Voltage Nodes

ECE 211 Circuits I

Lecture #5

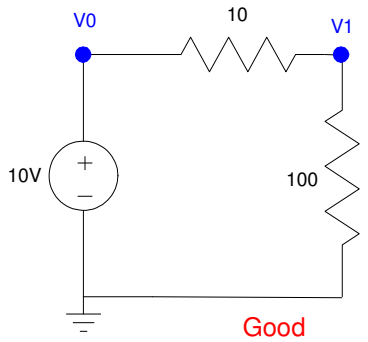
Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Background

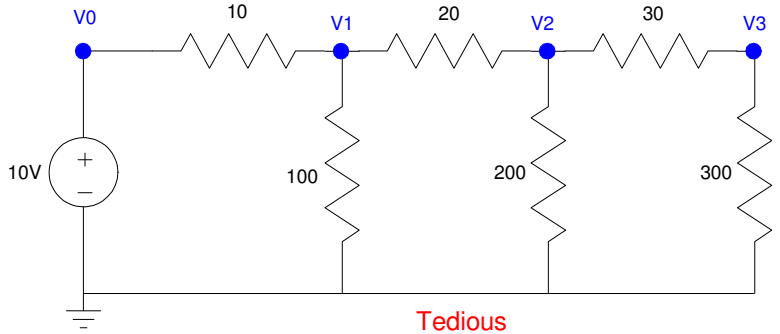
In the previous lecture, votlage division was used to find the voltages in a circuit

- For some circuits, this works really well
- For other, it's really tedious
- For still others, voltage division just doesn't work

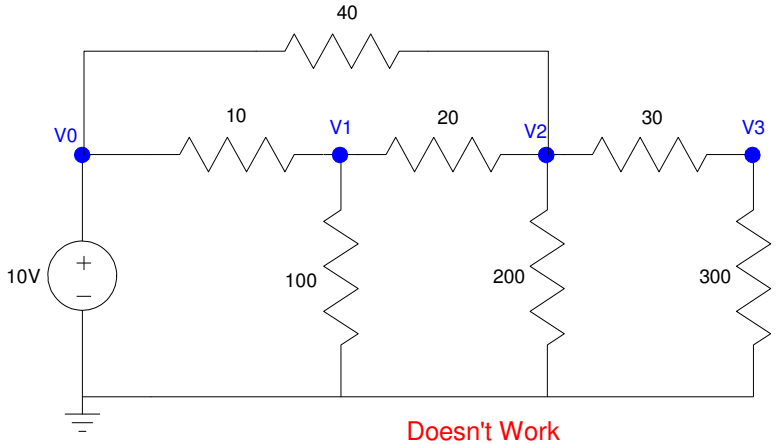
We need a better tool



Good



Tedious



Doesn't Work

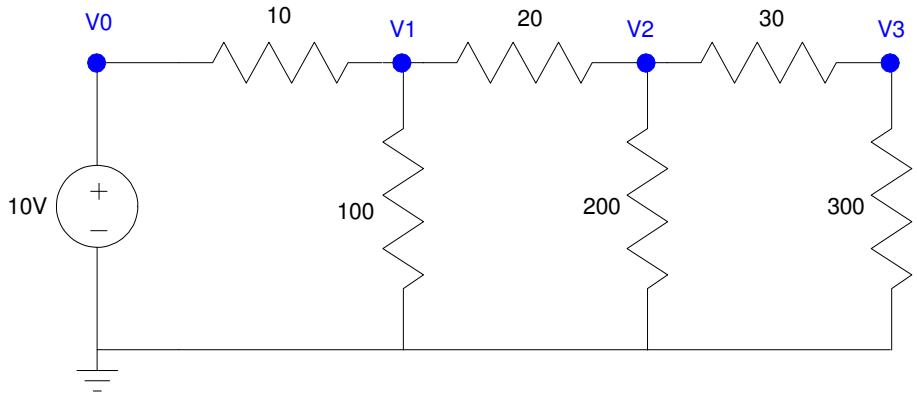
Main Tools for Circuit Analysis

To solve a circuit, we primarilly use three tools:

- Voltage Nodes (Kirchoff's current law)
- Current Loops (Kirchoff's voltage law)
- Thevenin Equivalents (stay tuned for these...)

The goal of the first two is to obtain N equations so solve for N unknowns.

- Topic for Math 129: Linear Algebra
- How to solve N equations for N unknowns



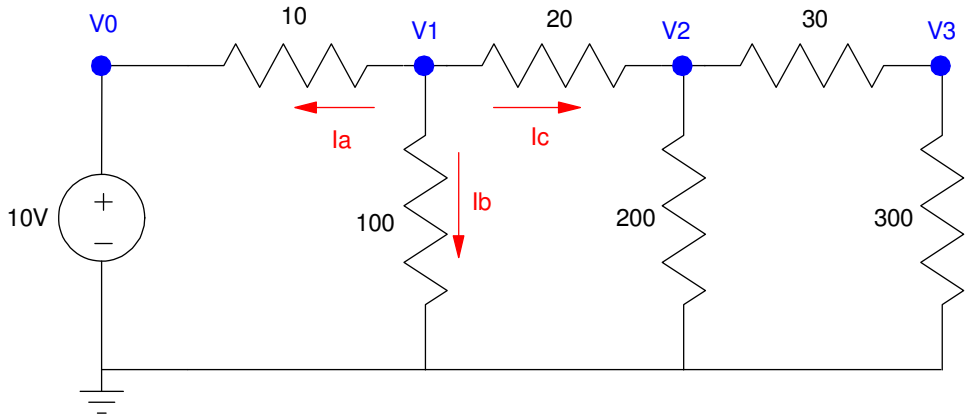
a0 V0 + a1 V1 + a2 V2 + a3 V3 = k1
b0 V0 + b1 V1 + b2 V2 + b3 V3 = k2
c0 V0 + c1 V1 + c2 V2 + c3 V3 = k3
d0 V0 + d1 V1 + d2 V2 + d3 V3 = k4

Voltage Nodes

- The voltages at each node are such that the current from each node sums to zero
- This holds at every node

Idea:

- I know how to solve N equations for N unknowns
  - Math 129 Linear Algebra
- If I can redude the probem to this form, I can solve the problem



Ia + Ib + Ic = 0

Note:

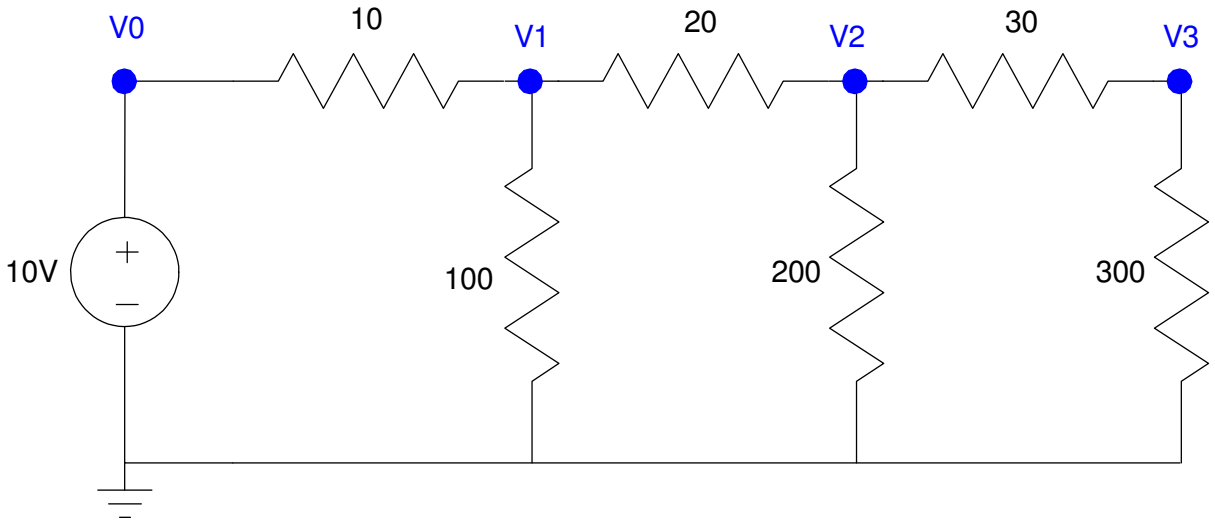
- This is a common theme throughout ECE
- What's important is getting the equations right
  - Matlab can solve 50 equations for 50 unknowns just as easily as 2 equations for 2 unkowns
  - Focus on getting the equations right.

Example #1

- Find the voltages V0, V1, V2, and V3
- Solved previously

  - CircuitLab

  - Voltage division



Result:

- V0 = 10.000V
- V1 = 8.553V
- V2 = 7.369V
- V3 = 6.699V

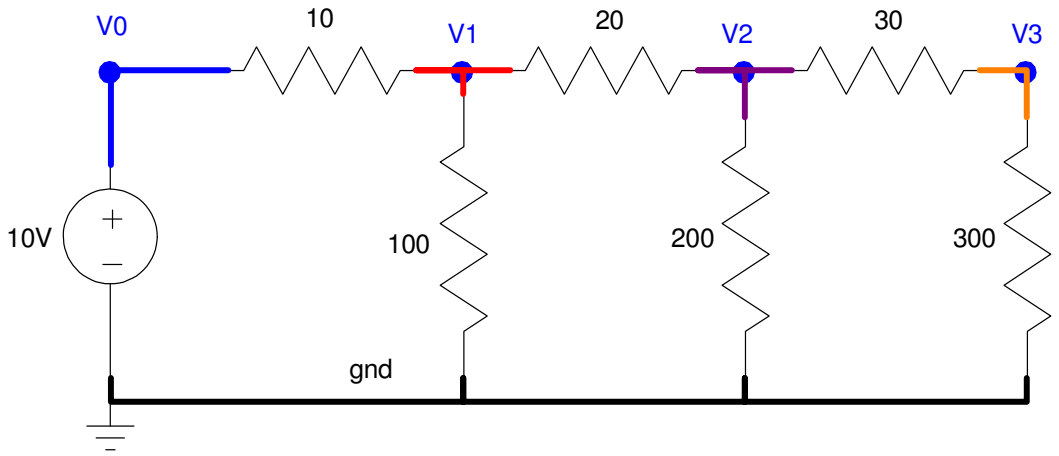
Now solve using Voltage Nodes

Step #1

- Define circuit ground
- Label each voltage node
- Count the number of voltage nodes

The number of voltage nodes is the number of equations you need

- 4 voltage nodes
- We need 4 equations



Step #2

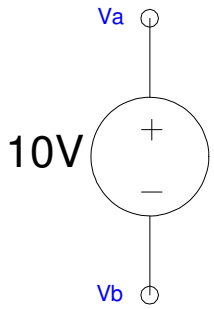
Start writing equations

Start with the easy equations

- Voltage sources

These define the voltage across two nodes

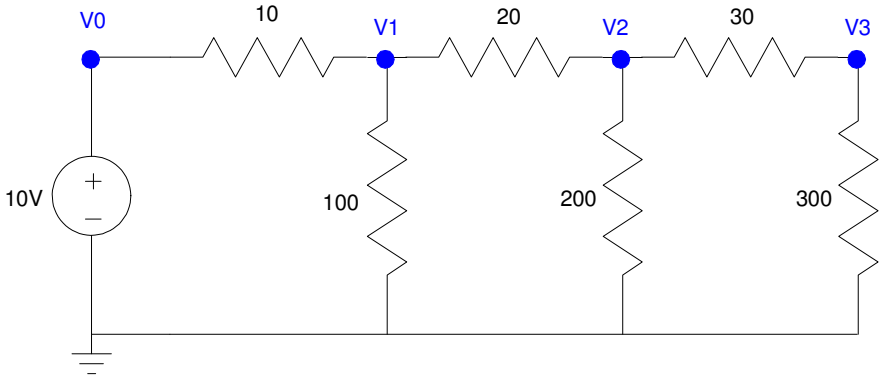
$$V_a - V_b = 10V$$



For our circuit

- Equation #1 of 4

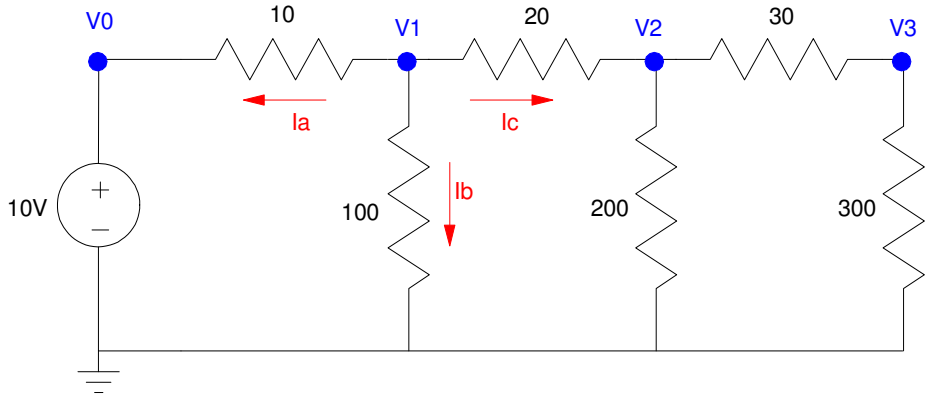
$$V_0 = 10V$$



Step #3

Write the voltage node equations

- Need 3 more equations
- At each voltage node, the currents flowing *from* the node must sum to zero



Node V1:

- Equation #2 or 4

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

Note:

- All the V1 terms are positive
- All the other terms are negative
- All units match (ohms + ohms + ohms)
- Quick way to check for sign errors

Node V2:

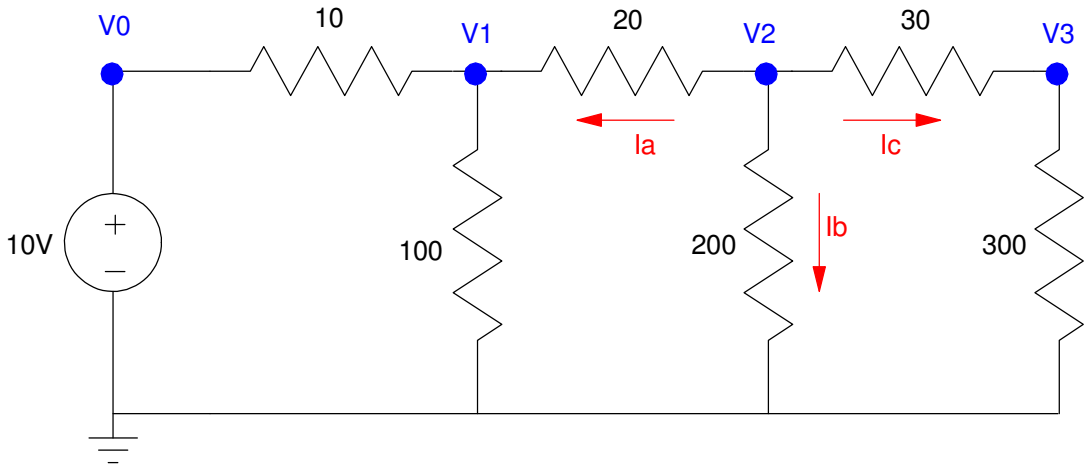
- Equation #3 of 4

$$I_a + I_b + I_c = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Note:

- All the V2 terms are positive
- All the other terms are negative
- All units match (ohms + ohms + ohms)



Node V3

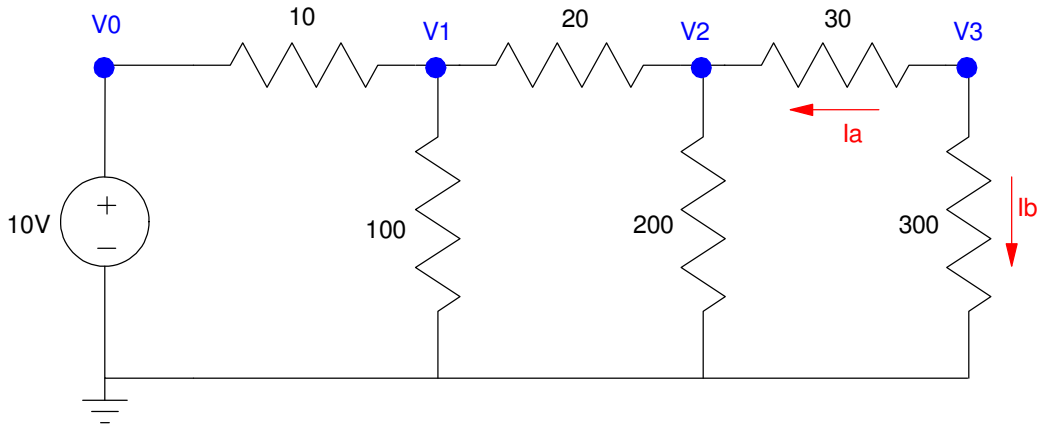
- Equation #4 of 4

$$I_a + I_b = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$

Note:

- All the V3 terms are positive
- All the other terms are negative
- All units match (ohms + ohms)



Net Result

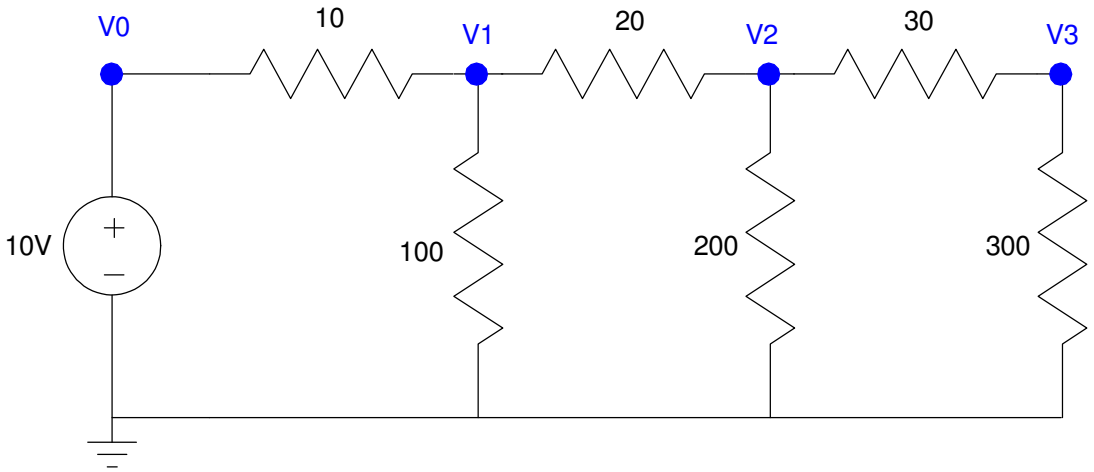
4 equations for 4 unknowns

$$V_0 = 10$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



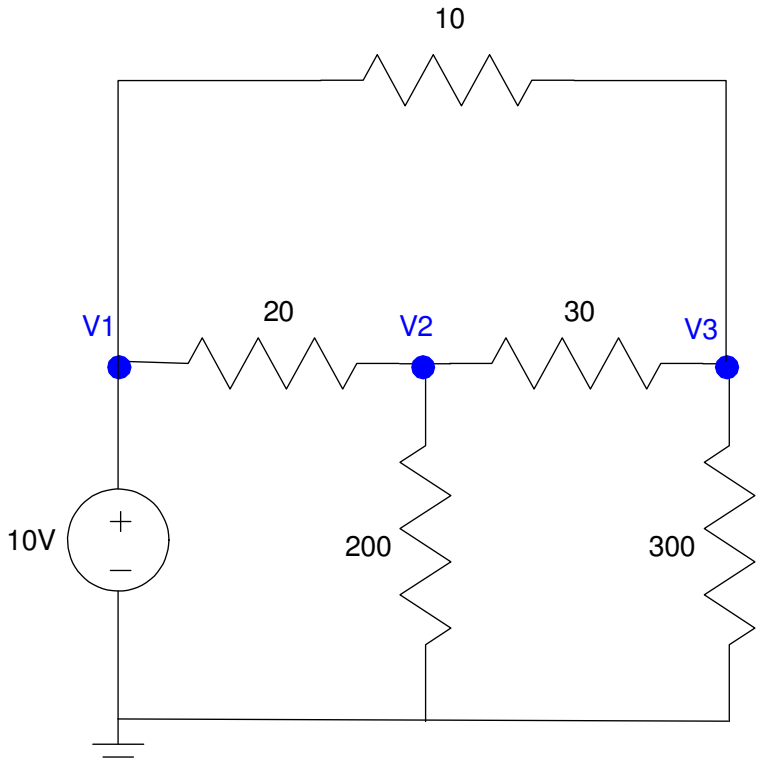
Note:

- This is typically where you stop on tests
- Solving requires Matlab (homework only)

Practice Problem

Write the voltage node equations for the following circuit

- 3 equations for the 3 unknown voltages



Step #4: Solve

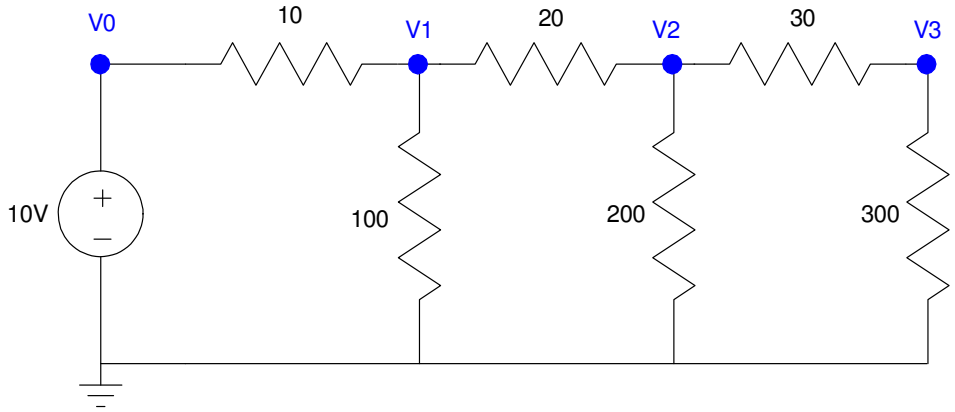
Previous equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_2}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{200}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



Group terms

$$V_0 = 10$$

$$\left(\frac{-1}{10}\right) V_0 + \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{20}\right) V_1 + \left(\frac{-1}{20}\right) V_2 = 0$$

$$\left(\frac{-1}{20}\right) V_1 + \left(\frac{1}{20} + \frac{1}{200} + \frac{1}{30}\right) V_2 + \left(\frac{-1}{30}\right) V_3 = 0$$

$$\left(\frac{-1}{30}\right) V_2 + \left(\frac{1}{30} + \frac{1}{300}\right) V_3 = 0$$

Place in matrix form

$$V_0 = 10$$

$$\left(\frac{-1}{10}\right) V_0 + \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{20}\right) V_1 + \left(\frac{-1}{20}\right) V_2 = 0$$

$$\left(\frac{-1}{20}\right) V_1 + \left(\frac{1}{20} + \frac{1}{200} + \frac{1}{30}\right) V_2 + \left(\frac{-1}{30}\right) V_3 = 0$$

$$\left(\frac{-1}{30}\right) V_2 + \left(\frac{1}{30} + \frac{1}{300}\right) V_3 = 0$$

becomes

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{20}\right) & \left(\frac{-1}{20}\right) & 0 \\ 0 & \left(\frac{-1}{20}\right) & \left(\frac{1}{20} + \frac{1}{200} + \frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30} + \frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
a1 = [1,0,0,0];
a2 = [-1/10, 1/10+1/100+1/20, -1/20, 0];
a3 = [0, -1/20, 1/20+1/200+1/30, -1/30];
a4 = [0, 0, -1/30, 1/30+1/300];
A = [a1;a2;a3;a4]

    1.0000         0         0         0
   -0.1000    0.1600   -0.0500         0
         0   -0.0500    0.0883   -0.0333
         0         0   -0.0333    0.0367

B = [10 ; 0 ; 0 ; 0]

    10
     0
     0
     0

V = inv(A) * B
```

V0 10.0000
V1 8.5529
V2 7.3694
V3 6.6994

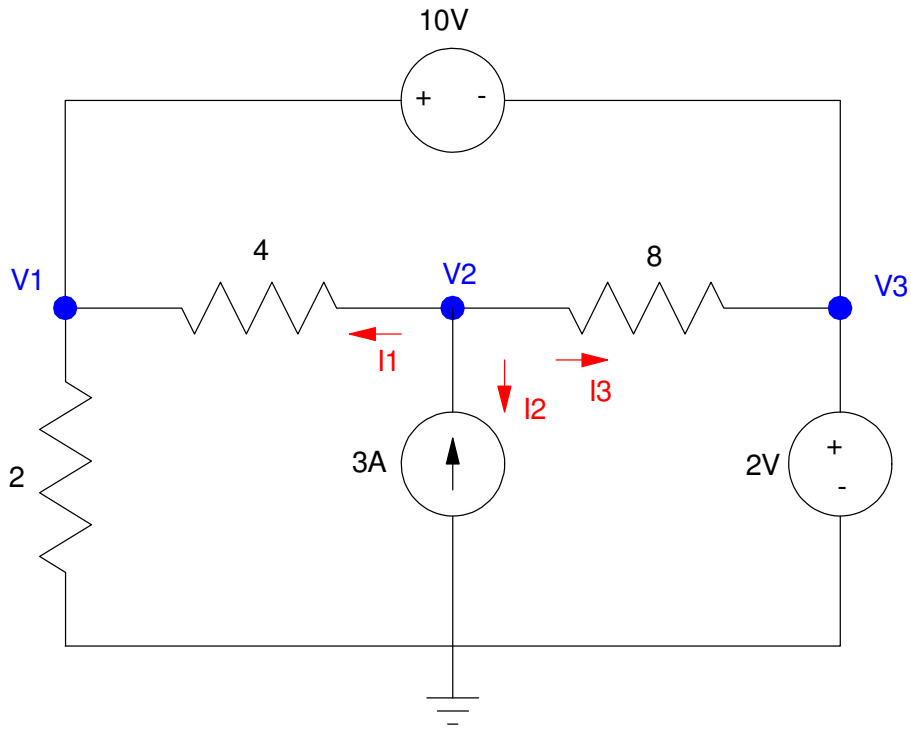
Current Sources

Current sources aren't an issue with voltage nodes

- They just define the current flowing

Example: Find {V1, V2, V3}

- 3 unknowns voltages
- Write 3 equations for 3 unknowns



Start with the easy ones

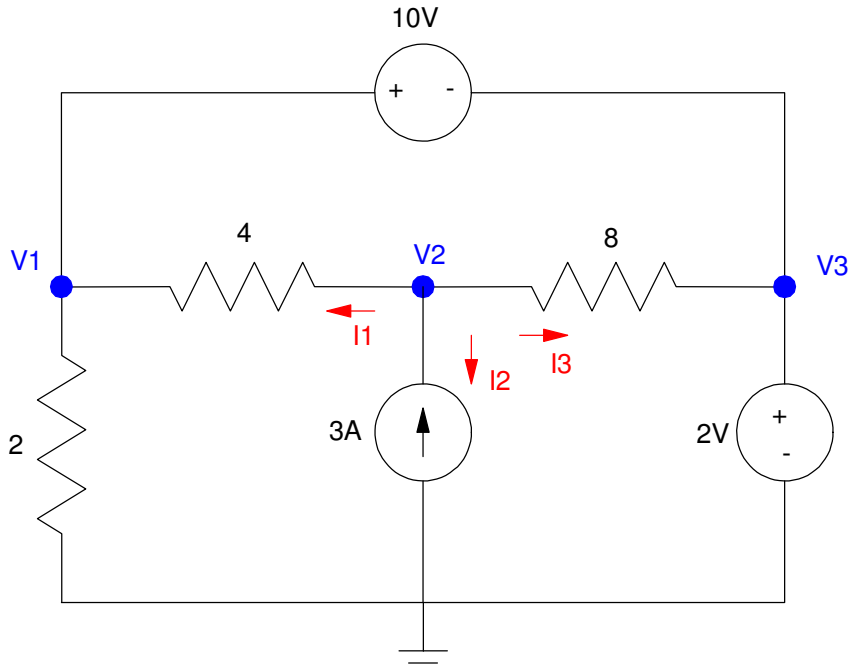
$$V_3 - 0 = 2$$

$$V_1 - V_3 = 10$$

Now we need one more equation.
Write the voltage node equation at V2

$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_2 - V_1}{4}\right) + (-3) + \left(\frac{V_2 - V_3}{8}\right) = 0$$



Group terms

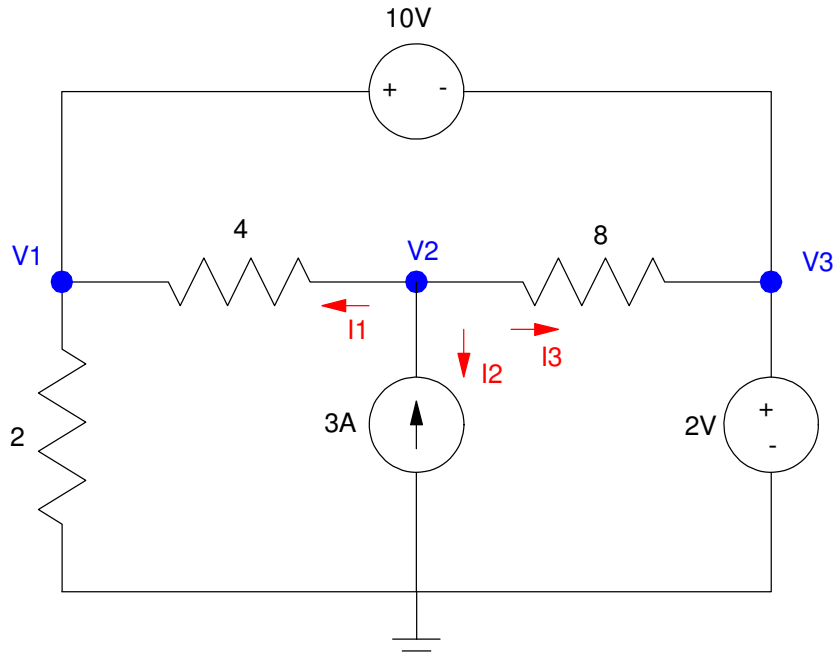
$$V_3 = 2$$

$$V_1 - V_3 = 10$$

$$-\left(\frac{1}{4}\right)V_1 + \left(\frac{1}{4}+\frac{1}{8}\right)V_2 - \left(\frac{1}{8}\right)V_3 = 3$$

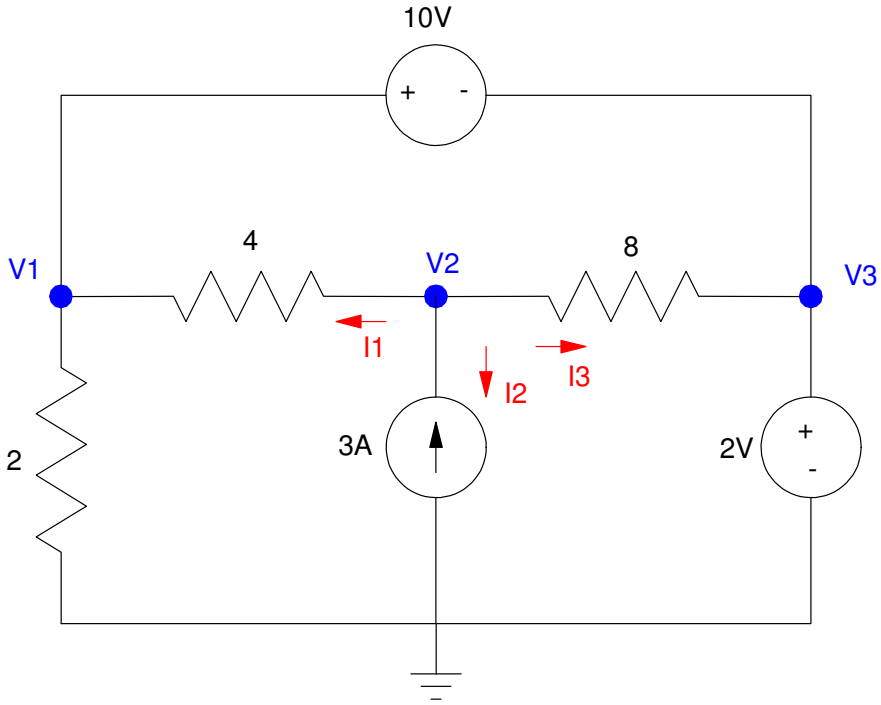
Place in matrix form:

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ -0.25 & 0.375 & -0.125 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 10 \\ 3 \end{bmatrix}$$

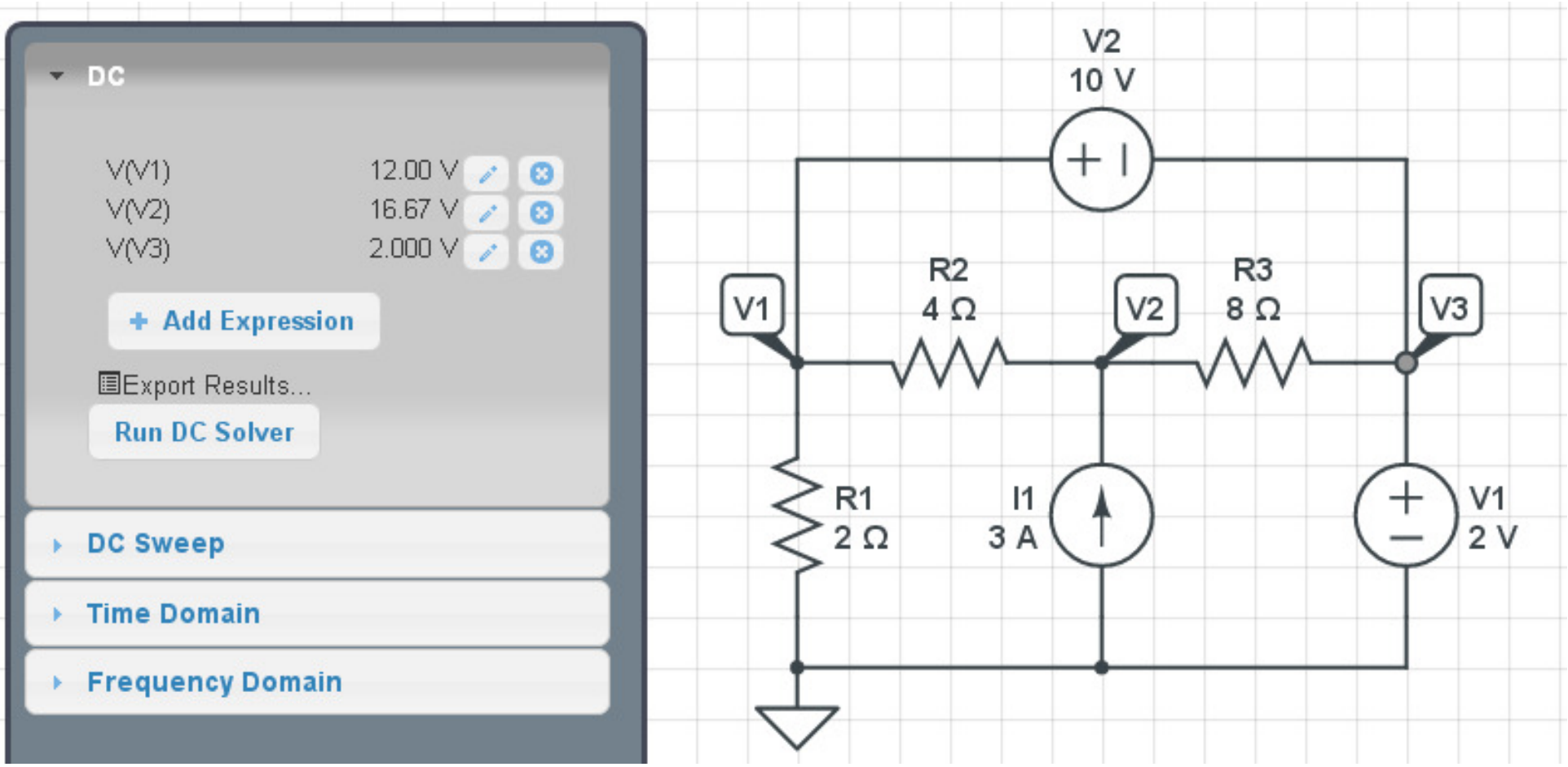


Solving in Matlab:

```
A = [0,0,1 ; 1,0,-1 ; -0.25,0.375,-0.125]

         0         0    1.0000
    1.0000         0   -1.0000
   -0.2500    0.3750   -0.1250

B = [2;10;3]

     2
    10
     3

V = inv(A)*B

V1   12.0000
V2   16.6667
V3    2.0000
```



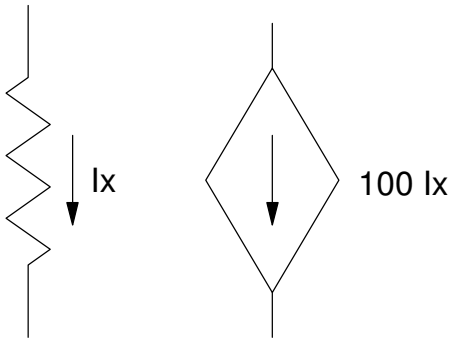
Check with CircuitLab



Dependent Sources

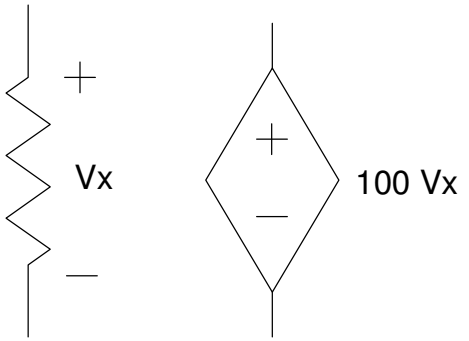
Dependent sources are

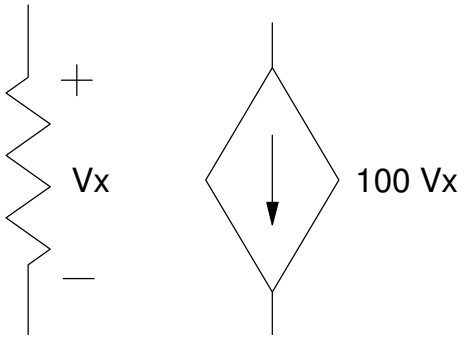
- Sources (voltage of current)
- Which depend upon another variable



Example:

- Current-controlled current source
  - BJT transistor
- Voltage-controlled voltage source
  - Amplifier
- Voltage-controlled current source
  - MOSFET





Devices which do this are covered in Electronics

- For this class, just treat these as sources

Voltage Node Equations with Dependent Sources

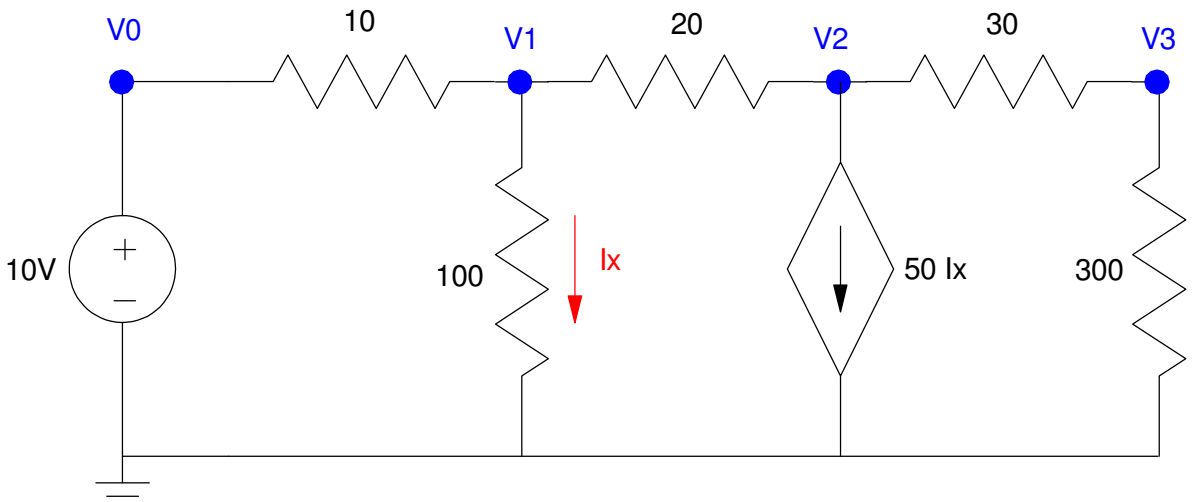
Each dependent source adds one more equation

- Define Vx or Ix

Example: Write the voltage node equations

- 4 voltages
- 1 dependent source
- = 5 unknowns

Write 5 equations for 5 unknowns



5 equations for 5 unknowns

10V Source:

$$V_0 = 10$$

Ix:

$$I_x = \left(\frac{V_1}{100}\right)$$

Node V1:

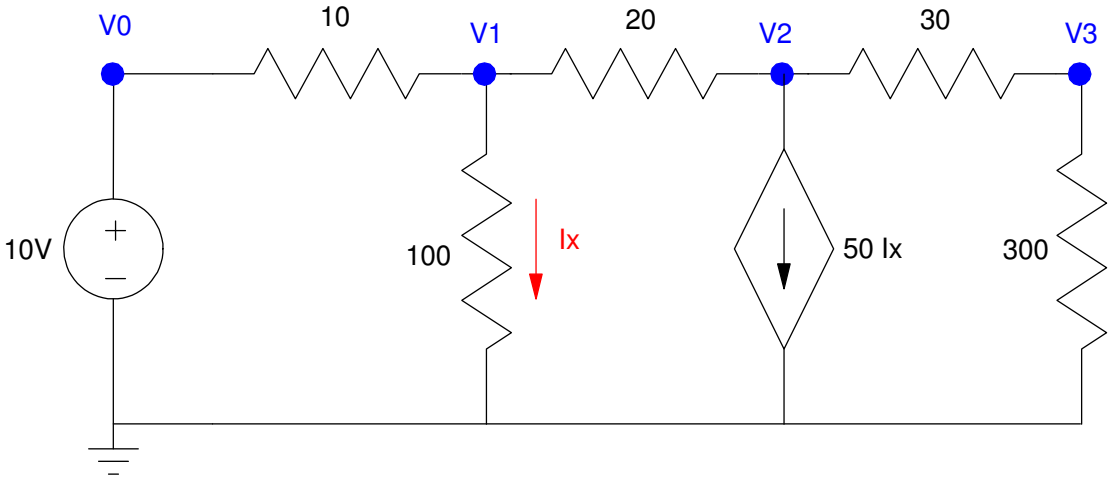
$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

Node V2:

$$\left(\frac{V_2 - V_1}{20}\right) + 50I_x + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Node V3:

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



Another Variation

Independent Source

$$V_0 = 10$$

Vx

$$V_x = V_1 - V_2$$

Node V1

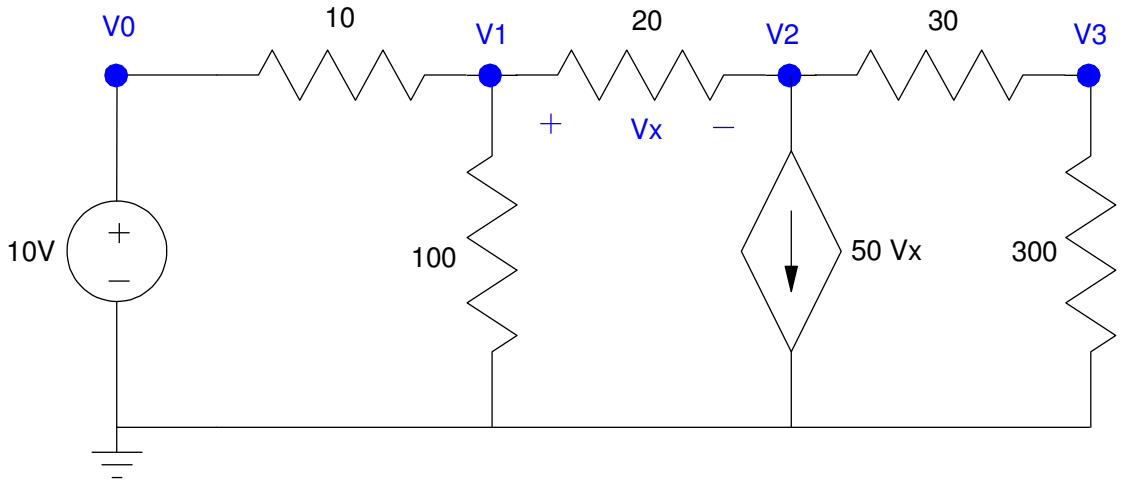
$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{100}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

Node V2

$$\left(\frac{V_2 - V_1}{20}\right) + 50V_x + \left(\frac{V_2 - V_3}{30}\right) = 0$$

Node V3

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{300}\right) = 0$$



Summary

Voltage Nodes is one way to compute the voltages in a circuit

- Define N voltage nodes
- Write N equations for N unknowns
- Solve using Matlab

This is based on conservation of current

- The sum of the currents from a node must be zero
- Ia + Ib + Ic = 0