Super-Nodes

ECE 211 Circuits I

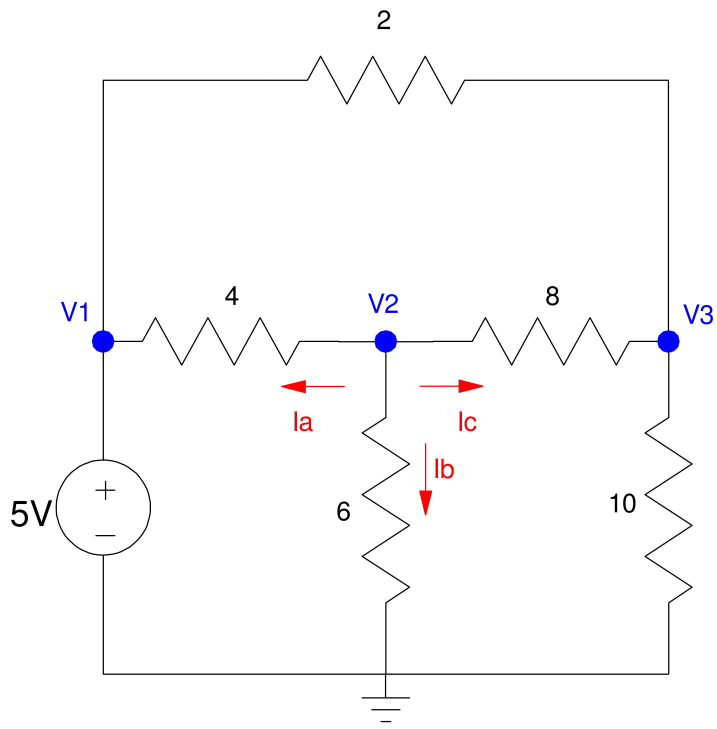
Lecture #6

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Background: Voltage Nodes

The previous lecture looked at finding the voltage in a circuit using Voltage Nodes

- Write N equations for N unknowns
- The sum of the currents from a given node must be zero



Voltage source to ground isn't a problem

- Defines the voltage at a given node

$$V_1 = 5$$

Resistors aren't a problem

- Sum the currents to zero from that node

$$\left(\frac{V_2 - V_1}{4}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_2 - V_3}{8}\right) = 0$$

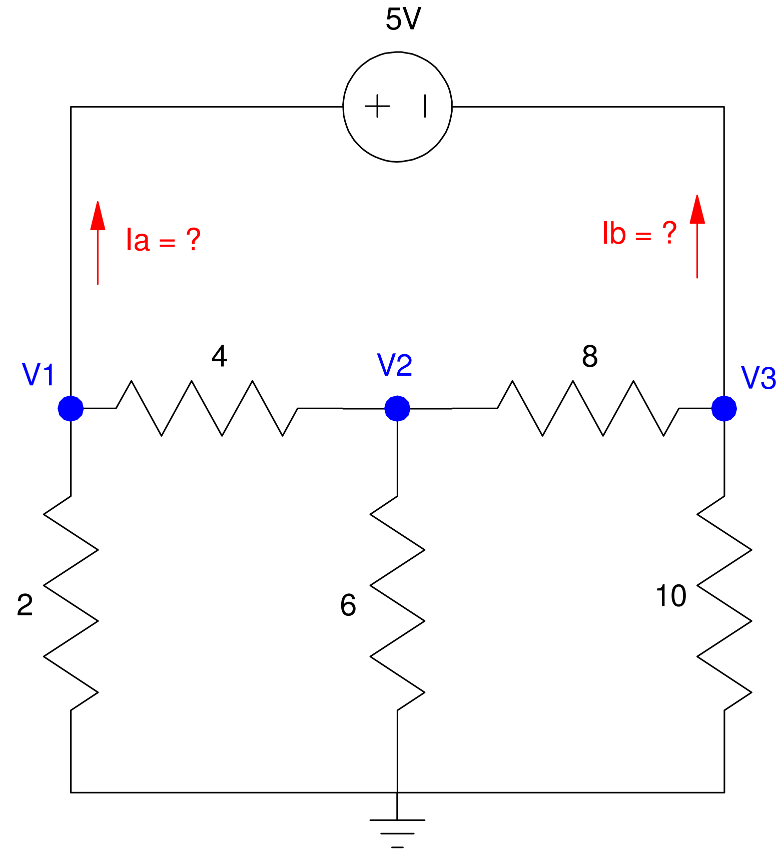
$$\left(\frac{V_3 - V_1}{2}\right) + \left(\frac{V_3 - V_2}{8}\right) + \left(\frac{V_3}{10}\right) = 0$$

This results in 3 equations for 3 unknowns.

Problem with Voltage Nodes

What do you do with voltage sources between nodes?

- The current in and out are unknown
- All you know is the voltage across the source



Equation #1

$$V_1 - V_3 = 5$$

Equation #2 (node V2)

$$\left(\frac{V_2 - V_1}{4}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_2 - V_3}{8}\right) = 0$$

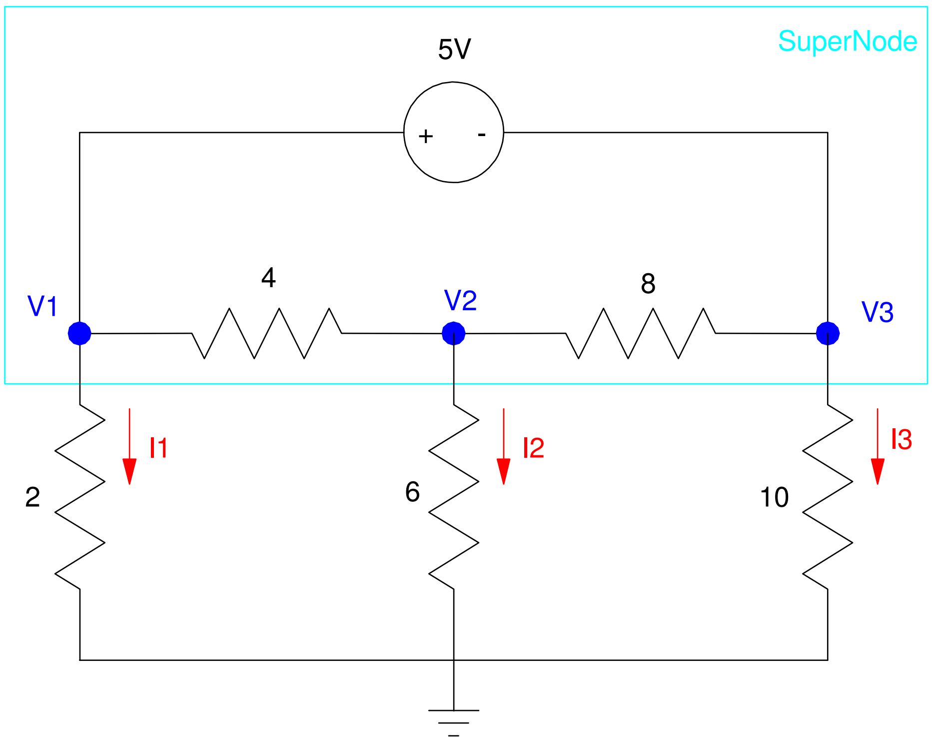
Equation #3

- Problem
- Ia is unknown
  - node V1 doesn't work
- Ib is unknown
  - node V3 doesn't work

Solution: Super-Nodes

Super-Nodes:

- A closed path the encloses 2+ nodes
- The current coming out of any closed path must sum to zero
  - Conservation of current



Equation #3 (super-node)

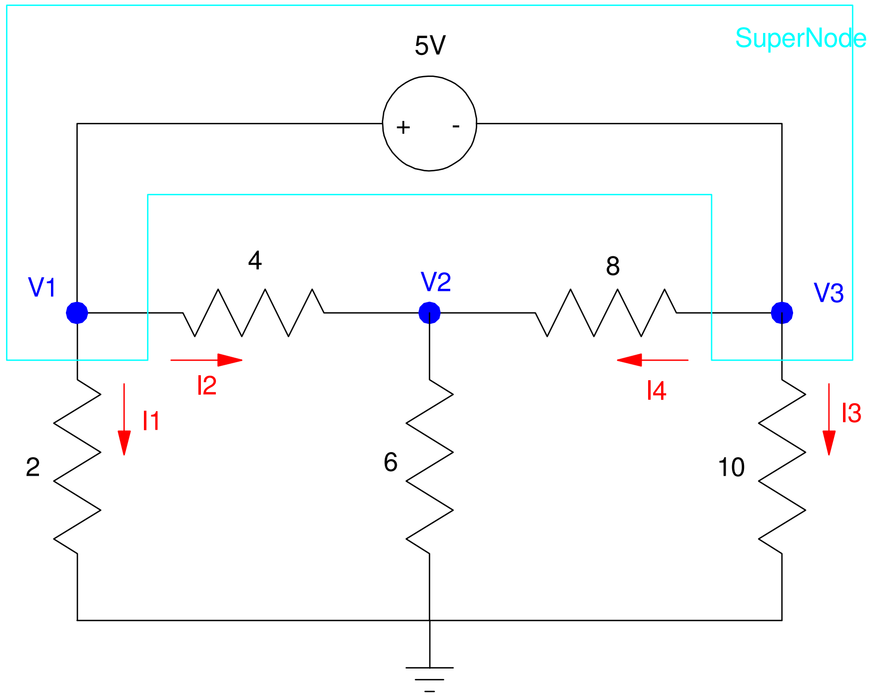
$$I_1 + I_2 + I_3 = 0$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 10$$

Super-Node (take 2):

Super-nodes are not unique

- Any closed surface works
- As long as you know the currents in each path



Equation #3

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_1 - V_2}{4}\right) + \left(\frac{V_3 - V_2}{8}\right) + \left(\frac{V_3}{10}\right) = 0$$

note: With super-nodes,

- The signs for the voltages for the nodes enclosed are positive
- The signs for other nodes are negative

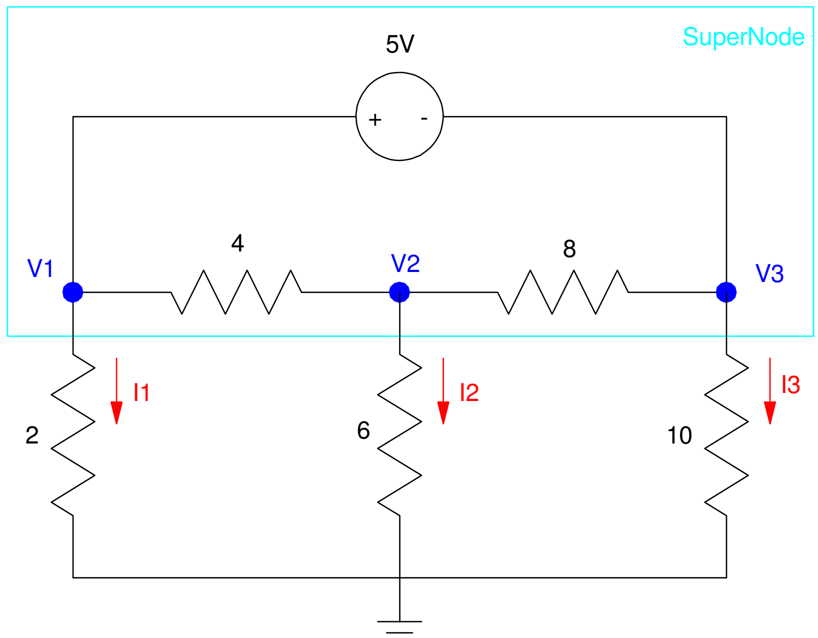
Solve:

Group terms:

$$V_1 - V_3 = 5$$

$$\left(\frac{-1}{4}\right)V_1 + \left(\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right)V_2 + \left(\frac{-1}{8}\right)V_3 = 0$$

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_2}{6}\right) + \left(\frac{V_3}{10}\right) = 0$$



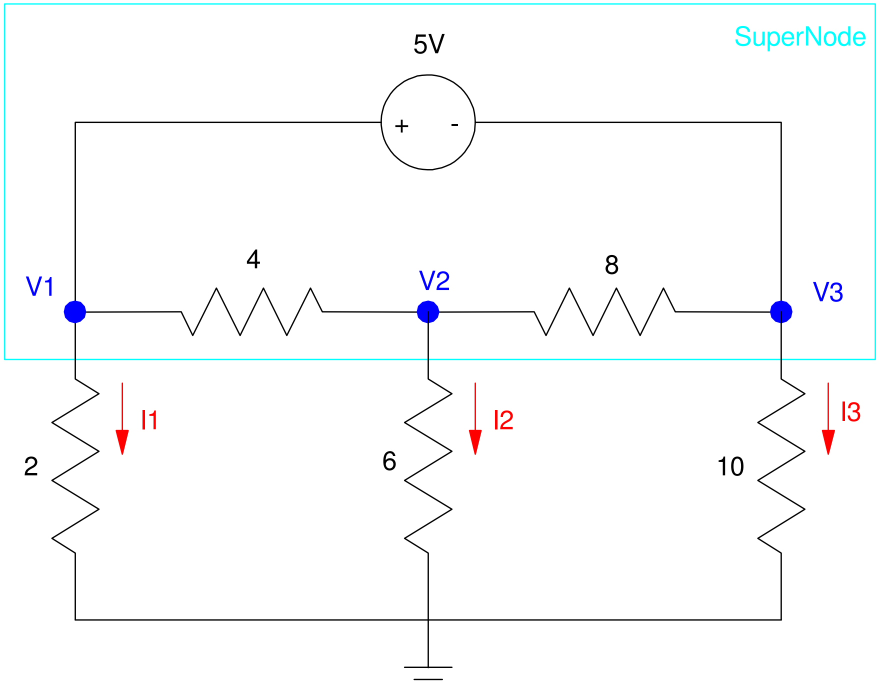
Place in matrix form

$$\begin{bmatrix} 1 & 0 & -1 \\ \left(\frac{-1}{4}\right) & \left(\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right) & \left(\frac{-1}{8}\right) \\ \left(\frac{1}{2}\right) & \left(\frac{1}{6}\right) & \left(\frac{1}{10}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

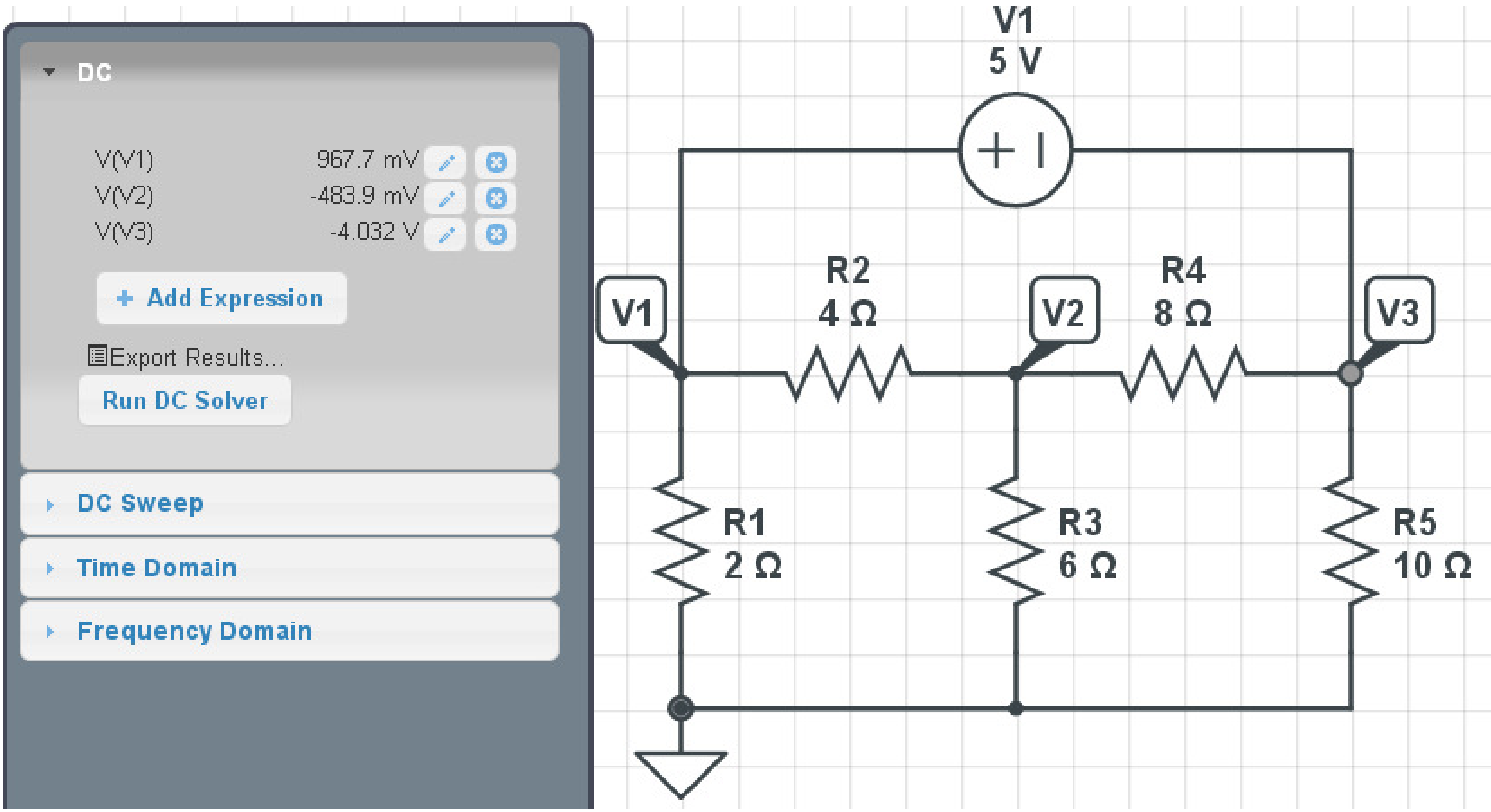
Solve in Matlab

```
A = [1,0,-1 ; -1/4,1/4+1/6+1/8,-1/8 ; 1/2,1/6,1/10]

    1.0000         0   -1.0000
   -0.2500    0.5417   -0.1250
    0.5000    0.1667    0.1000

B = [5;0;0]

     5
     0
     0

V = inv(A)*B
```

V1 0.9677
V2 -0.4838
V3 -4.0323



Same as CircuitLab



Practice Problem

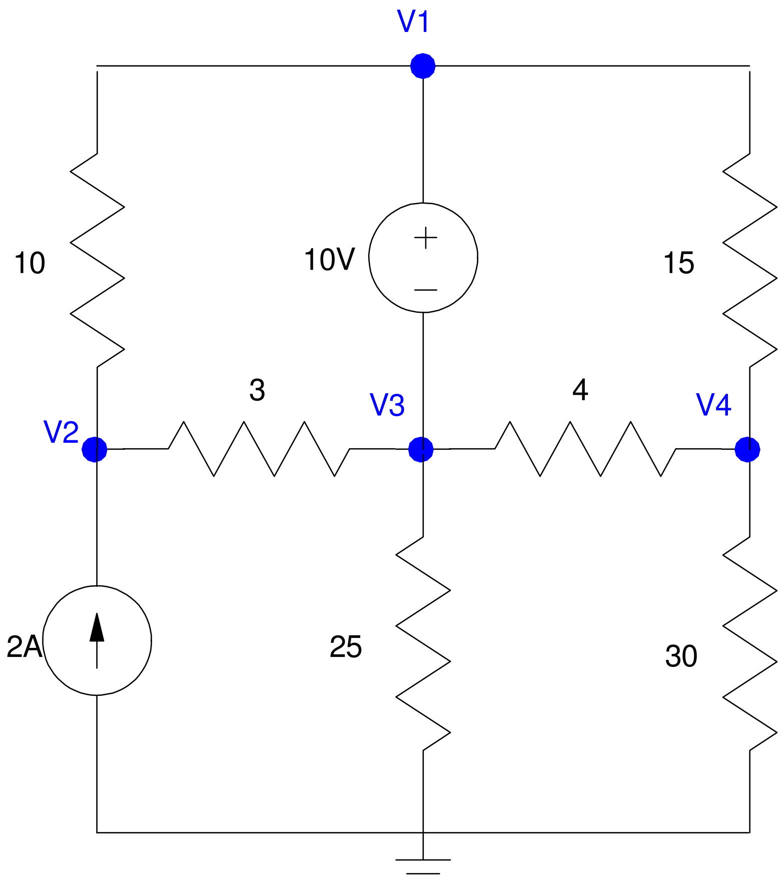
Write a 4th equation to find {V1, V2, V3, and V4}

(1) $V_1 - V_3 = 10$

(2) $-2 + \left(\frac{V_2 - V_1}{10}\right) + \left(\frac{V_2 - V_3}{3}\right) = 0$

(3) $\left(\frac{V_4 - V_1}{15}\right) + \left(\frac{V_4 - V_3}{4}\right) + \left(\frac{V_4}{30}\right) = 0$

(4) ?



When is a Super-Node Equation Valid?

- Some super-node equations are valid
- Some are not
- How do you tell if your equation is valid?

Method #1: All circuit elements are included

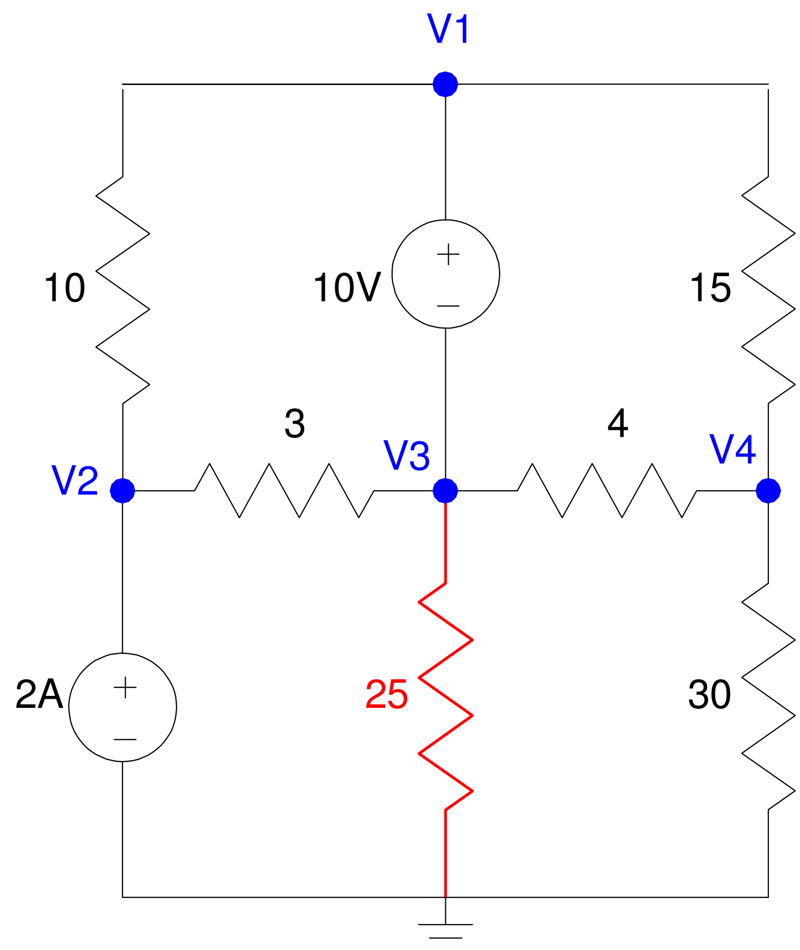
(1) $V_1 - V_3 = 10$

(2) $-2 + \left(\frac{V_2 - V_1}{10}\right) + \left(\frac{V_2 - V_3}{3}\right) = 0$

(3) $\left(\frac{V_4 - V_1}{15}\right) + \left(\frac{V_4 - V_3}{4}\right) + \left(\frac{V_4}{30}\right) = 0$

Note that the 25 Ohm resistor doesn't show up in any equation

- The super-node needs to include this resistor



Method #2: Matlab

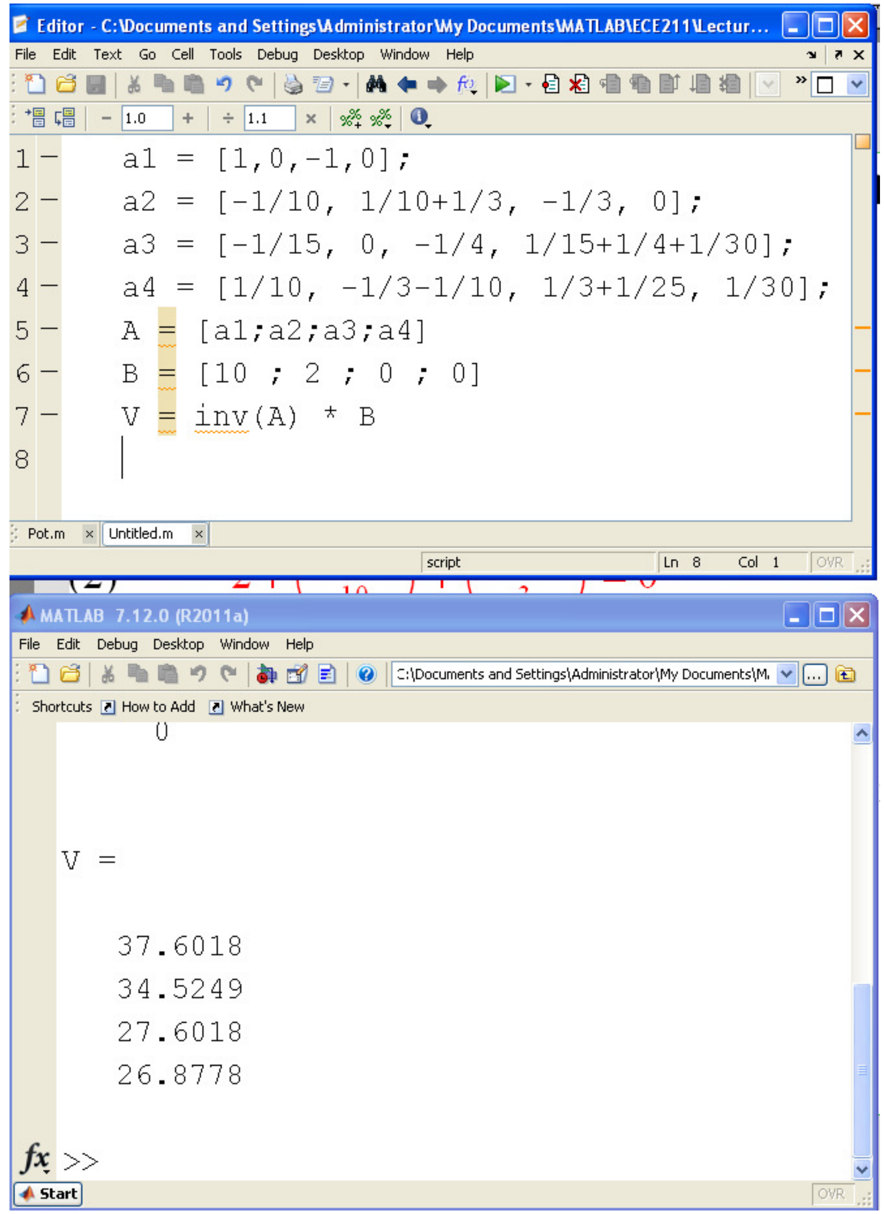
If the super-node equation is valid, you will be able to invert the matrix

$$AV = B$$

$$V = A^{-1}B$$

If the super-node equation is invalid, you will get an error

```
>> V = inv(A)*B
Warning: Matrix is singular to
working precision.

V =

    NaN
    NaN
    NaN
    NaN
```

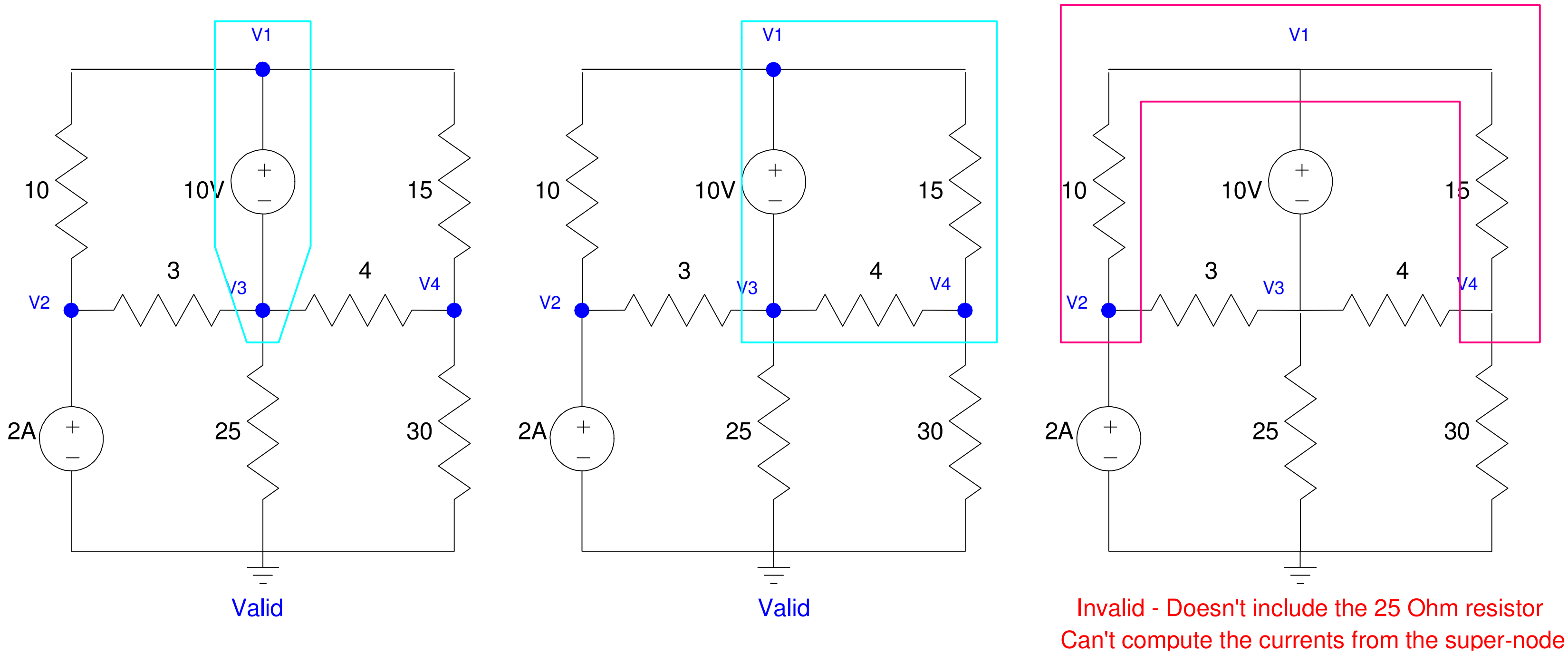


What's Happening?

To solve for 4 unknown voltages, you need 4 linearly independent equations

If the 4th equation is redundant (linear combination for the first three),

- There's no new information
- There's not enough information to solve
- The A matrix is not invertable

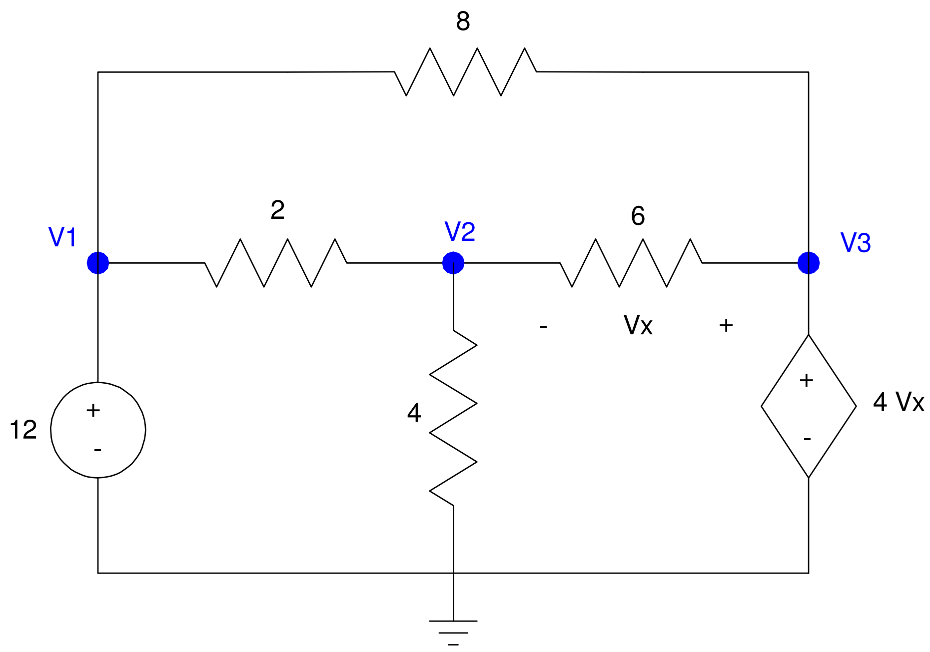


Voltage Nodes with Dependent Sources

- Same as voltage nodes
- Plus one equation for each dependent source

Example: Find V1, V2, V3, Vx

- 4 equations for 4 unknowns



Easy ones:

$$V_x = V_3 - V_2$$

$$V_1 = 12$$

$$V_3 = 4V_x$$

Node equation at V2

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_2 - V_3}{6}\right) = 0$$

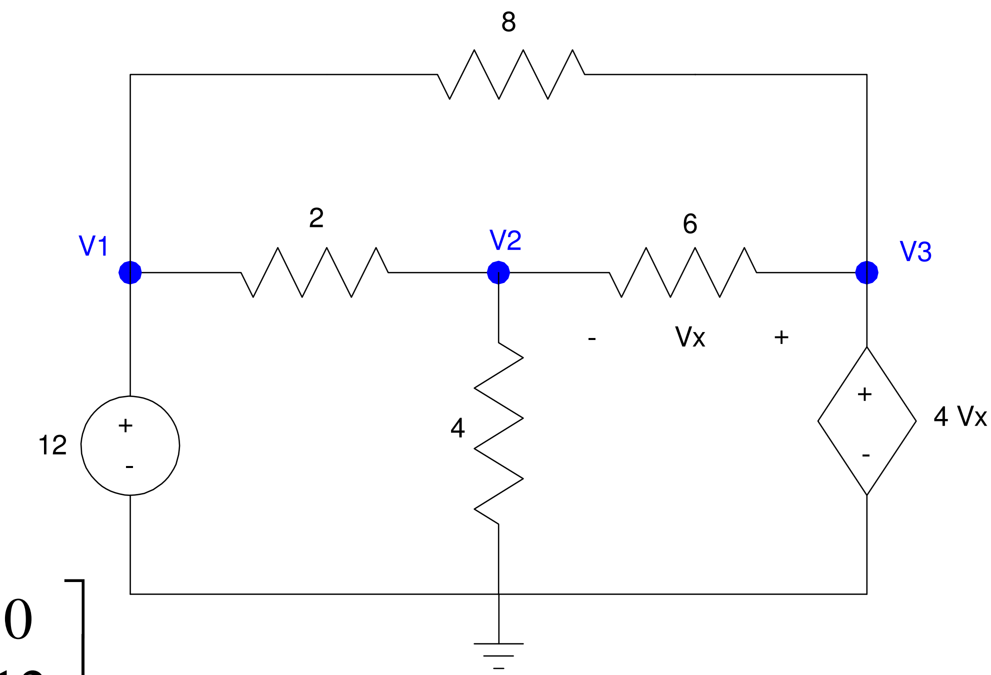
Solve: Group terms

$$V_x - V_3 + V_2 = 0$$

$$V_1 = 12$$

$$V_3 - 4V_x = 0$$

$$\left(\frac{-1}{2}\right)V_1 + \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right)V_2 + \left(\frac{-1}{6}\right)V_3 = 0$$



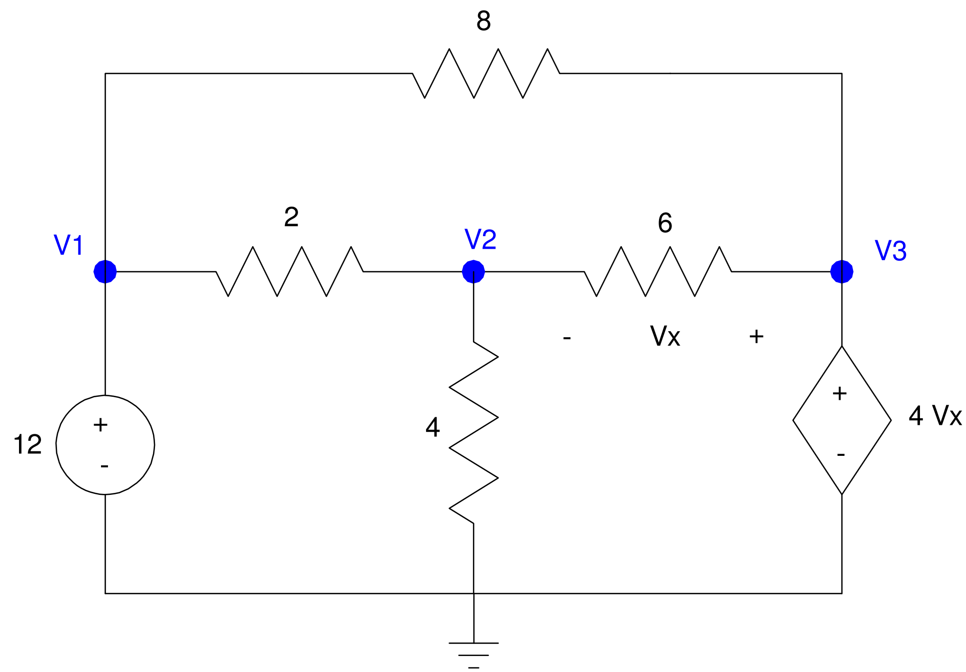
Placing in matrix form

$$\begin{bmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 \\ \left(\frac{-1}{2}\right) & \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\right) & \left(\frac{-1}{6}\right) & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

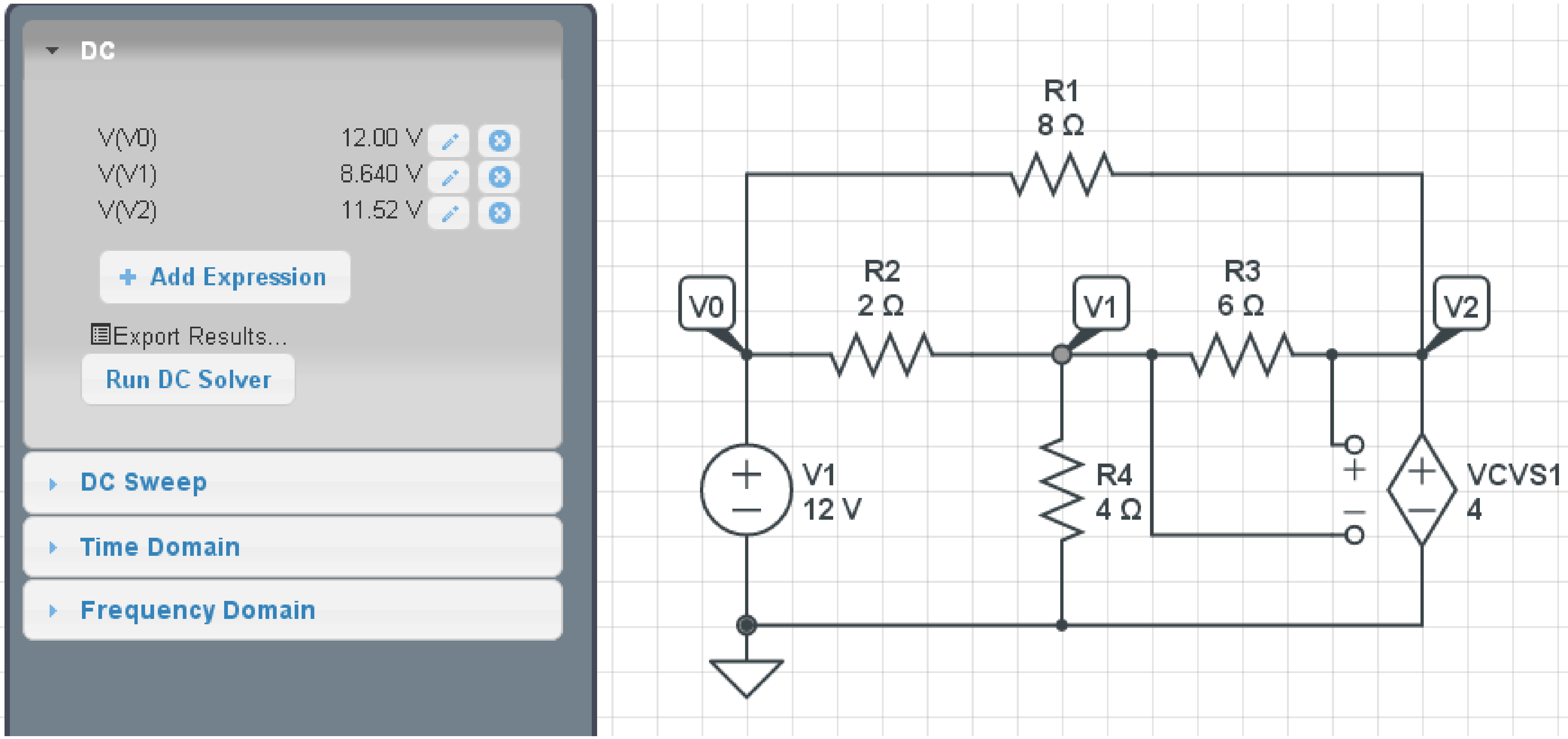
Solve in Matlab

```
A = [0,1,-1,1 ; 1,0,0,0 ; 0,0,1,-4 ; -1/2,1/2+1/4+1/6,-1/6,0]

         0    1.0000   -1.0000    1.0000
    1.0000         0         0         0
         0         0    1.0000   -4.0000
   -0.5000    0.9167   -0.1666         0

B = [0;12;0;0]

     0
    12
     0
     0


V = inv(A)*B
```

V1   12.0000
V2   8.6385
V3   11.5180
Vx   2.8795



Checking in Circuitlab - the voltages match



Super-Nodes and Dependent Sources

- If needed, define a closed-path (i.e. a Super-node) to give the rest of the N equtions needed

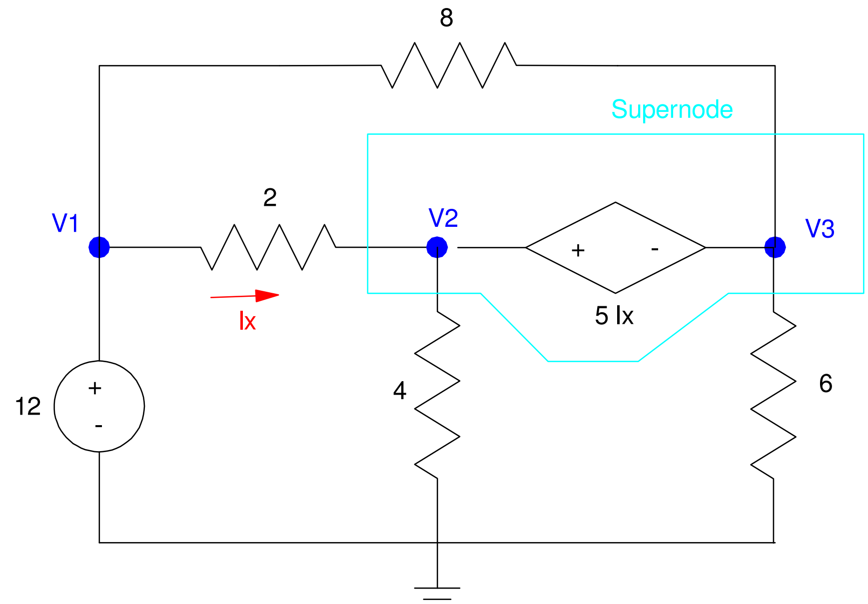
Example: Find { V1, V2, V3, Ix }

Easy Equations:

$$I_x = \left(\frac{V_1 - V_2}{2}\right)$$

$$V_1 = 12$$

$$V_2 - V_3 = 5I_x$$



Define a Super-Node

- Current out of the Super-Node = 0

$$\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_3}{6}\right) + \left(\frac{V_3 - V_1}{8}\right) = 0$$

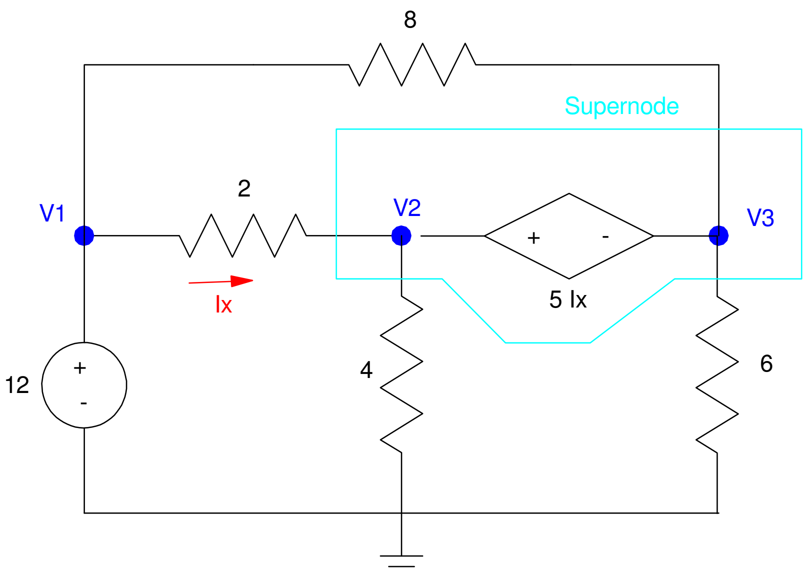
Group Terms

$$V_1 - V_2 - 2I_x = 0$$

$$V_1 = 12$$

$$V_2 - V_3 - 5I_x = 0$$

$$\left(\frac{-1}{2} + \frac{-1}{8}\right)V_1 + \left(\frac{1}{2} + \frac{1}{4}\right)V_2 + \left(\frac{1}{6} + \frac{1}{8}\right)V_3 = 0$$



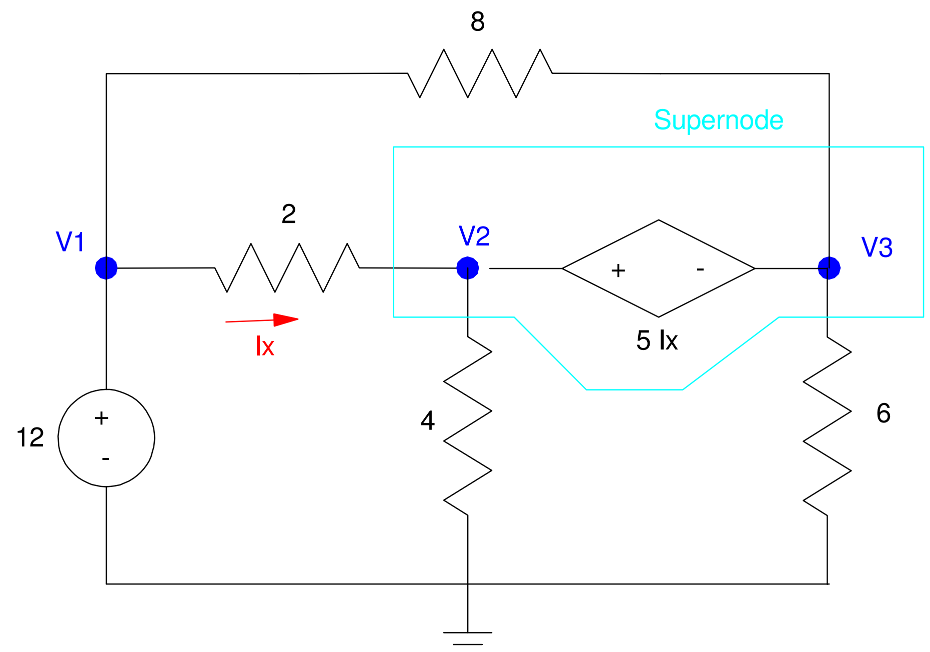
Place in matrix form

$$\begin{bmatrix} 1 & -1 & 0 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ -0.625 & 0.75 & 0.2917 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

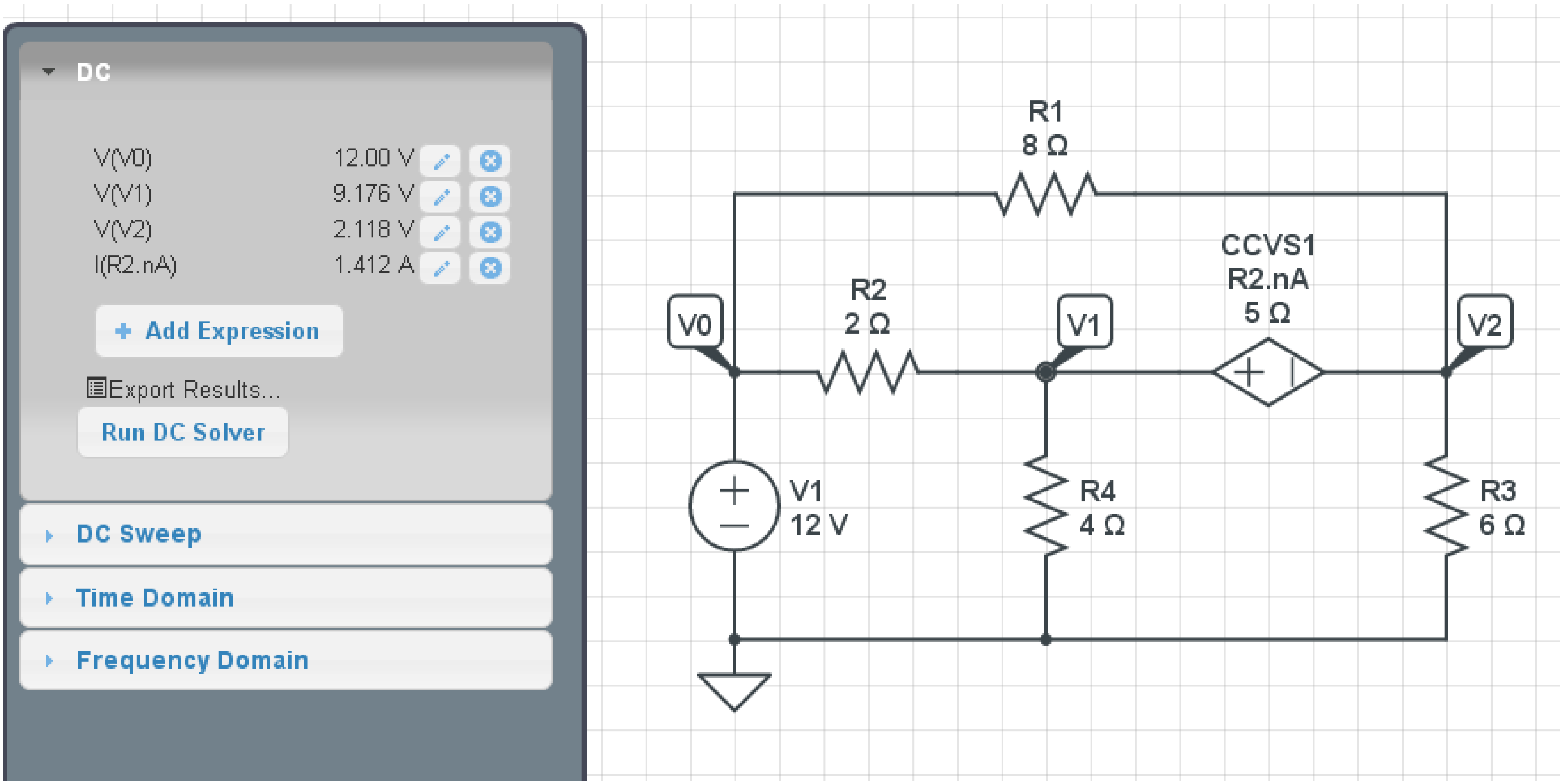
Solve in Matlab

```
A = [1,-1,0,-2 ; 1,0,0,0 ; 0,1,-1,-5 ; -0.625,0.75,0.2917,0]

    1.0000   -1.0000         0   -2.0000
    1.0000         0         0         0
         0    1.0000   -1.0000   -5.0000
   -0.6250    0.7500    0.2917         0

B = [0;12;0;0]

     0
    12
     0
     0

V = inv(A)*B
```

V1   12.0000
V2   9.1764
V3   2.1175
Ix   1.4118

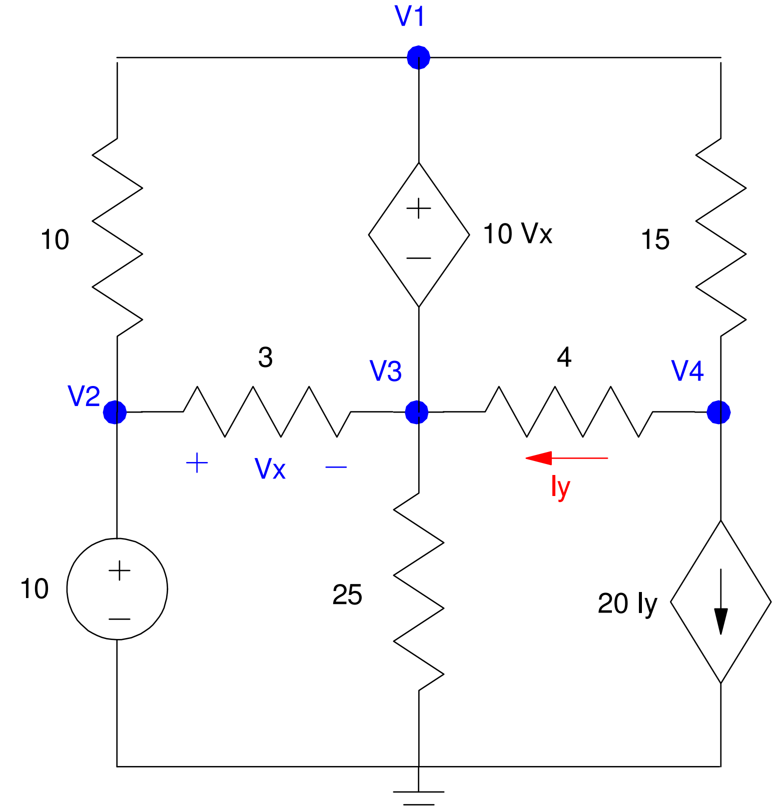


Verify using Circuitlab



Practice Problem

Write the voltage node equations for the following circuit



Summary

Conservation of current applies to any closed path

- The sum of all currents from a closed region is zero

If you are having problems coming up with N equations for N unknowns, add a super-node

- Define a closed-path
- Such that all currents from the super-node are known

Super-nodes are not unique

- Several closed paths could be used
- Each should give the same result
- It helps the grader if you show what super-node you're using in homework sets and exams