Inductors

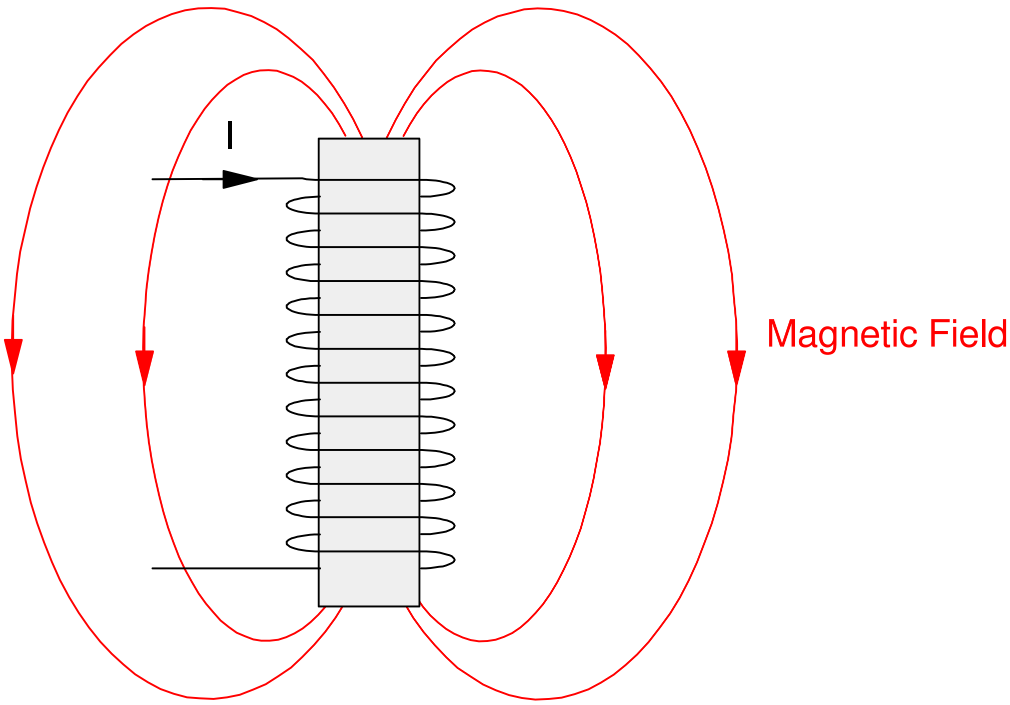
ECE 211 Circuits I

Lecture #18

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

Inductors

- Capacitors deal with voltage and store energy in an electric field.
- Inductors deal with current and store energy in a magnetic field.

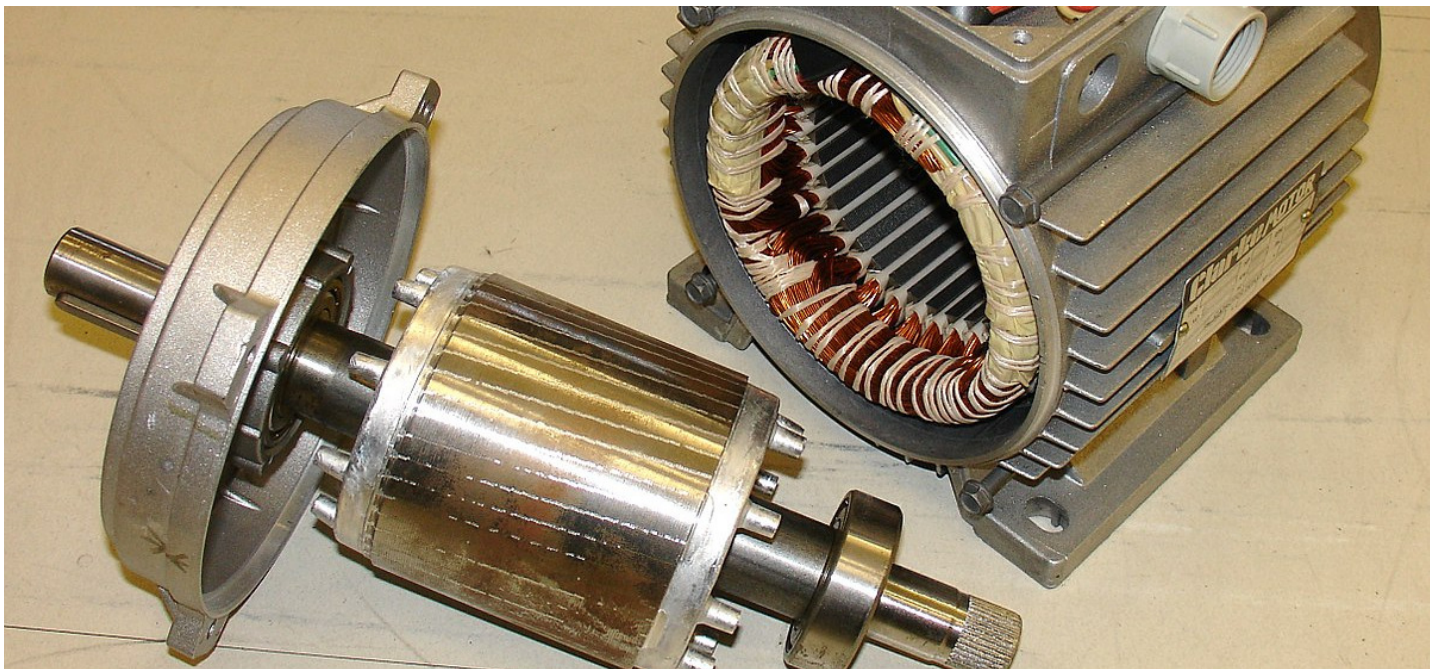


In general, inductors are avoided if at all possible:

- Unlike electric fields and capacitors, magnetic fields do not have starting point and an ending point. That means that the magnetic fields extend outside the inductor and affect other components in a circuit.
- The core of an inductor almost has to be iron. Iron is about the only material which is strongly magnetic. This dependence upon iron makes inductors heavy, lossy, and only capable of supporting a limited amount of current until the iron saturates.
- The large amount of windings necessary to make an inductor creates a large resistive element with inductors, also reducing the efficiency.

Sometimes, you just can't avoid using inductors:

- Speakers rely upon electromagnets to work. An electromagnet is inherently an inductor.
- Motors rely upon electromagnetics to work. Again, any time you have an electromagnet you inherently have an inductor.
- Transmission lines are wires that carry electricity large distances. Current through a wire creates a magnetic field. This magnetic field stores energy, and again, created inductance.



Example 1: Determine R & L

- 100 windings of 36 gage wire copper wire (1.38 Ohms / meter)[1]
- Cross sectional area = 25mm^2
- Iron core with relative permeability of 800

Solution: From Electronics Tutorials[2]

$$L = \mu N^2 A / l = 0.0101H$$

$$R = (100 \text{ windings})\left(0.02\frac{\text{m}}{winding}\right)(1.38\tfrac{\Omega}{m}) = 2.77\Omega$$

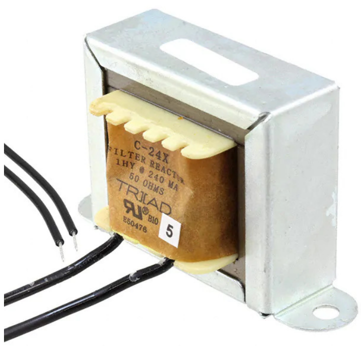
You can't avoid resistance with inductors

1 https://www.engineeringtoolbox.com/copper-wire-d_1429.html

2 https://www.electronics-tutorials.ws/inductor/inductor.html

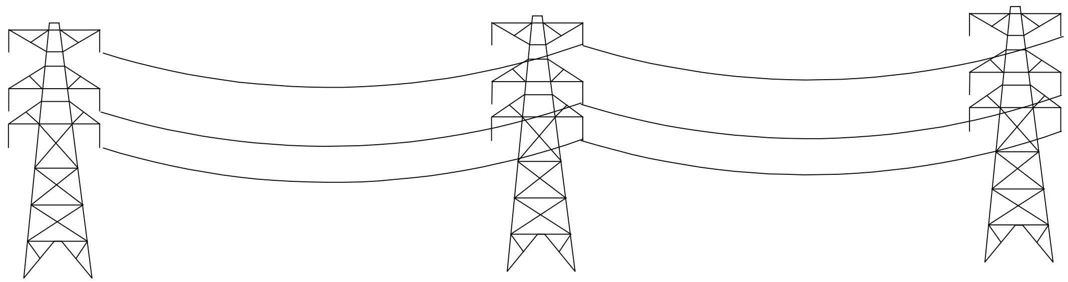
This shows up in inductors you can get from Digikey:

- If you want large inductance, you have high resistance
- If you want low resistance, you get low inductance
- If you want a small inductor, you get both: low inductance and high resistance

CX24	5250-RC
	
1 H	0.0001H
50 Ohms	0.216 Ohms
240mA (max)	2 A
$10.23 ea	$2.11

Example 2: Determine the inductance of a copper transmission line:

- Length = 1km
- radius = 1cm
- frequency = 60Hz



From Wikipedia[3]

$$L = \frac{\mu_0}{2\pi} l(A - B + C) = 2.30 \frac{mH}{km}$$

$$A = \ln\left(\frac{l}{r} + \sqrt{\left(\frac{l}{r}\right)^2 + 1}\right) = 12.20$$

$$B = \frac{1}{\frac{r}{l} + \sqrt{1 + \left(\frac{r}{l}\right)^2}} \approx 1$$

$$C = \frac{1}{4 + r\sqrt{\frac{2}{\rho}\omega\mu}} = 0.1569$$

[3] https://en.wikipedia.org/wiki/Inductor#Inductance_formulas

VI Characteristics for an Inductor

$$V = L\frac{dI}{dt}$$

$$E = \frac{1}{2}LI^2$$

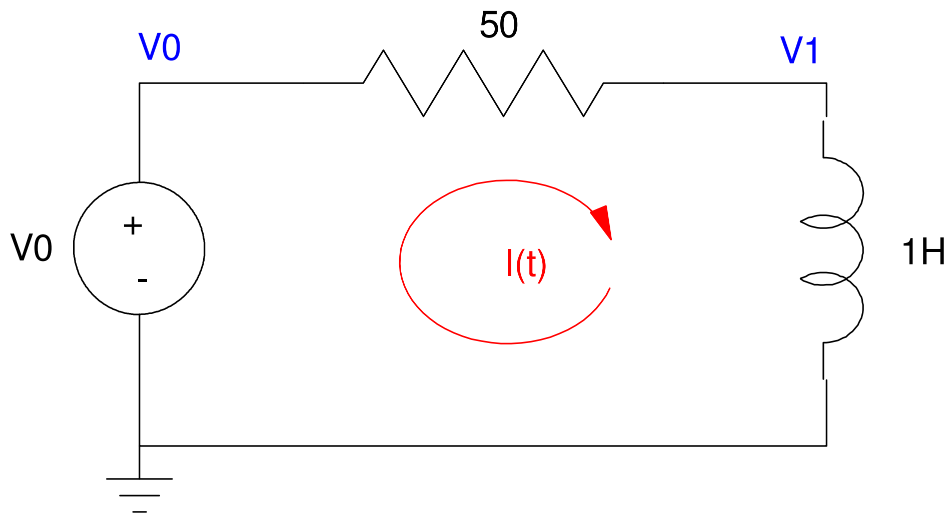
- Each inductor adds a 1st-order differential equation to a circuit
- The differential equation is in terms of current.

Example 3:

$$V_0(t) = 10\,u(t)$$

$$V_1 = L\frac{dI}{dt} = V_0 - 50I$$

$$\frac{dI}{dt} = V_0 - 50I$$



Solve using numerical integration like we did with capacitors:

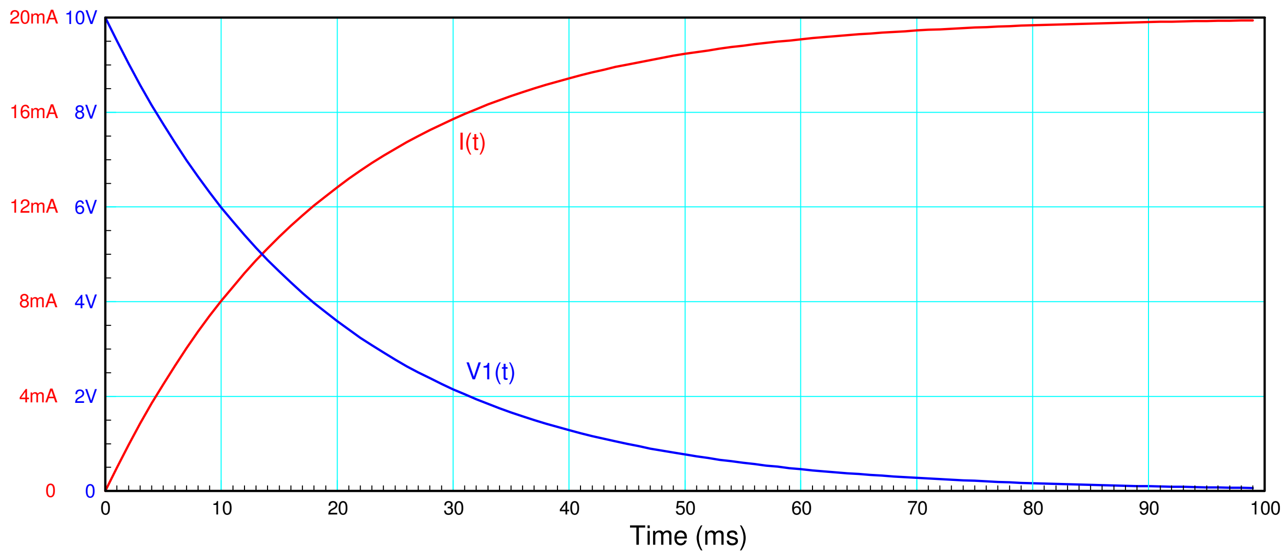
```
V0 = 10;
I = 0
dt = 0.001;
t = 0;
Y = [];

while(t < 0.1)
   dI  = V0 - 50*I;

   I = I + dI * dt;
   t = t + dt;
   V1 = V0 - 50*I;

   Y = [Y ; [I, V1] ];
   end

t = [0:length(Y)-1]' * dt;
plot(t,Y)
```



Voltage Amplification

- With inductors, you can get more voltage out than you put in
- If the current suddenly goes to zero, the voltage goes to infinity
- This is how alternators and spark plugs work in your car.

$$V = L\frac{dI}{dt}$$

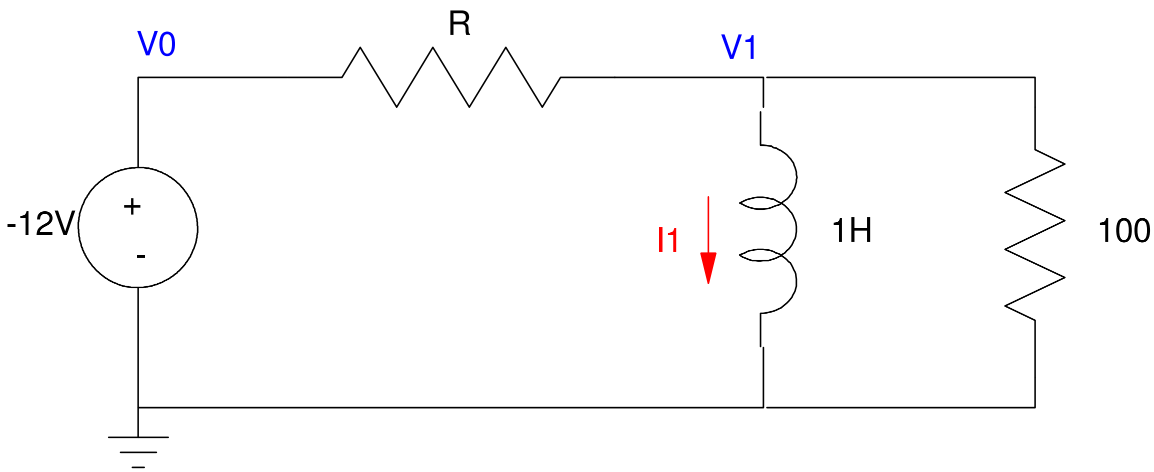
Example 4:

- Let R varies from 5 Ohms to 1000 Ohms (switch closed or switch open)

Solution: Write the differential equation for this circuit

$$\left(\frac{V_1 - V_0}{R}\right) + \left(\frac{V_1}{100}\right) + I_1 = 0$$

$$V_1 = L\frac{dI_1}{dt}$$



giving

$$\left(\frac{1}{R} + \frac{1}{100}\right)V_1 = \left(\frac{V_0}{R}\right) - I_1$$

$$\left(\frac{1}{R} + \frac{1}{100}\right)\left(\frac{dI_1}{dt}\right) = \left(\frac{V_0}{R}\right) - I_1$$

$$\frac{dI_1}{dt} = \left(\frac{1}{R} + \frac{1}{100}\right)^{-1}\left(\left(\frac{V_0}{R}\right) - I_1\right)$$

Solve numerically:

```
V0 = -12;
I = 0
dt = 0.001;
t = 0;
Y = [];

while(t < 2)
   if(sin(2*pi*t) > 0)
       R = 5;
   else
       R = 1000;
   end

   dI=(V0/R-I)/(1/R+1/100);

   I = I + dI * dt;
   t = t + dt;
   V1 = V0 - I*R;

   Y = [Y ; [I, V1] ];
end
```