Current Loops

a.k.a. Current Mesh

ECE 211 Circuits I

Lecture #7

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

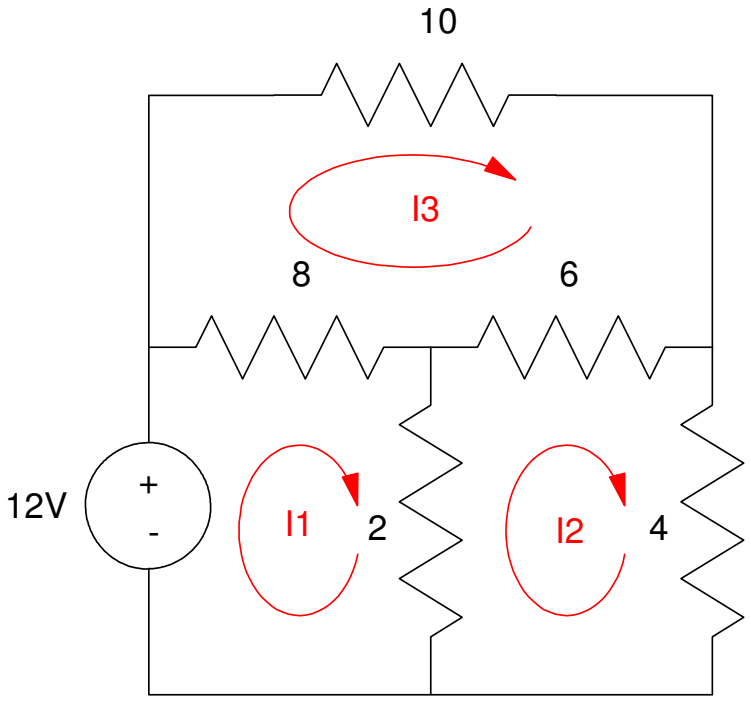
Current Loops

- Current Mesh
- Kirchoff's Current Loops
- Kirchoff's Voltage Law

Goal:

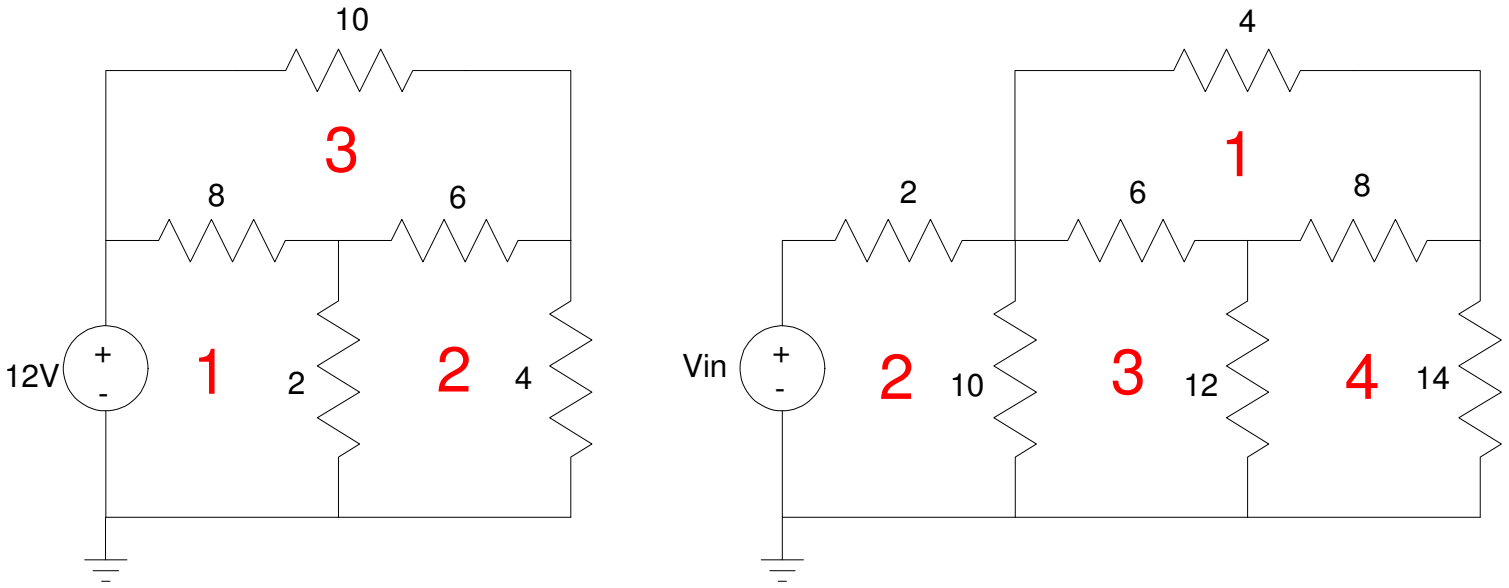
- Write N equations to solve for N unknown currents
- Uses conservation of voltage

  - The sum of the voltages around any closed path mush sum to zero
  - a.k.a. Kirchoff's Voltage law



Step 1: Count Windows

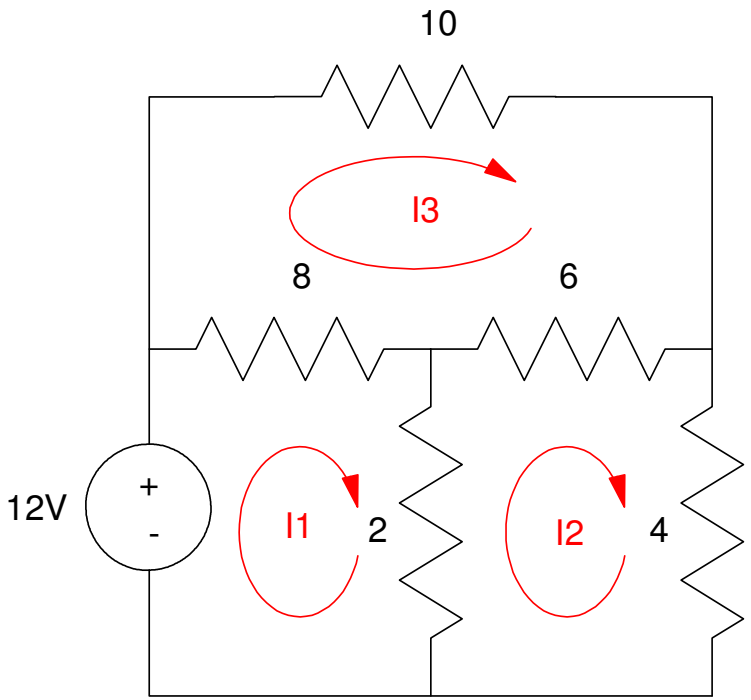
- # of loop equations needed = # windows
- Not always possible



A circuit with 3 windows (left) and 4 windows (right).

Step 2: Label Currents

- Path
- Direction



Step 3: Write N equations for N unknowns

- Sum the voltage around each current loop
- Be consistent: Subtract if you hit the - sign first
- Make sure you end up where you started
- The voltages must add up to zero

$$-12 + (I_1 - I_3)8 + (I_1 - I_2)2 = 0$$

$$(I_2 - I_1)2 + (I_2 - I_3)6 + (I_2)4 = 0$$

$$(I_3 - I_1)8 + (I_3)10 + (I_3 - I_2)6 = 0$$

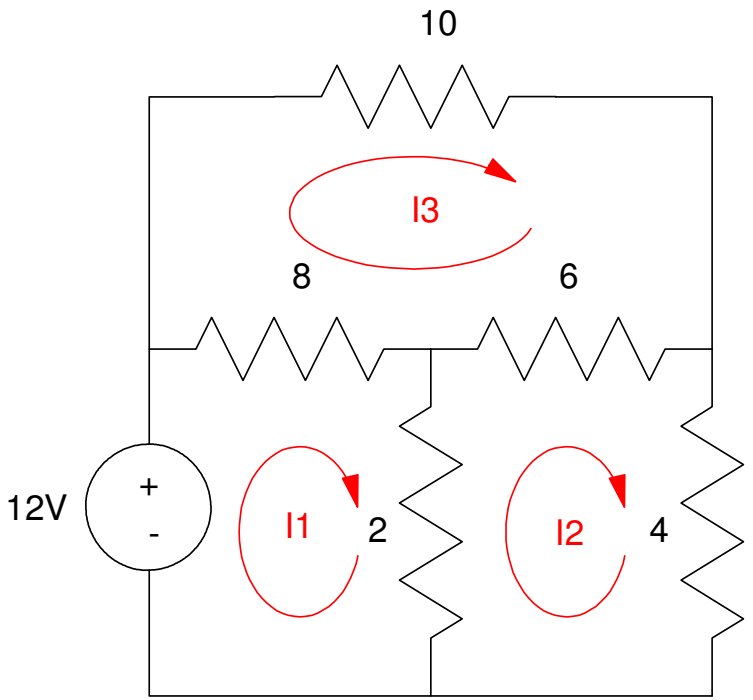
Note the signs of each term

- When you go arount loop I1
  - All of the I1 terms are positive
  - All of the other terms are negative
- When you go around loop I2
  - All of the I2 terms are positive
  - All of the other terms are negative

$$-12 + (I_1 - I_3)8 + (I_1 - I_2)2 = 0$$

$$(I_2 - I_1)2 + (I_2 - I_3)6 + (I_2)4 = 0$$

$$(I_3 - I_1)8 + (I_3)10 + (I_3 - I_2)6 = 0$$



Also note the units match up

volts + volts + volts = 0

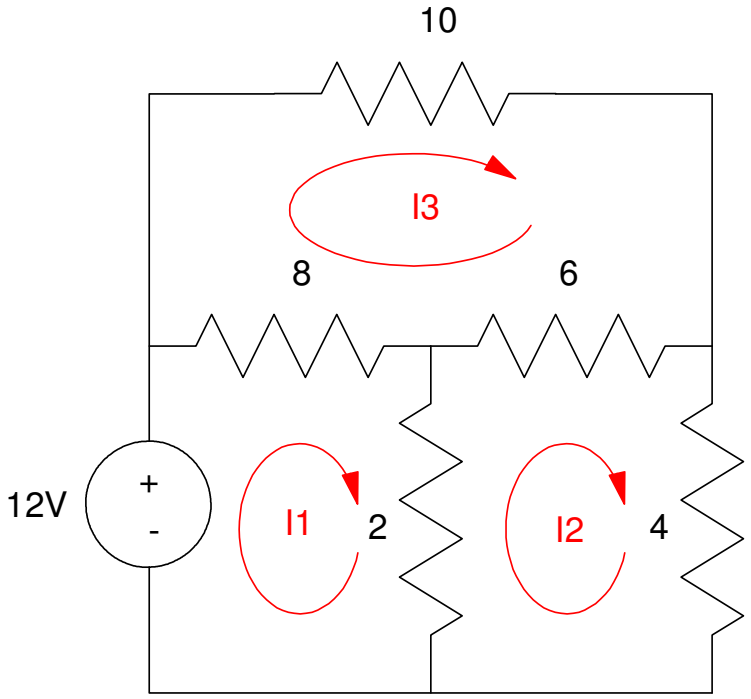
Step 4: Solve

Group terms

$$10\,I_1 - 2\,I_2 - 8\,I_3 = 12$$
$$-2\,I_1 + 12\,I_2 - 6\,I_3 = 0$$
$$-8\,I_1 - 6\,I_2 + 24\,I_3 = 0$$

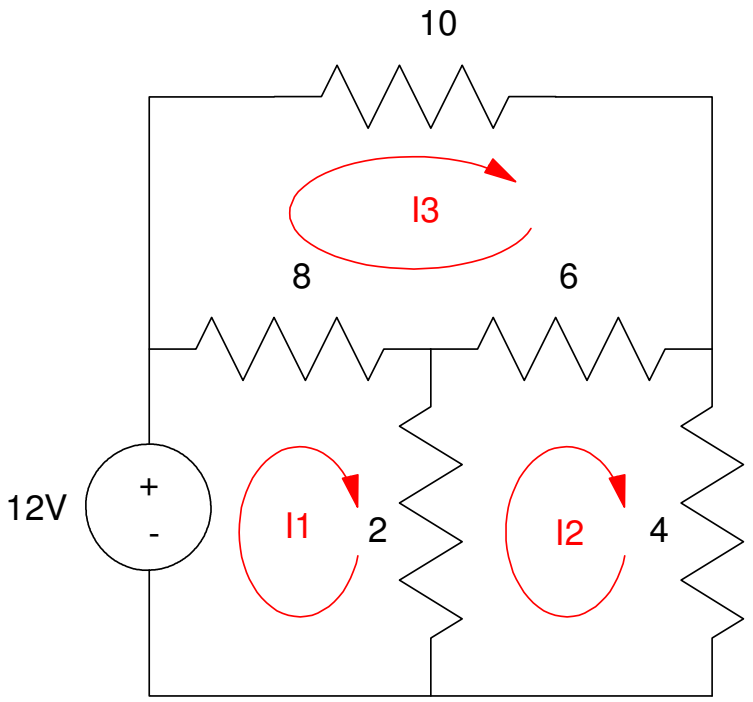
Place in matrix form

$$\begin{bmatrix} 10 & -2 & -8 \\ -2 & 12 & -6 \\ -8 & -6 & 24 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$



Solve using Matlab:

```
A = [10,-2,-8;-2,12,-6;-8,-6,24]

    10.   - 2.    - 8.
  - 2.      12.   - 6.
  - 8.    - 6.      24.

B = [12;0;0]

    12.
    0.
    0.

inv(A)*B

I1     2.0655738
I2     0.7868852
I3     0.8852459
```



Verification in CircuitLab

In lab, currents are hard to measure

- Fuses blow as soon as someone tries to use an ammeter as a voltmeter
- Also requires inserting an ammeter in series with an element

Easier to measure the voltages and compute the currents

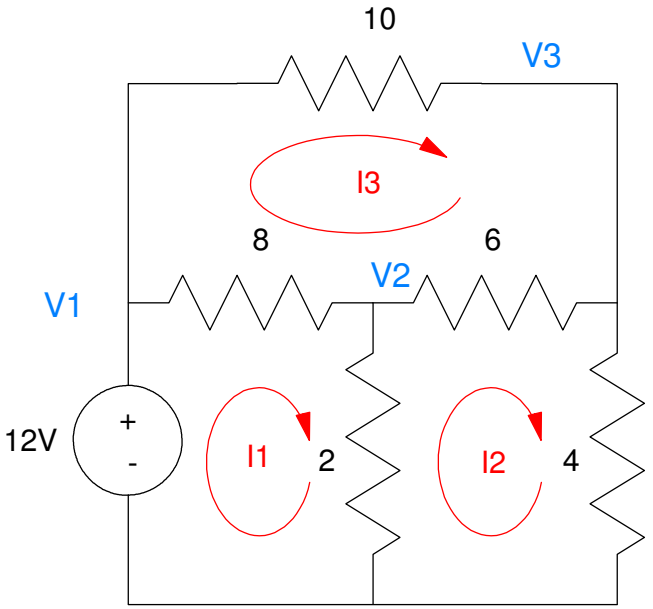
- V1 = 12.00V
- V2 = 2.557V
- V3 = 3.148V

so

$$I_3 = \left(\frac{V_1 - V_3}{10}\right) = 885.2mA$$

$$I_2 = \left(\frac{V_3}{4}\right) = 787.0mA$$

$$I_1 - I_3 = \left(\frac{V_1 - V_2}{8}\right) = 1.1804A$$



Example 2: Find the currents for the 4-window circuit. Assume Vin = +12V.

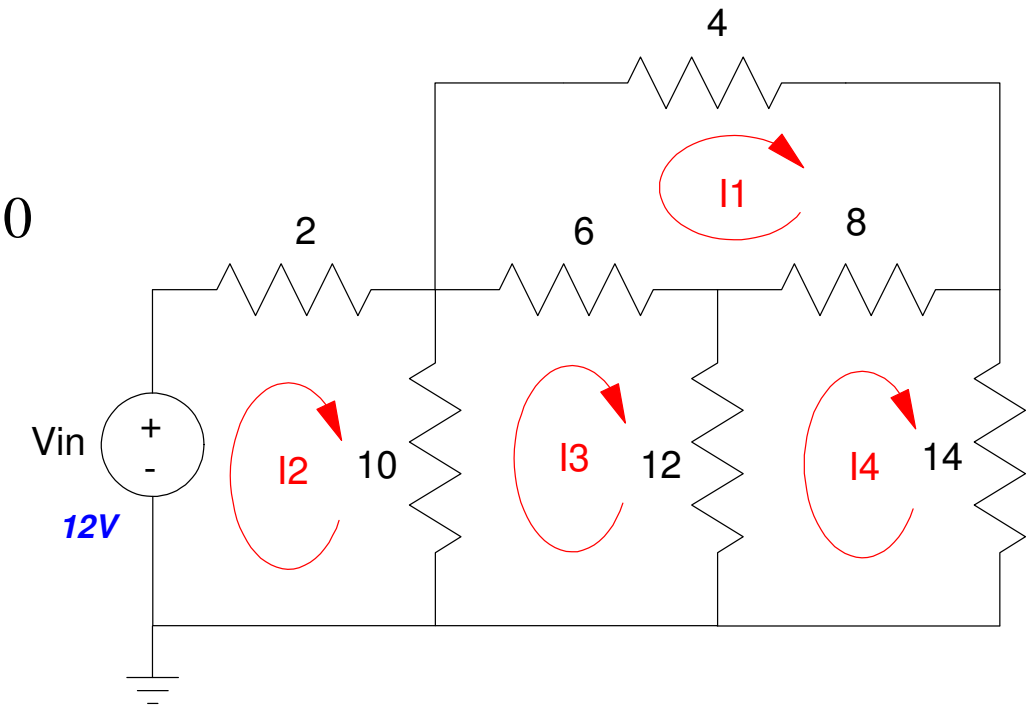
First, write 4 equations for 4 unknowns:

$$4I_1 + 8(I_1 - I_4) + 6(I_1 - I_3) = 0$$

$$-12 + 2I_2 + 10(I_2 - I_3) = 0$$

$$10(I_3 - I_2) + 6(I_3 - I_1) + 12(I_3 - I_4) = 0$$

$$12(I_4 - I_3) + 8(I_4 - I_1) + 14(I_4) = 0$$



Group terms:

$$18I_1 - 6I_3 - 8I_4 = 0$$

$$12I_2 - 10I_3 = 12$$

$$-6I_1 - 10I_2 + 28I_3 - 12I_4 = 0$$

$$-8I_1 - 12I_3 + 34I_4 = 0$$

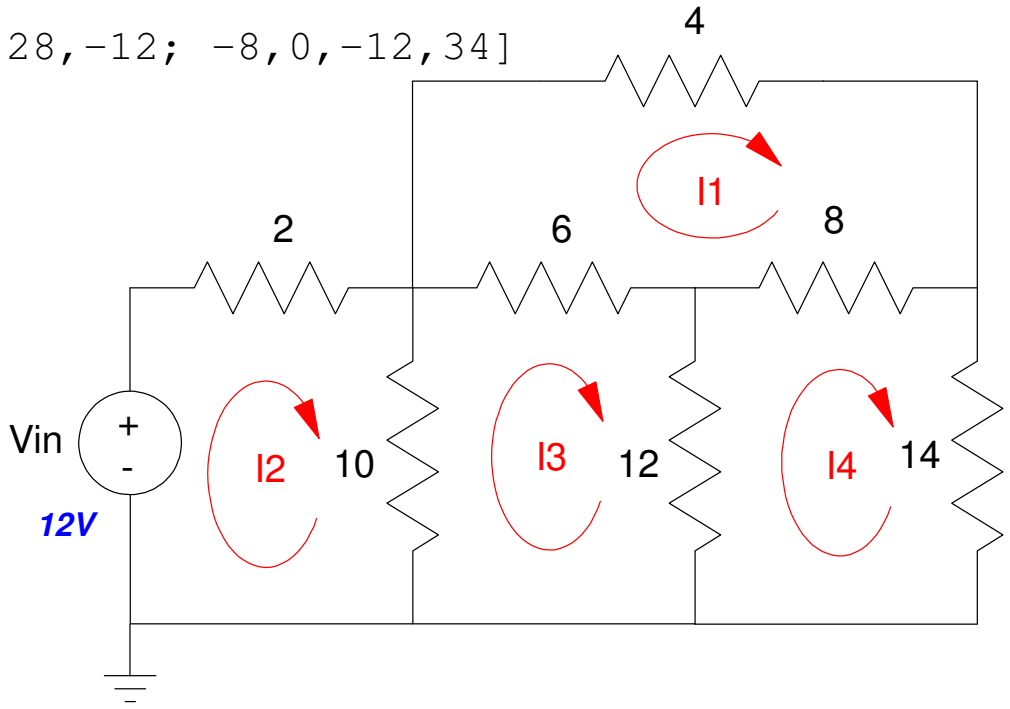
Place in matrix form:

$$\begin{bmatrix} 18 & 0 & -6 & -8 \\ 0 & 12 & -10 & 0 \\ -6 & -10 & 28 & -12 \\ -8 & 0 & -12 & 34 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \end{bmatrix}$$

Solving in MATLAB:

```
A = [18,0,-6,-8; 0,12,-10,0; -6,-10,28,-12; -8,0,-12,34]
B = [0;12;0;0]
inv(A)*B
```

```
I1:     0.5164104
I2:     1.7860913
I3:     0.9433096
I4:     0.4544411
```



From Matlab

```
I1:     0.5164104
I2:     1.7860913
I3:     0.9433096
I4:     0.4544411
```

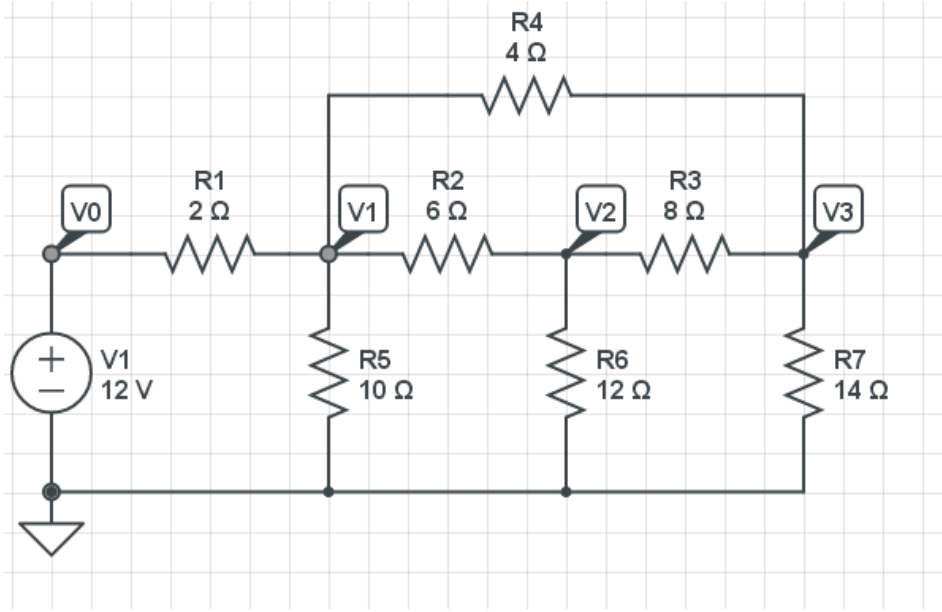
From CircuitLab

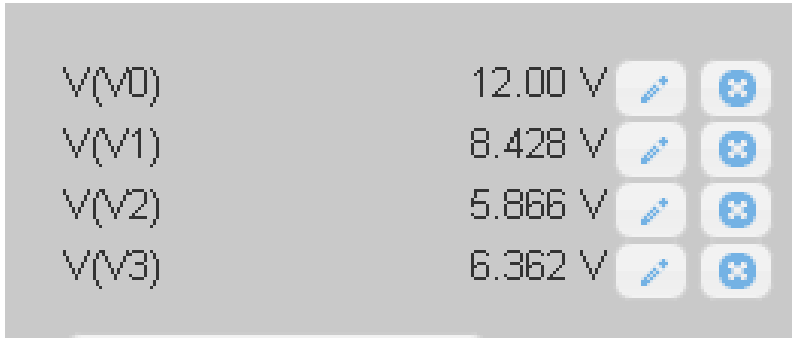
$$I_1 = \left(\frac{V_1 - V_3}{R_4}\right) = 0.5165A$$

$$I_2 = \left(\frac{V_0 - V_1}{R_1}\right) = 1.7860A$$

$$I_2 - I_3 = \left(\frac{V_1}{R_5}\right) = 0.8428A$$

$$I_4 = \left(\frac{V_3}{R_7}\right) = 0.4544A$$





Current Loops with Voltage Sources

Not a problem

- If you encounter the + sign first, add the voltage.
- If you encounter the - sign first, subtract the voltage.

Example: Write 3 equations for 3 unknowns:

$$-12 + 8(I_1 - I_3) + 2(I_1 - I_2) = 0$$

$$2(I_2 - I_1) + 6 + 4(I_2) = 0$$

$$-10 + 6 + 8(I_3 - I_1) = 0$$