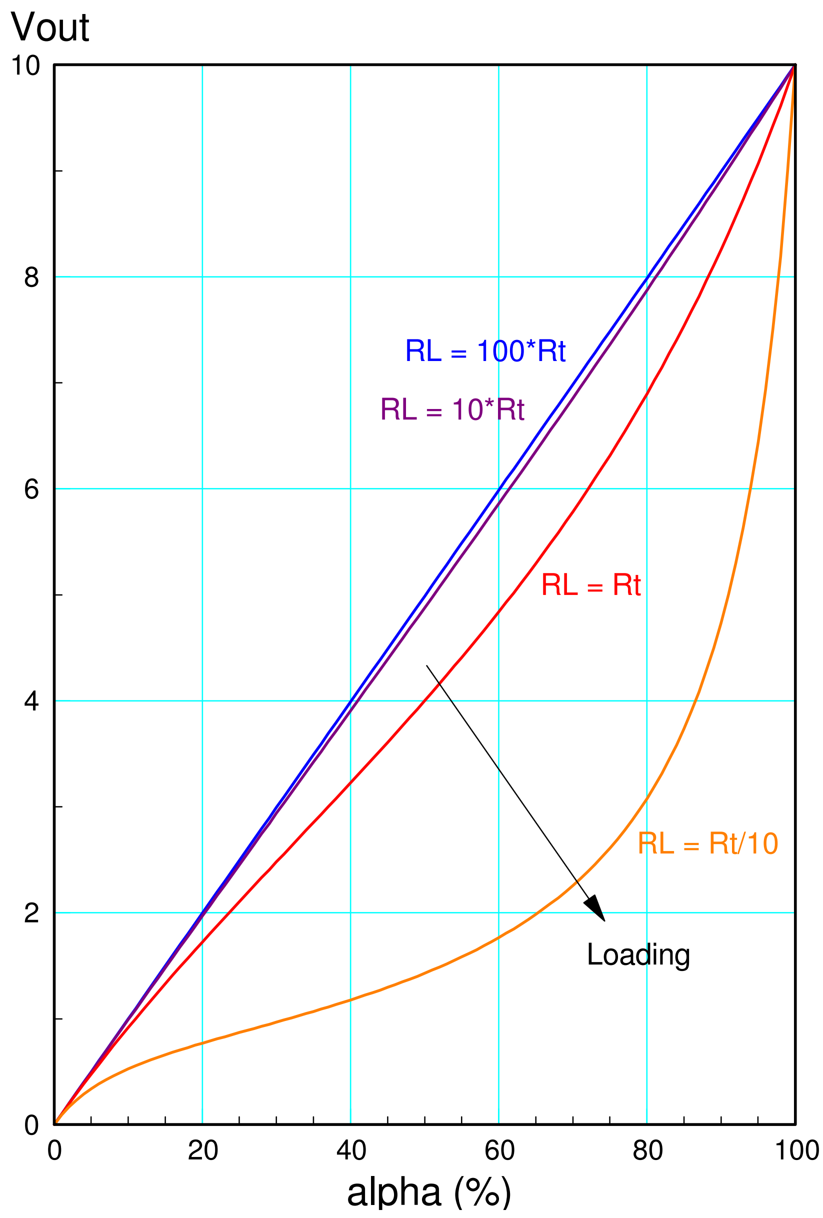
$$V_{out} = \left(\frac{R_2 || R_L}{R_2 || R_L + R_1} \right) V_{in}$$

When $R_L = \infty$

- Vout is proportional to α

When R_L decreases

- Vout droops



Matlab Code:

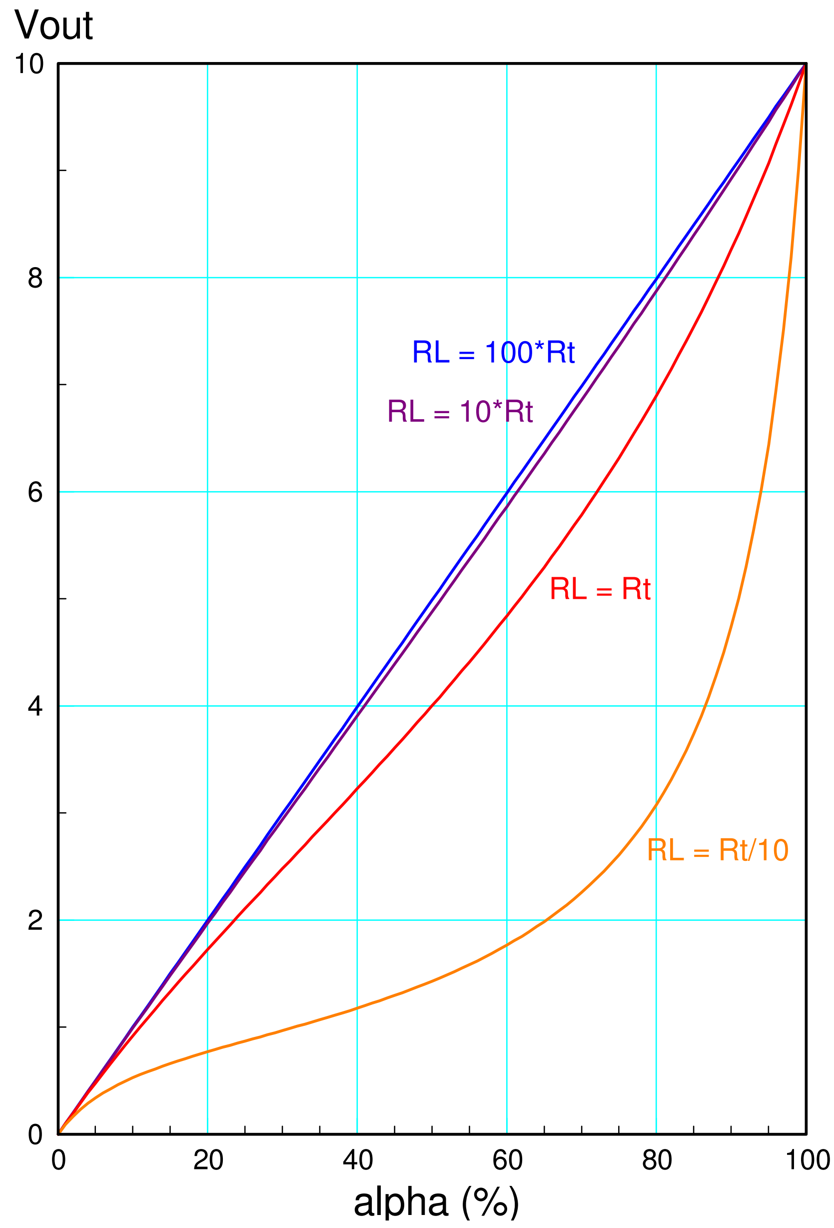
- Vin = 10V
- R(total) = 1k
- RL = 0.1 to 100 x R(total)

```
a = [0:0.01:1]';
Rt = 1e3;
RL = 1e2;

R1 = (1-a) * Rt;
R2 = a * Rt;
R2 = 1 ./ (1 ./ R2 + 1/RL);

Vo = R2 ./ (R1 + R2) * 10;

plot(a, Vo);
```



Potiometers & Efficiency

Note: Potentiometers are really inefficient ways to change votlages

- Only use when power isn't an issue
- Other methods exist (covered in Electronics)

Example: Convert 10V to 5V @ 1W

- R_L = 25 Ohms (1W)

Let R(pot) = 25 Ohms

$$\alpha = 0.6180 \qquad \textit{found using numerical methods}$$

$$R_1 = (1-\alpha)\cdot 25 = 9.549\Omega$$

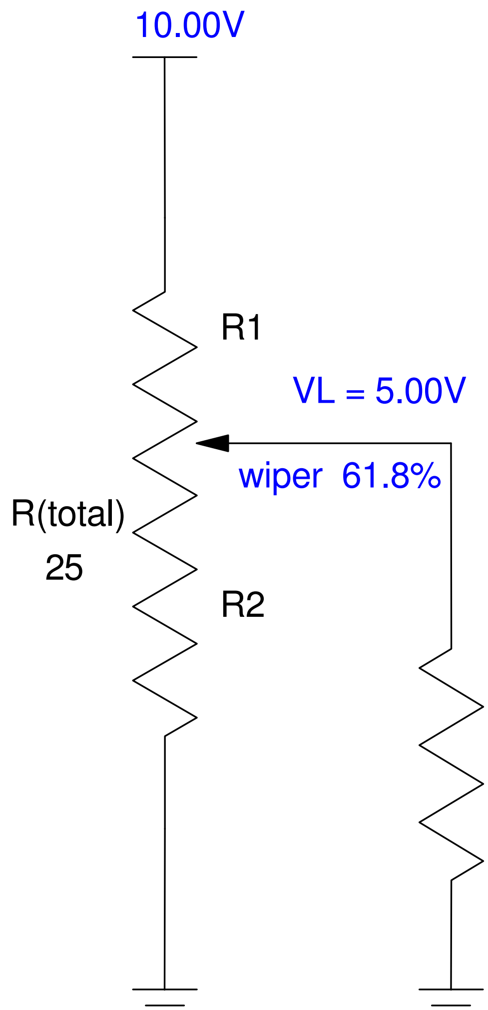
$$R_2 = \alpha \cdot 25 = 15.451\Omega$$

$$P_L = \frac{V^2}{R} = \frac{(5V)^2}{25\Omega} = 1W$$

$$P_1 = \frac{V^2}{R} = \frac{(10V-5V)^2}{9.549\Omega} = 2.6181W$$

$$P_2 = \frac{V^2}{R} = \frac{(5V)^2}{15.451\Omega} = 1.618W$$

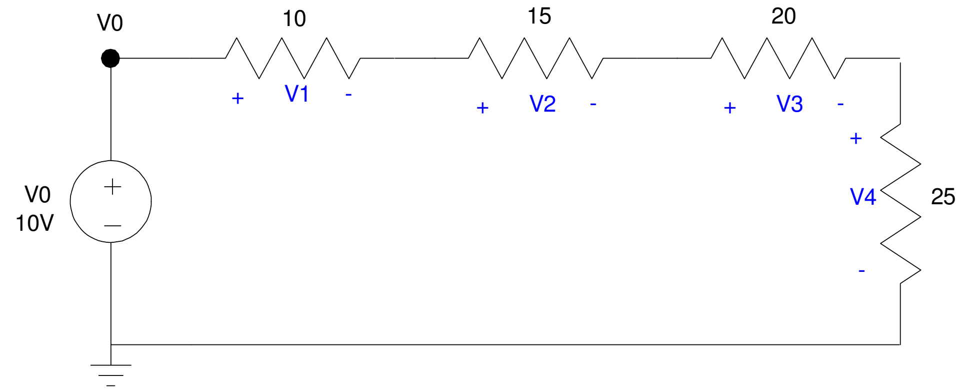
$$eff = \frac{P_L}{P_1+P_2+P_L} = 19.10\%$$



Summary

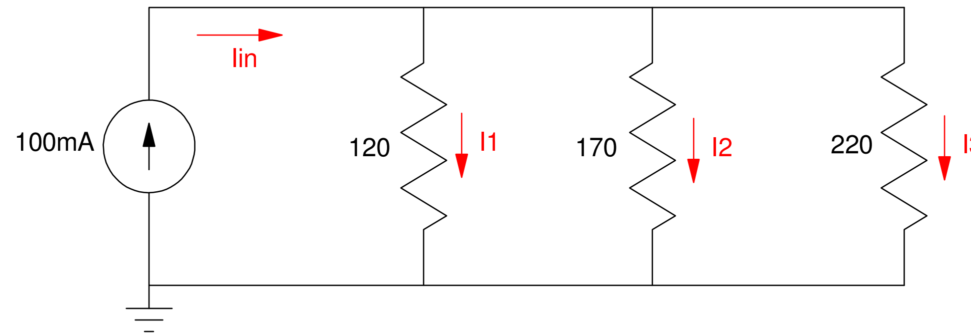
Voltage division is for resistors in series

$$V_x = \left(\frac{R_x}{R_1+R_2+R_2+R_4}\right) V_{in}$$



Current division is for resistors in parallel

$$I_x = \left(\frac{\frac{1}{R_x}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right) I_{in}$$



Potentiometers provide

- A variable voltage
- A variable resistance