
Thevenin Equivalents & Max Power Transfer

ECE 211 Circuits I

Lecture #11

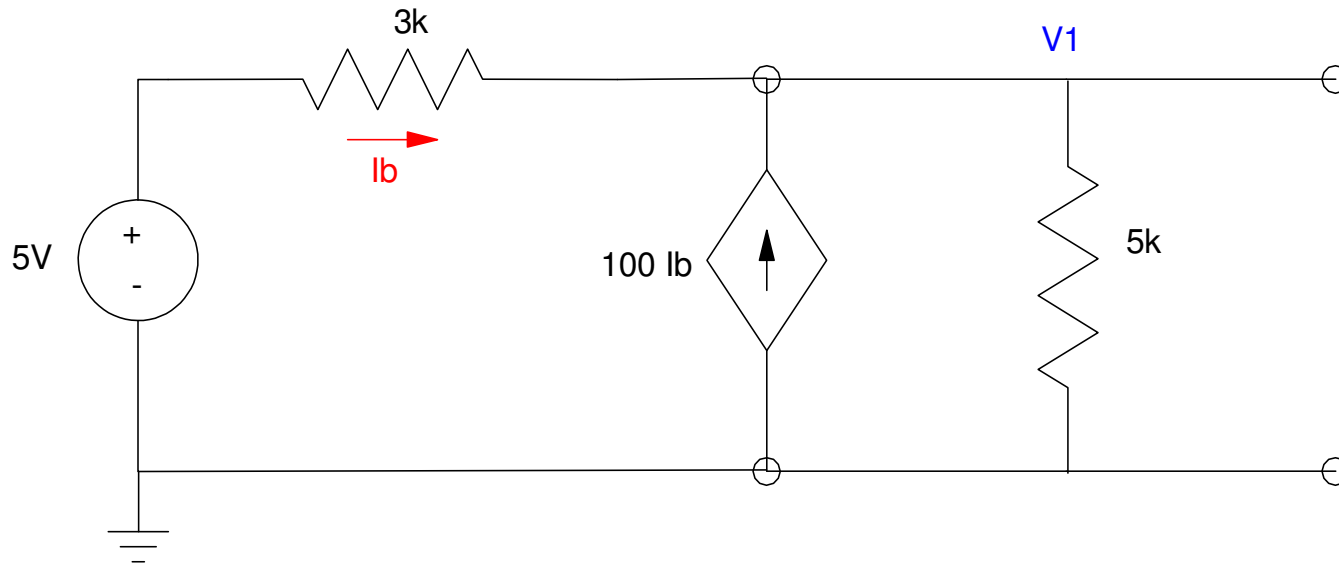
Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Thevenin and Norton Equivalents (take 2)

Sometimes, the Thevenin resistance isn't obvious.

- If so, apply a test voltage and compute the current draw
- The Thevenin resistance looking in is V_{in} / I_{in}

Example 1: Determine the Thevenin equivalent for the following circuit



V_{th} : Determine the open-circuit voltage. Write the voltage node equation at V1

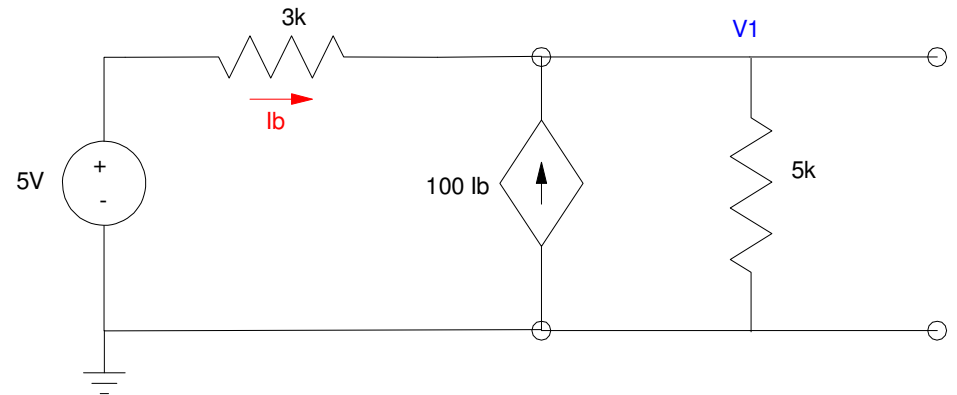
$$I_b = \left(\frac{5 - V_1}{3k} \right)$$

$$\left(\frac{V_1 - 5}{3k} \right) - 100I_b + \left(\frac{V_1}{5k} \right) = 0$$

Substitute and solve

$$V_1 = \left(\frac{\left(\frac{101}{3k} \right)}{\left(\frac{101}{3k} \right) + \left(\frac{1}{5k} \right)} \right) 5V = 4.9705V$$

This is V_{th} .



R_{th} :

- Turn off the voltage source
- Measure the resistance

This isn't obvious. So

- Apply a 1V test voltage
- Compute the current drwa (I_{in})

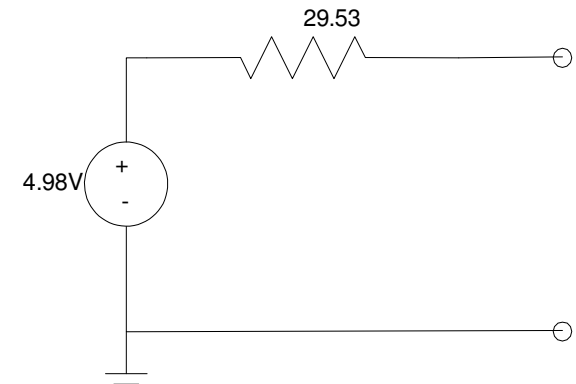
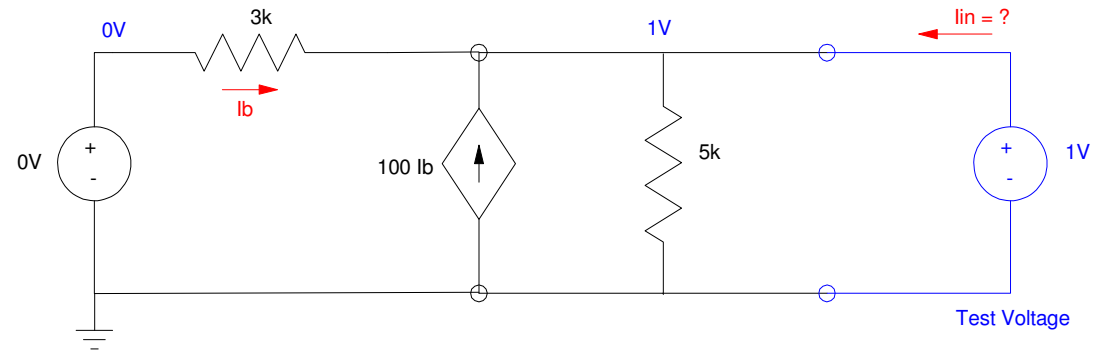
$$I_b = \left(\frac{0V - 1V}{3k} \right)$$

$$I_{in} = \left(\frac{1V - 0V}{3k} \right) - 100I_b + \left(\frac{1V}{5k} \right)$$

$$I_{in} = 33.87mA$$

So

$$R_{th} = \frac{V_{in}}{I_{in}} = \frac{1V}{33.87mA} = 29.53\Omega$$



Example 2: Determine the Thevenin equivalent

V_{th} : Find V_3 (open circuit voltage)

$$V_2 = -1000V_1$$

$$\left(\frac{V_1-1}{1k}\right) + \left(\frac{V_1-V_3}{10k}\right) = 0$$

$$\left(\frac{V_3-V_1}{10k}\right) + \left(\frac{V_3-V_2}{100}\right) = 0$$

Solve (time passes....)

$$A = [1000, 1, 0; 11, 0, -1; -1, -100, 101];$$

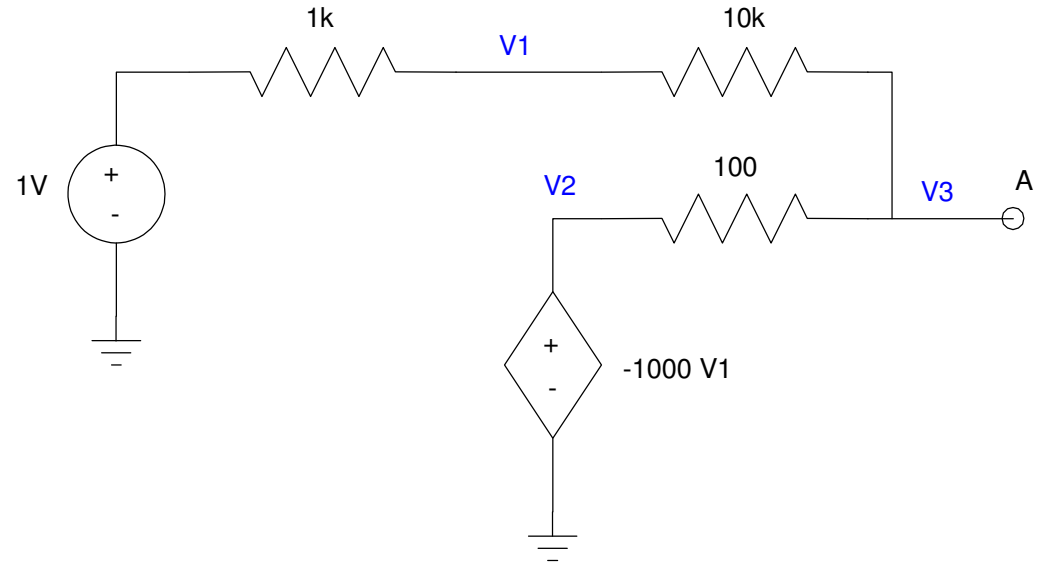
$$B = [0; 10; 0];$$

$$V = \text{inv}(A) * B$$

$$V_1 = 0.0100$$

$$V_2 = -9.9891$$

$$V_3 = -9.8901 = V_{th}$$



R_{th} :

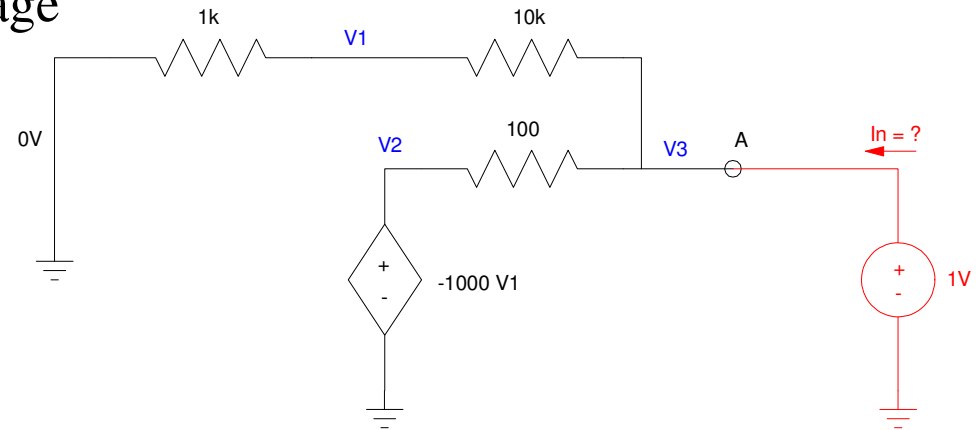
- Turn off voltage sources and measure the resistance
- Since this isn't obvious, apply a 1V test voltage

$$V_1 = \left(\frac{1k}{1k+10k} \right) \cdot 1V = 90.91mV$$

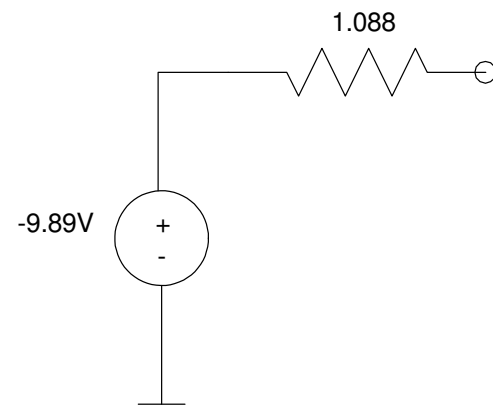
$$V_2 = -1000V_1 = -90.91V$$

$$I_{in} = \left(\frac{1V}{11k} \right) + \left(\frac{1V - (-90.91V)}{100} \right) = 919.2mA$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{1V}{919.2mA} = 1.088\Omega$$



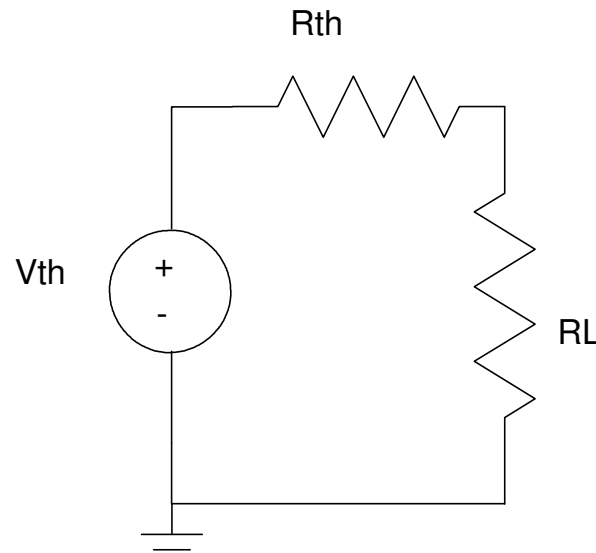
So, the Thevenin equivalent is...



Max Power Transfer

What resistance (R_L) maximizes the power to the load?

- (V_{th} , R_{th}) models a solar panel. What load maximizes the power the solar cell produces?
- (V_{th} , R_{th}) models a stereo. What speaker (R_L) maximizes the output power?



Case 1: R_{th} is fixed. Find R_L to maximize the power to the load.

Note that there is a maximum point:

- If $R_L = 0$, the power to the load is zero
- If $R_L = \text{infinity}$, $I = 0$ and the power to the load is again zero.

Somewhere between $R_L = 0$ and $R_L = \text{infinity}$ is a maximum power transfer.

$$I = \left(\frac{V_{th}}{R_{th} + R_L} \right)$$

$$P = I^2 R_L$$

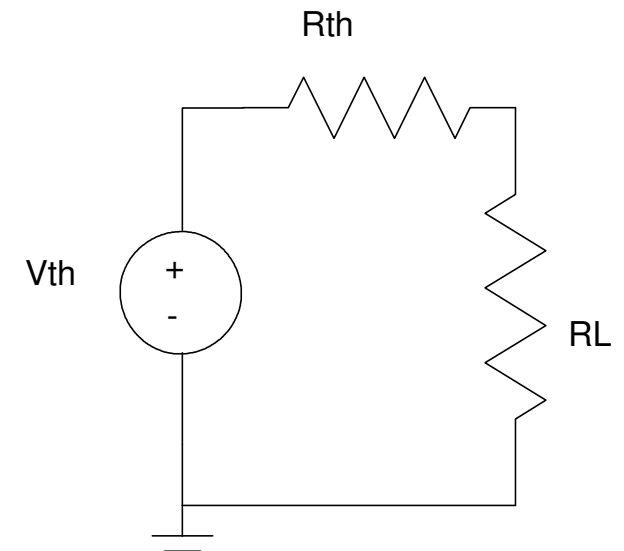
$$P = \left(\frac{R_L}{(R_{th} + R_L)^2} \right) V_{th}^2$$

$$\frac{d}{dR_L} \left(\frac{R_L}{(R_{th} + R_L)^2} \right) = 0$$

$$(R_L + R_{th})(R_{th} - R_L) = 0$$

$$R_L = R_{th} \quad \text{maximum}$$

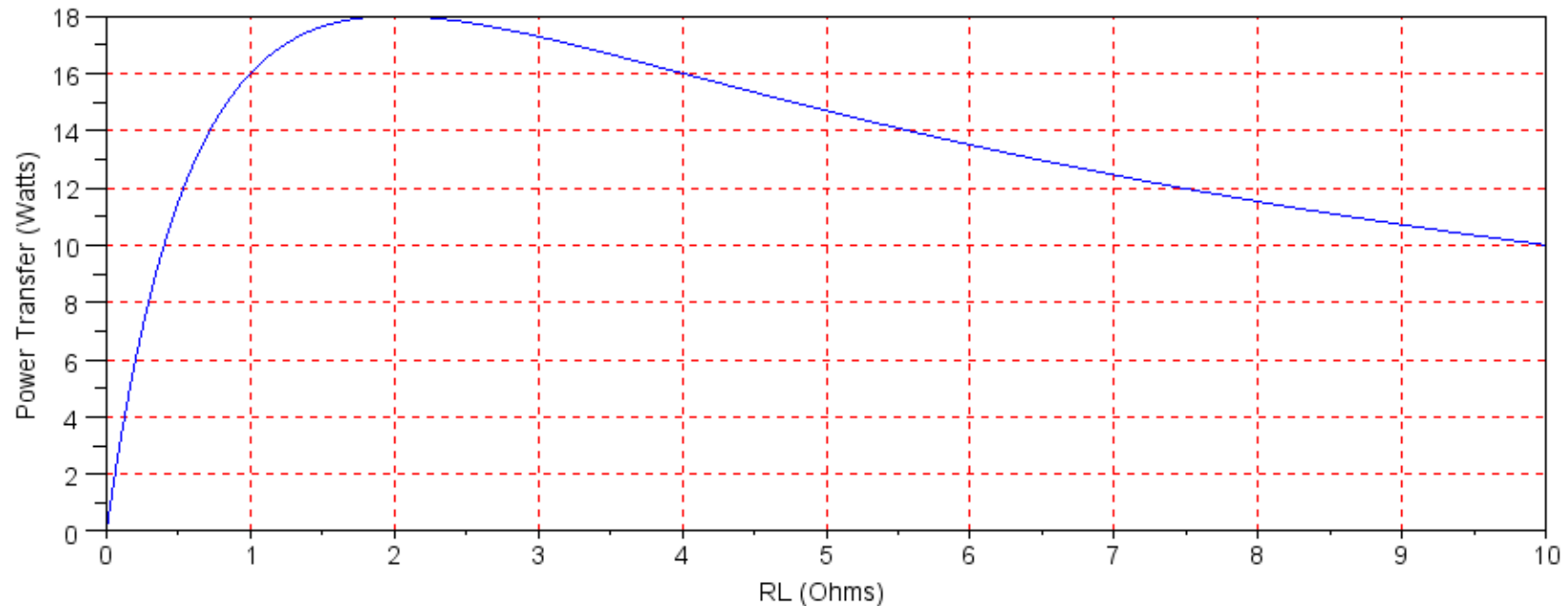
$$R_L = -R_{th} \quad \text{minimum}$$



Assume for instance that $V_{th} = 12V$ and $R_{th} = 2 \text{ Ohms}$:

```
Vin = 12;  
Rth = 2;  
RL = [0:0.01:10]';  
I = Vin ./ (Rth + RL);  
P = (I .^ 2) .* RL;
```

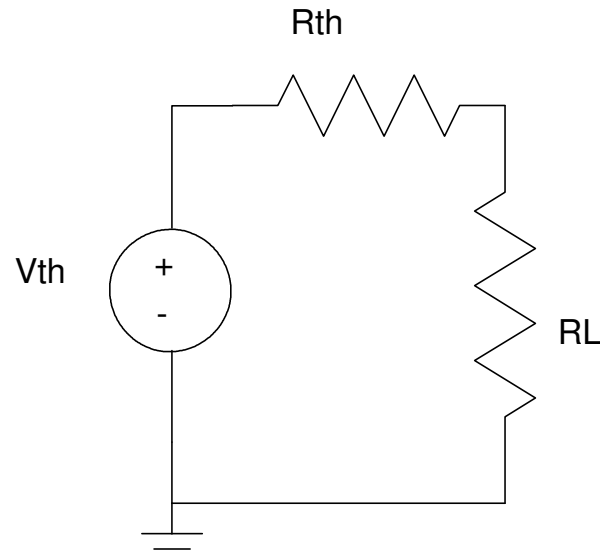
Note that at maximum power transfer, you are 50% efficient



Case 2: R_L is fixed. Find R_{th} to maximize the power to the load.

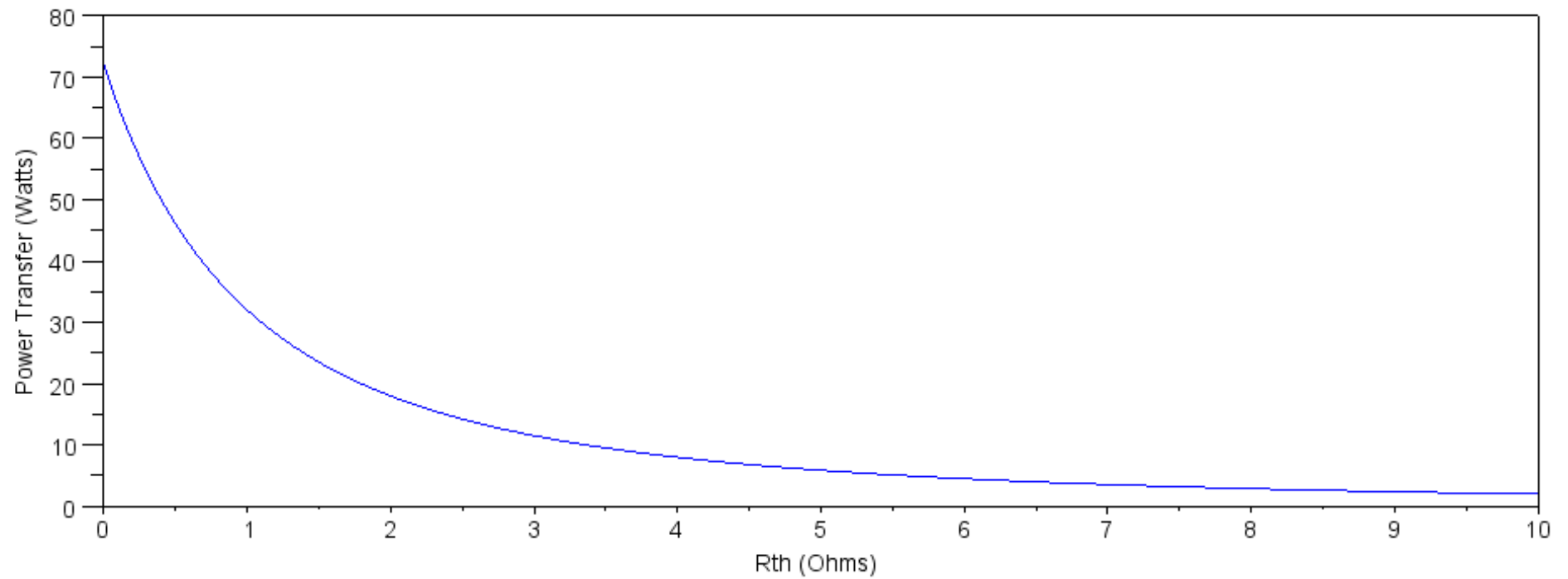
The solution in this case is *not* $R_{th} = R_L$

- Maximum is when $R_{th} = -R_L$
- Closest you can get is $R_{th} = 0$



$V_{th} = 12V$ and $R_L = 2 \text{ Ohms}$

```
Vth = 12;  
RL = 2;  
Rth = [0:0.01:10]';  
I = Vth ./ (RL + Rth);  
P = ( I .^ 2 ) .* RL;  
plot(Rth, P);
```



Case 3: Find R_L to maximize the efficiency of the system.

Efficiency is the power to the load divided by the total power dissipated.

$$eff = \left(\frac{P_{Load}}{P_{total}} \right)$$

or

$$eff = \frac{I^2 R_L}{I^2 (R_{th} + R_L)} = \left(\frac{R_L}{R_L + R_{th}} \right)$$

Max efficiency is

- $R_L = \text{infinity}$
- Power = 0

That's one of the problems of delivering power to a load

- For high efficiency, you want $R_L \gg R_{th}$
- For maximum power, you want $R_L = R_{th}$

