Thevenin Equivalent and Load Lines

ECE 211 Circuits I

Lecture #10

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Thevenin Equivalent

So far, we have two tools:

- Voltage Nodes: Find the voltages so that the currents balance (sum to zero)
- Current Loops: Find the currents so that the votlages balance (sum to zero)

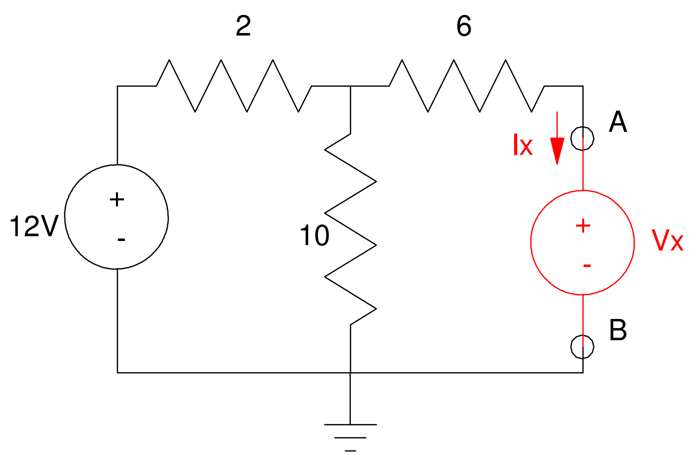
Thevenin Equivalents is tool #3

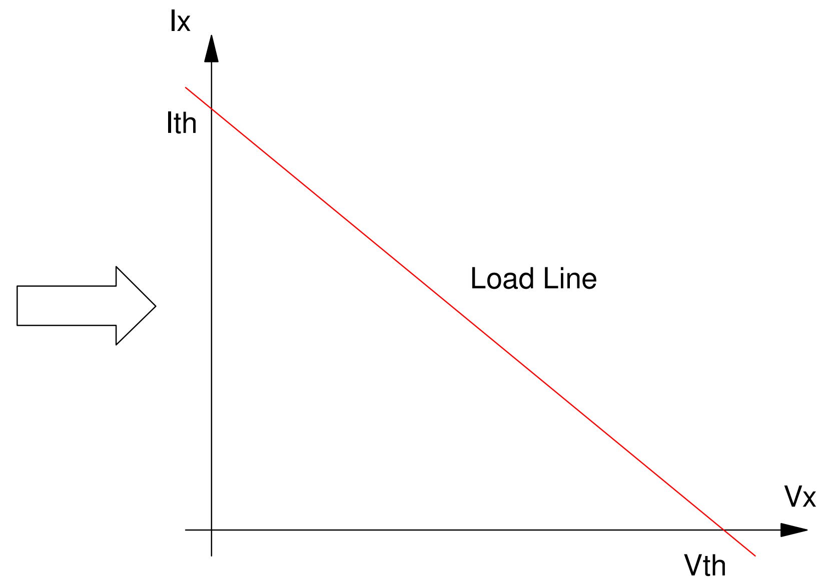
- Harder to find
- More difficult to comprehend
- Not necessary: you can solve any circuit using current loops and voltage nodes
- But, Thevenin equivalents can make some circuits much easier to analyze

Load Lines:

Idea:

- The voltage-current relationship for a linear circuit follows a straight line.
- Any circuit which produces the same load line behaves the same
- The simplest circuit which does this is
  - A voltage source & resistor (Thevenin equivalent)
  - A current source & a resistor (Norton equivalent)



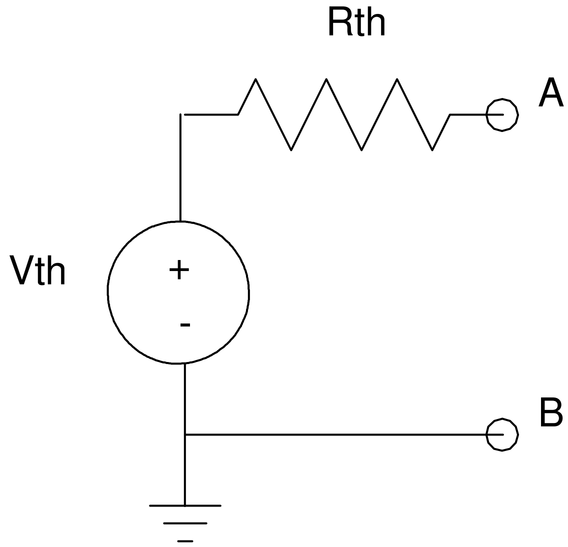


Thevenin and Norton Equivalent

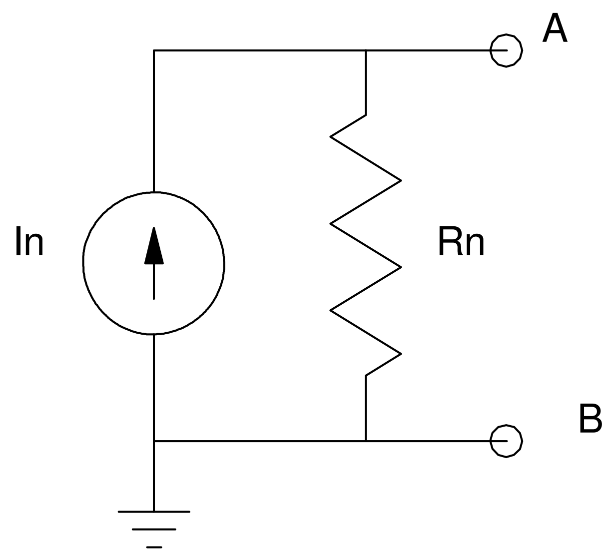
Simplest circuit to produce a load line

- A voltage source and resistance in series (termed a Thevenin equivelent), or
- A current source and resistance in parallel (termed a Norton equivelent).

The trick is to find the values ot Vth, Rth, or Ith



Thevenin Equivalent



Norton Equivalent

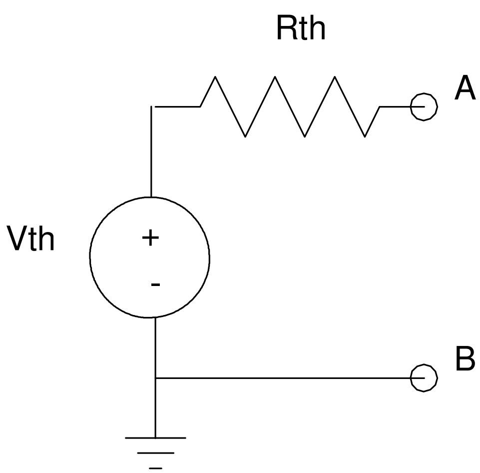
Finding Vth, Rth, In, Rn

Circuits are equivalent

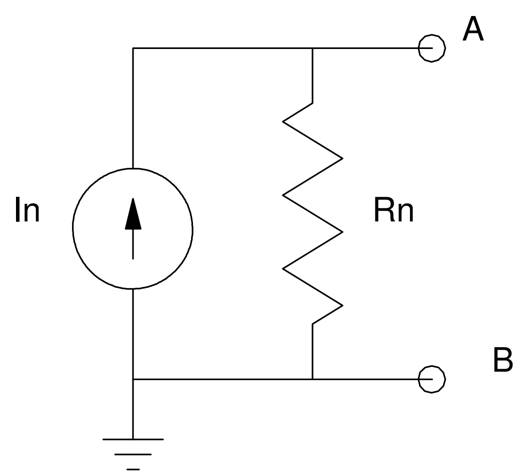
- Whatever works for your original circuit works for the Thevenin / Norton equivalent

Procedure:

- Vth: Measure the open-circuit voltage of your circuit.
- Rth = Rn: Turn off all sources (V = 0, I = 0). Measure the resulting resistance.
- In: Measure the short-circcuit current



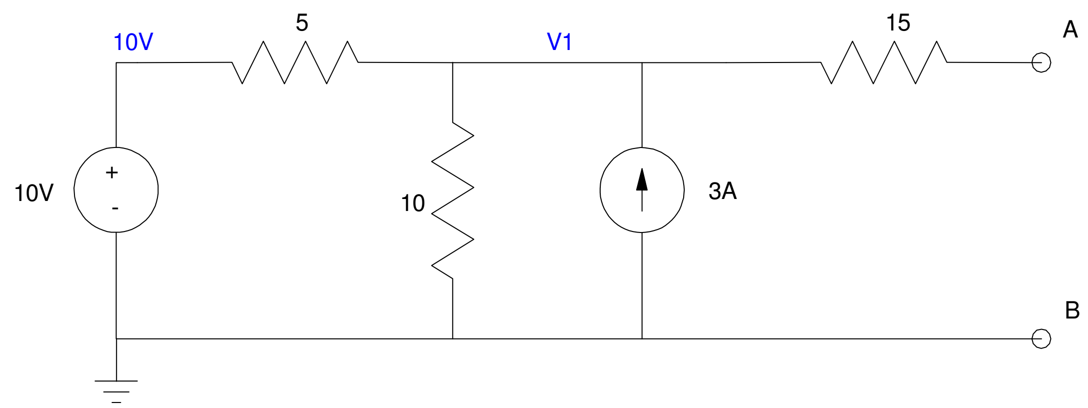
Thevenin Equivalent



Norton Equivalent

Example 1: Determine the Thevenin and Norton equivalent for the following circuit:

Vth: Measure the open-circuit voltage.



$$\left(\frac{V_1 - 10}{5}\right) + \left(\frac{V_1}{10}\right) - 3 = 0$$

$$V_{th} = V_1 = 16.67V$$

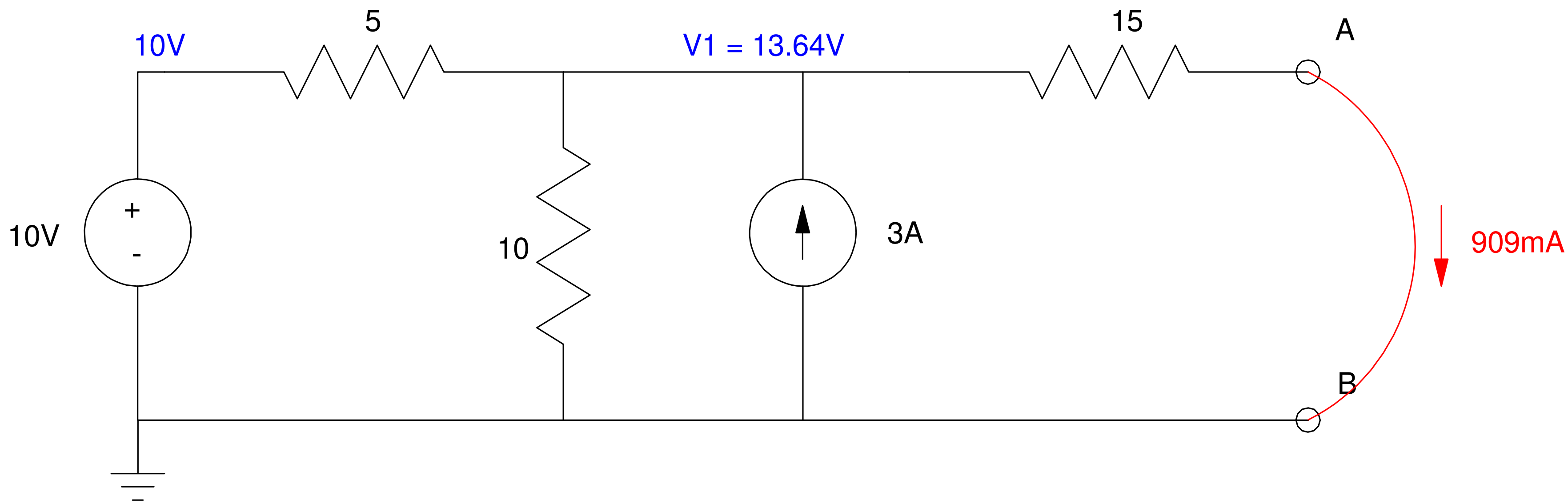
This is Vth

In: Short AB and measure the current. Again, this isn't obvious so write the node equation at V1

$$\left(\frac{V_1-10}{5}\right)+\left(\frac{V_1}{10}\right)-3+\left(\frac{V_1}{15}\right)=0$$

$$V_1 = 13.64V$$

$$I_{short} = I_N = \frac{13.64V}{15\Omega} = 909.1mA$$



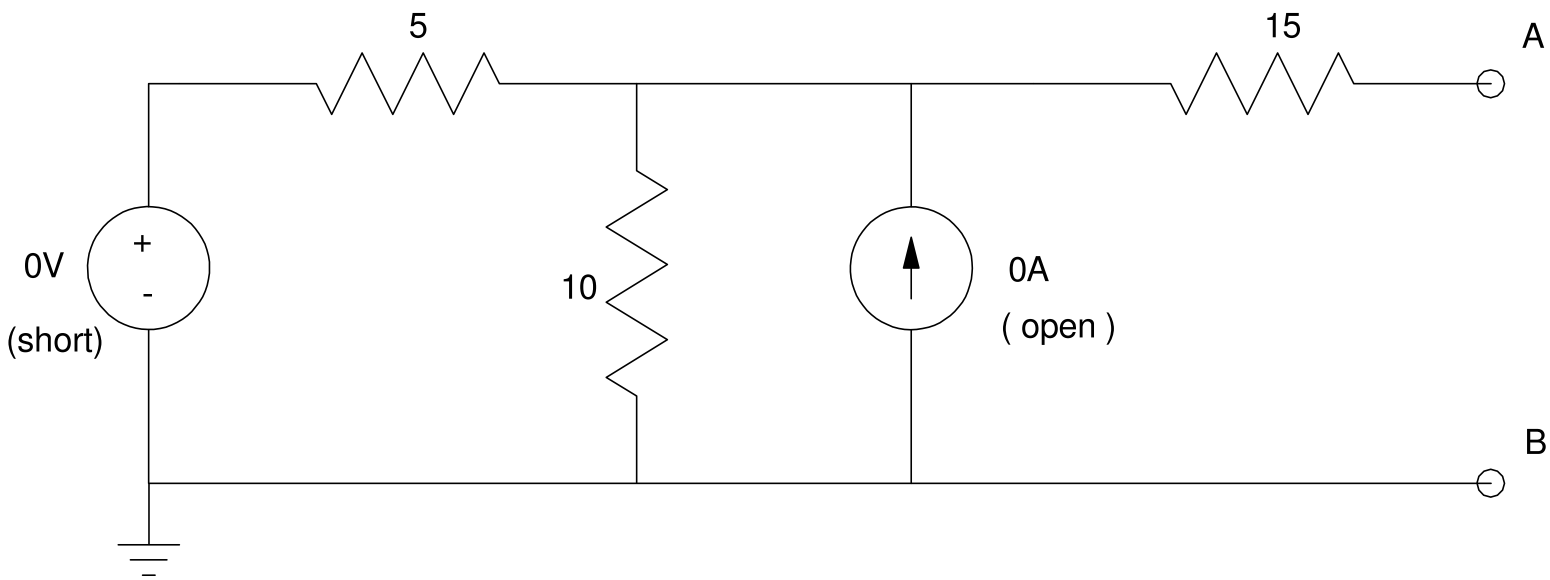
Rth: Turn off the sources (V = 0, I = 0). Measure the resistance between A and B

$$R_{AB} = 15 + 5||10$$

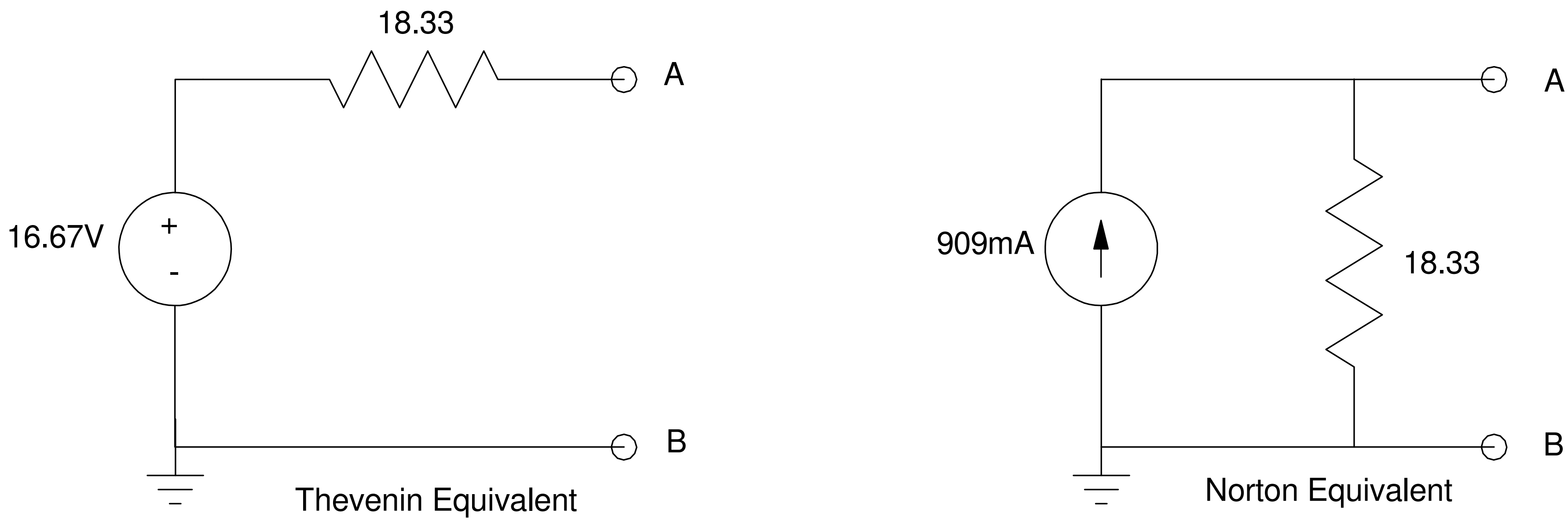
$$R_{AB} = 18.333\Omega$$

Note that you only need to compute two of these: the third redundant.

$$R_{AB} = \frac{V_{tn}}{I_n} = \frac{16.67V}{909mA} = 18.33\Omega$$



Result:



Resulting Thevenin and Norton equivalents of Example 1

Circuit Simplification using Thevenin and Norton Equivalent

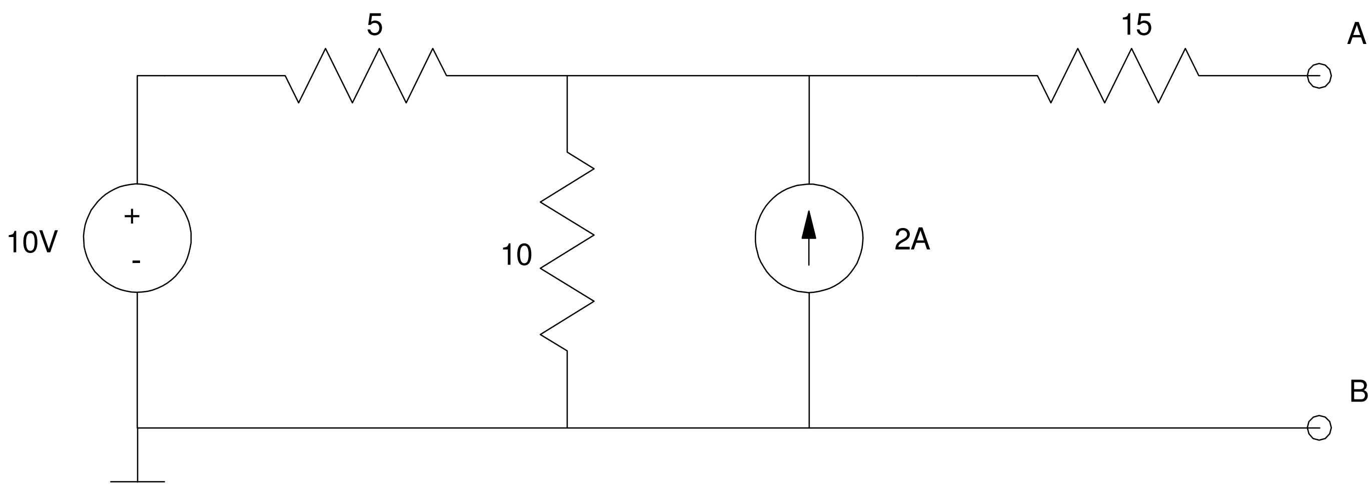
Sometimes you can simplify a circuit by flipping between Thevenin and Norton equivalents

$$R_{Thevenin} = R_{Norton}$$

$$V_{Thevenin} = I_{Norton} \cdot R$$

$$I_{Norton} = \frac{V_{Thevenin}}{R}$$

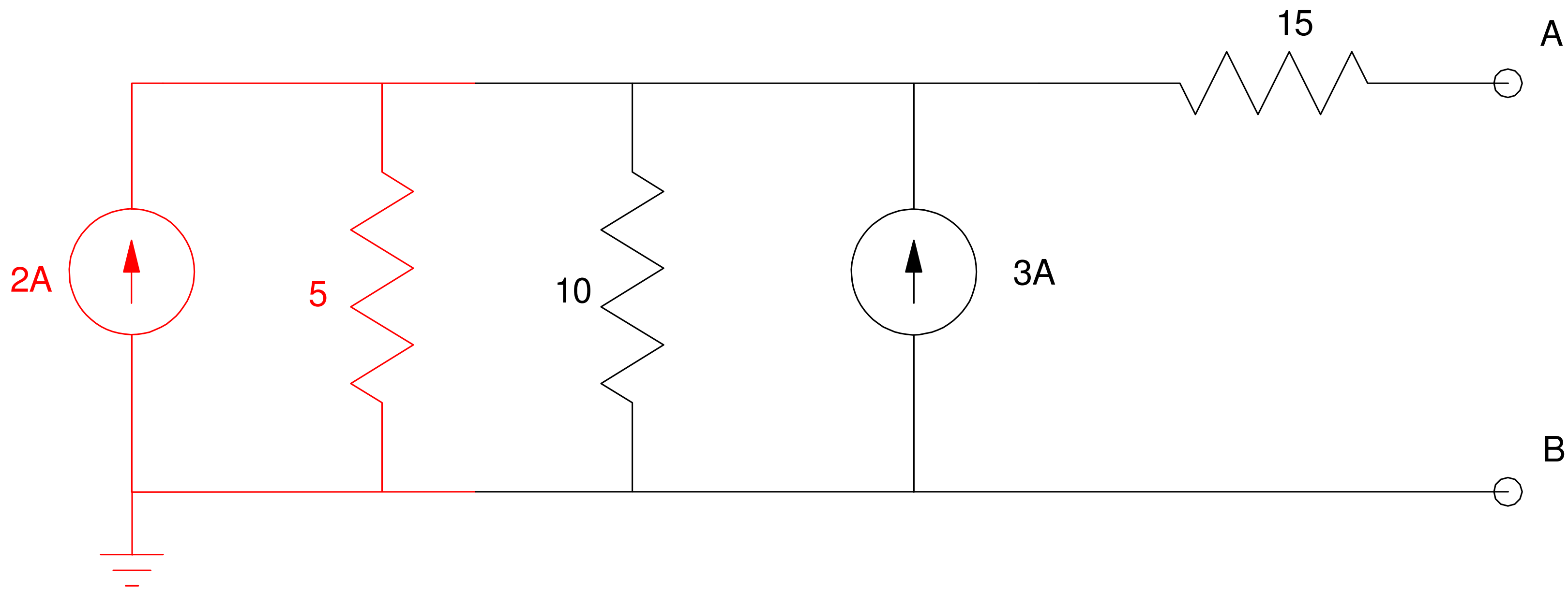
Eample: Find the Thevenin equivalent



Step 1: Convert the 10V / 5 Ohm resistor to its Norton equivalent

$$I_N = \frac{10V}{5\Omega} = 2A$$

$$R_N = R_{Th} = 5\Omega$$



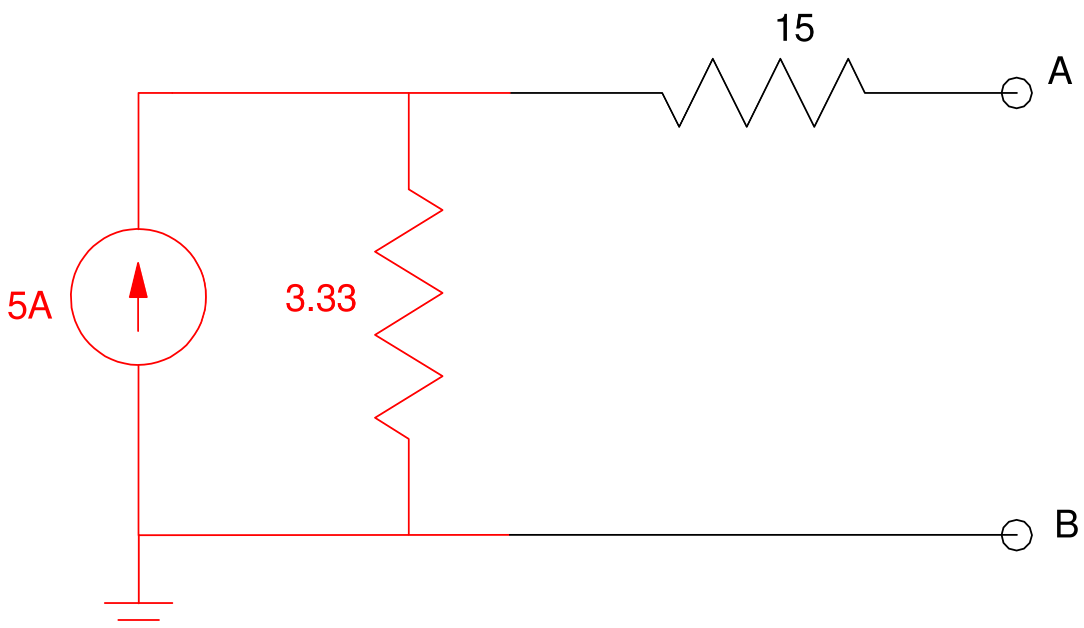
Convert the 10V : 5 Ohm source to its Norton equivalent (shown in red)

Add the resistors in parallel

$$R_{net} = 5||10 = \left(\frac{1}{5} + \frac{1}{10}\right)^{-1} = 3.333\Omega$$

Add the current soruces

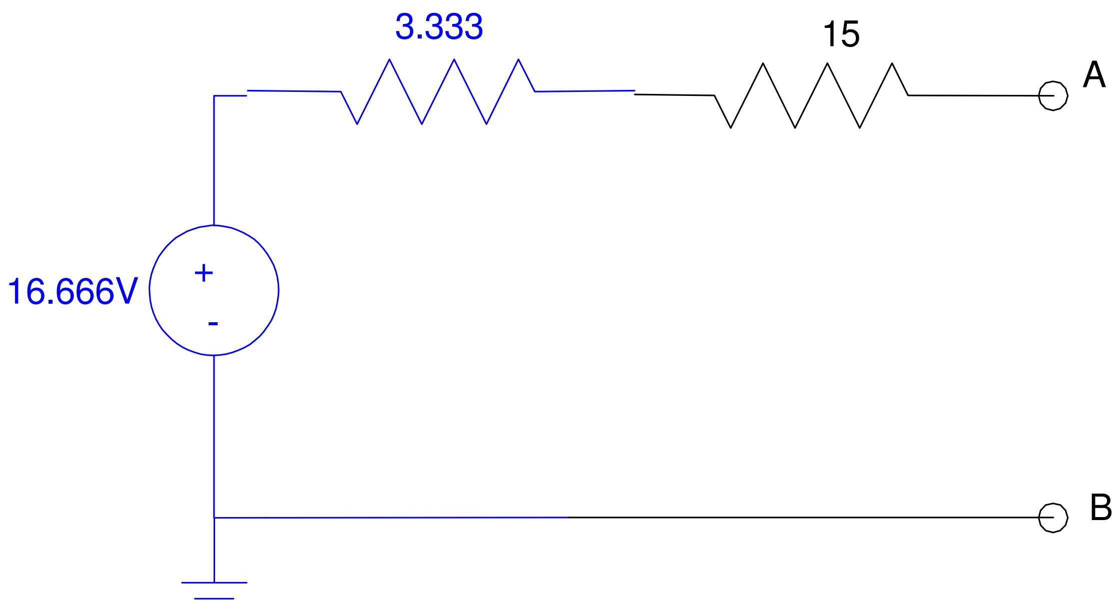
$$I_{net} = 2 + 3 = 5$$



Add the resistors and current sources in parallel:

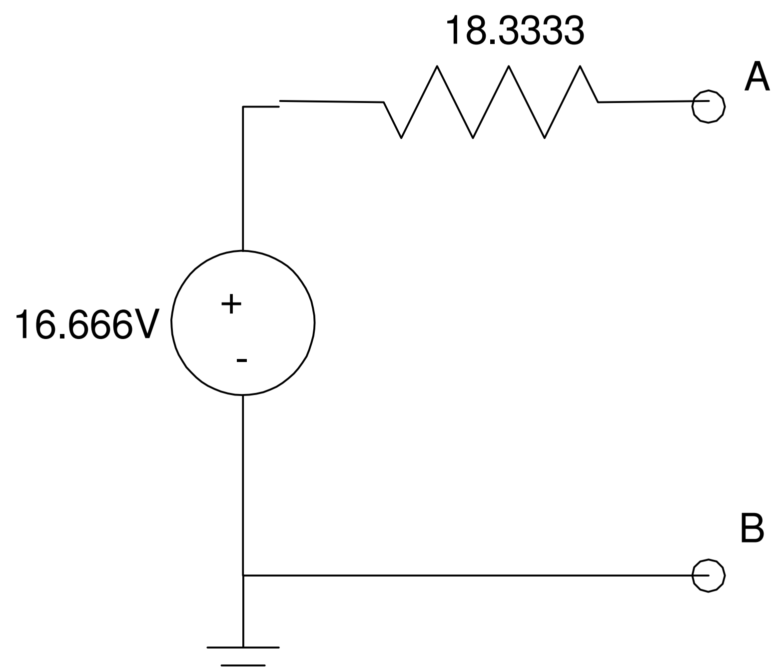
Convert back to a Thevenin equivalent

$$V_{th} = 5A \cdot 3.3333\Omega = 16.666V$$



Convert to a Thevenin equivalent (shown in blue)

Add the resistors in series and you have the Thevenin equivalent looking in from terminals AB

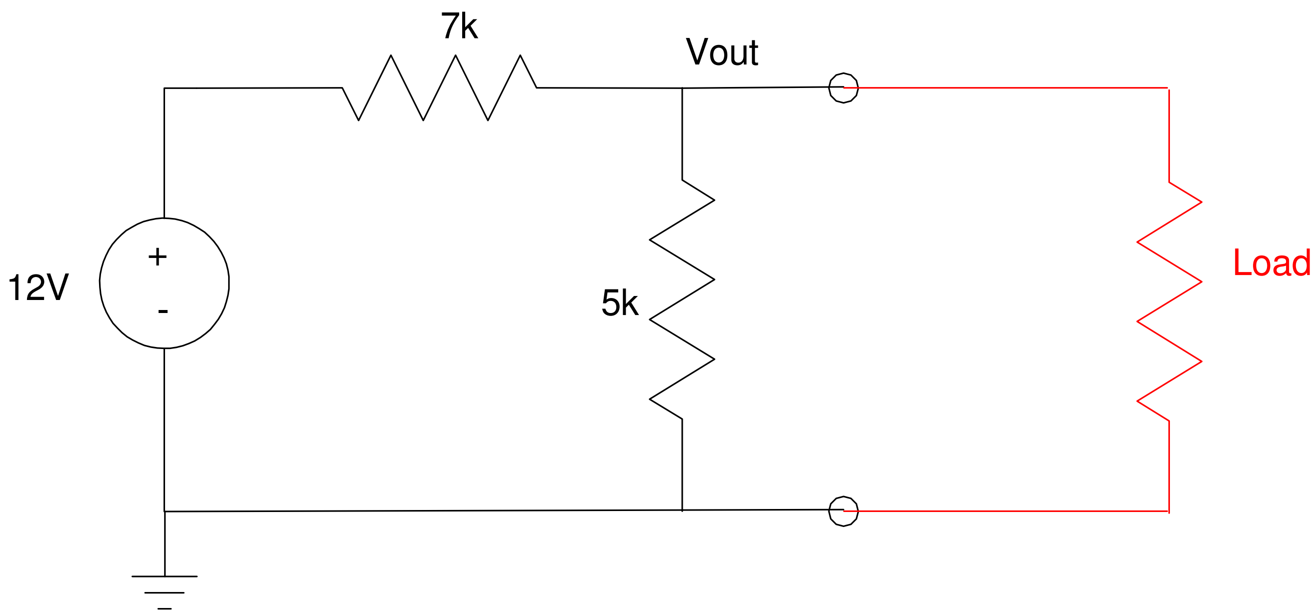


Thevenin Equivalet of Example 2

Example 3: Thevenin equivalents can provide insight.

- The following voltage divider works (Vout = 5V) if there is no load
- When you add a load (100mA @ 5V) the output voltage goes to zero.

Why?



Voltage divider used to convert 12V to 5V

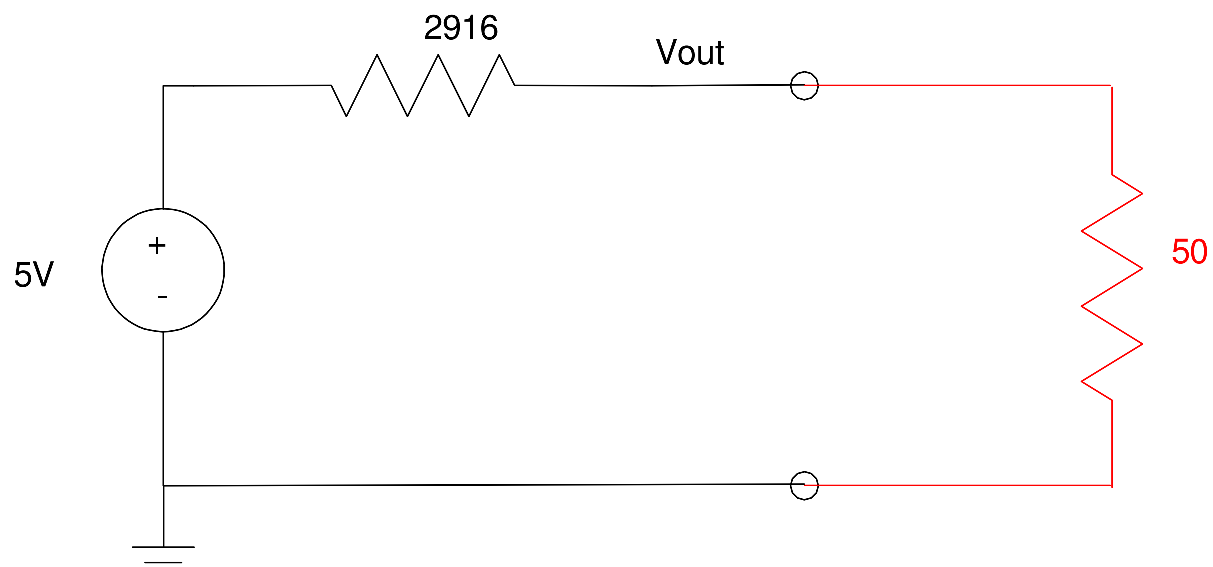
Convert to a Thevenin equivalent

$$V_{th} = V_{open} = \left(\frac{5k}{5k+7k}\right) 12V = 5V$$

$$R_{th} = 7k || 5k = 2916\Omega$$

If your load draws 100mA @ 5V, it looks like a 50 Ohm resistor

$$R_{load} = \frac{5V}{100mA} = 50\Omega$$



By voltage division, Vout is now

$$V_{out} = \left(\frac{50}{50+2916}\right) \cdot 5V$$

$$V_{out} = 0.0843V$$

This circuit works as a 5V source as long as you don't use it. In Electronics, we'll cover other circuits which *do* work.