Complex Numbers

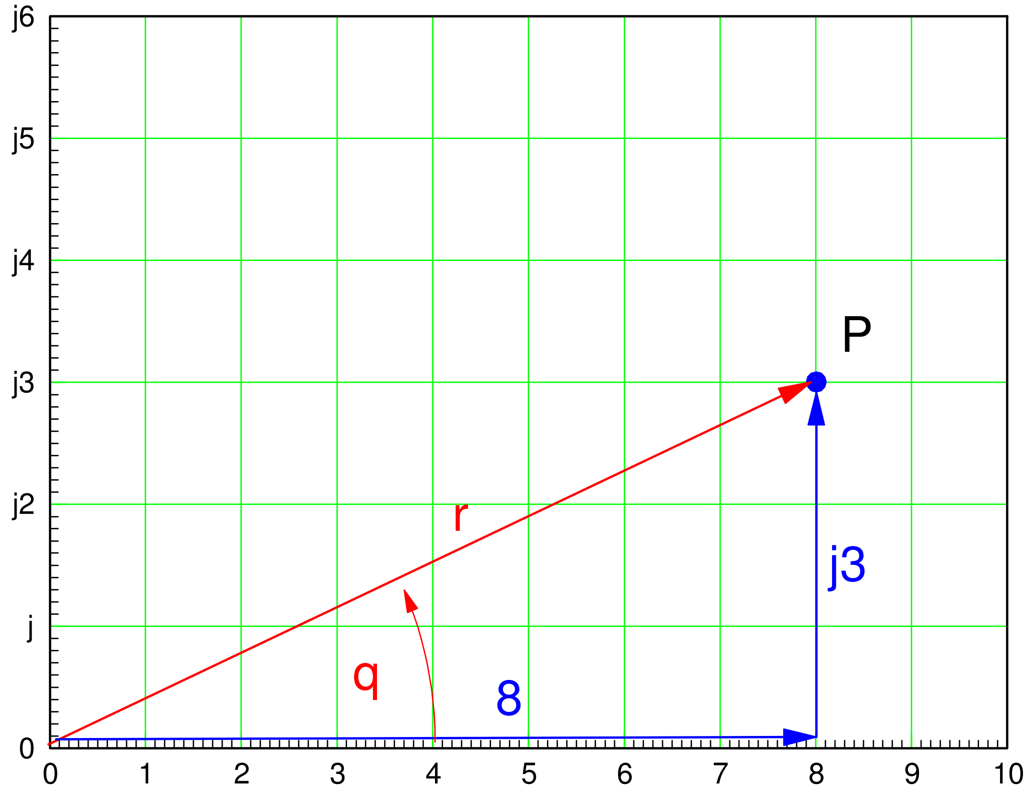
ECE 211 Circuits I

Lecture #21

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

Objective:

- Become familiar with using complex numbers for addition, subtraction, multiplication, and division



The number Zero:

- Zero is an odd concept: something that represents nothing
- Zero isn't needed: the Romans had an extensive economy without the number zero.
- Without zero, addition becomes difficult.
- Without zero, multiplication becomes difficult

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      MXXIII                MXXIII
+        CVI           *       CVI
=          ?           =        ?
```

Negative Numbers:

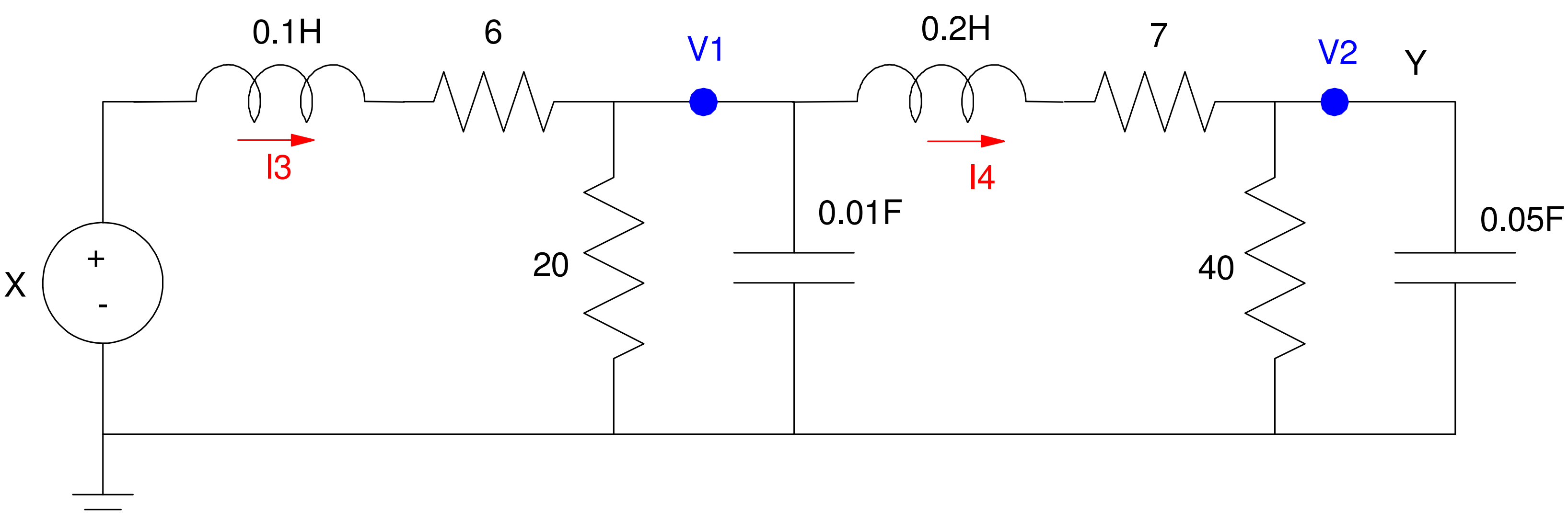
- Negative numbers are even more starange
- Their invention allowed Holland to become a world power

By keep tracking of credits (+) and debits (-), the double-entry book-keeping system allowed Dutch merchants to understand what ventures were profitable and which were not.

Complex Numbers

To solve differential equations (i.e. circuits) with sinudoidal inputs

- Solve 2N equations for 2N unknowns
  - Using sine() and cosine() functions
- Solve N equations for N unknowns
  - Using complex numbers

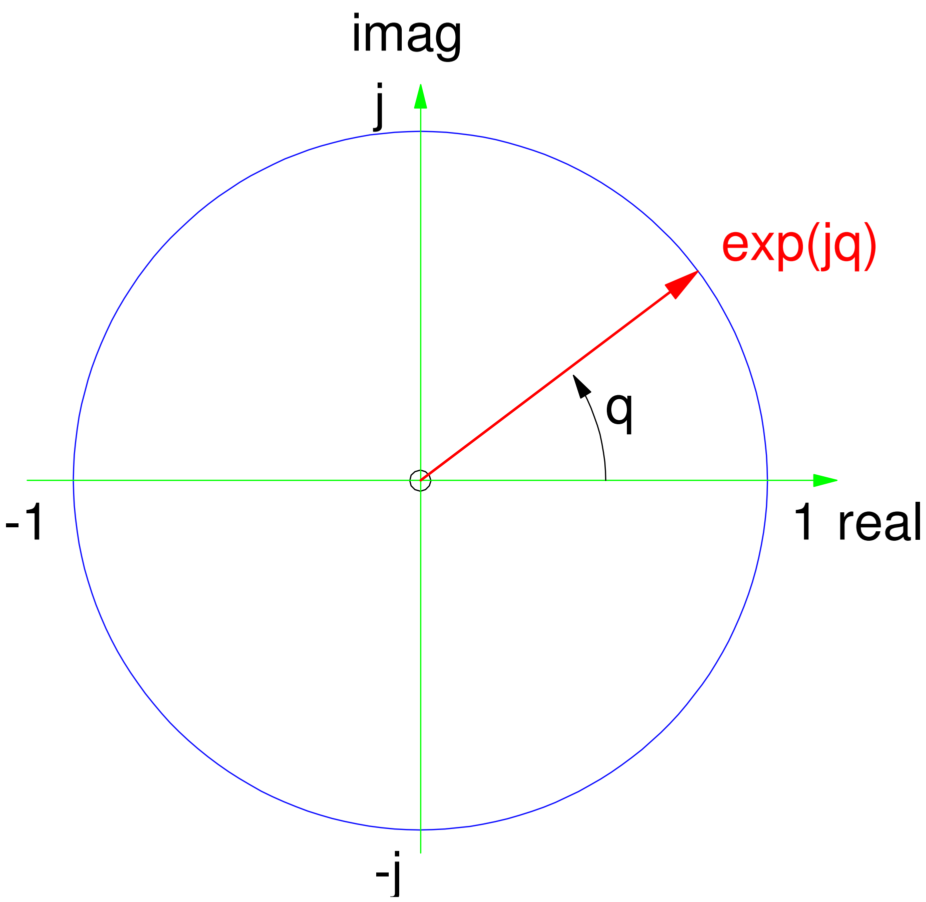


Definition of Complex Numbers

Two basic definitions for complex numbers are

$$j^2 = -1$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$



Polar and Rectangular Form:

A complex number can be represented in rectangular or polar form

$$x + jy \ \ r \cdot e^{j\theta} = r\angle\theta$$

The relationship is

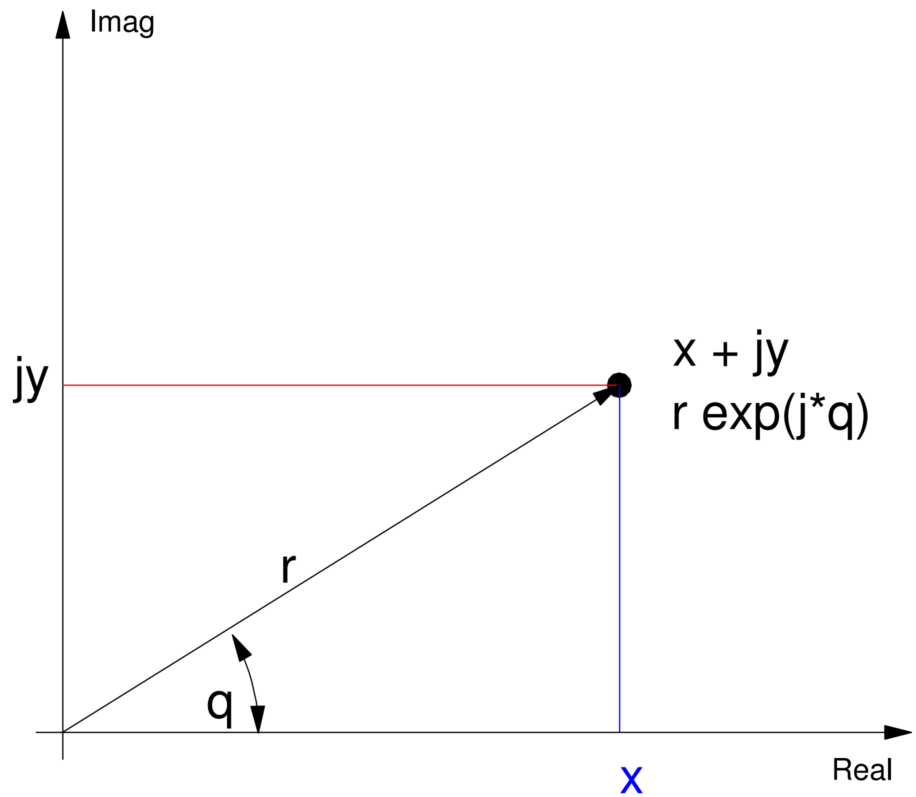
$$r = \sqrt{x^2 + y^2}$$

$$\tan(\theta) = \frac{y}{x}$$

or

$$x = r \cdot \cos(\theta)$$

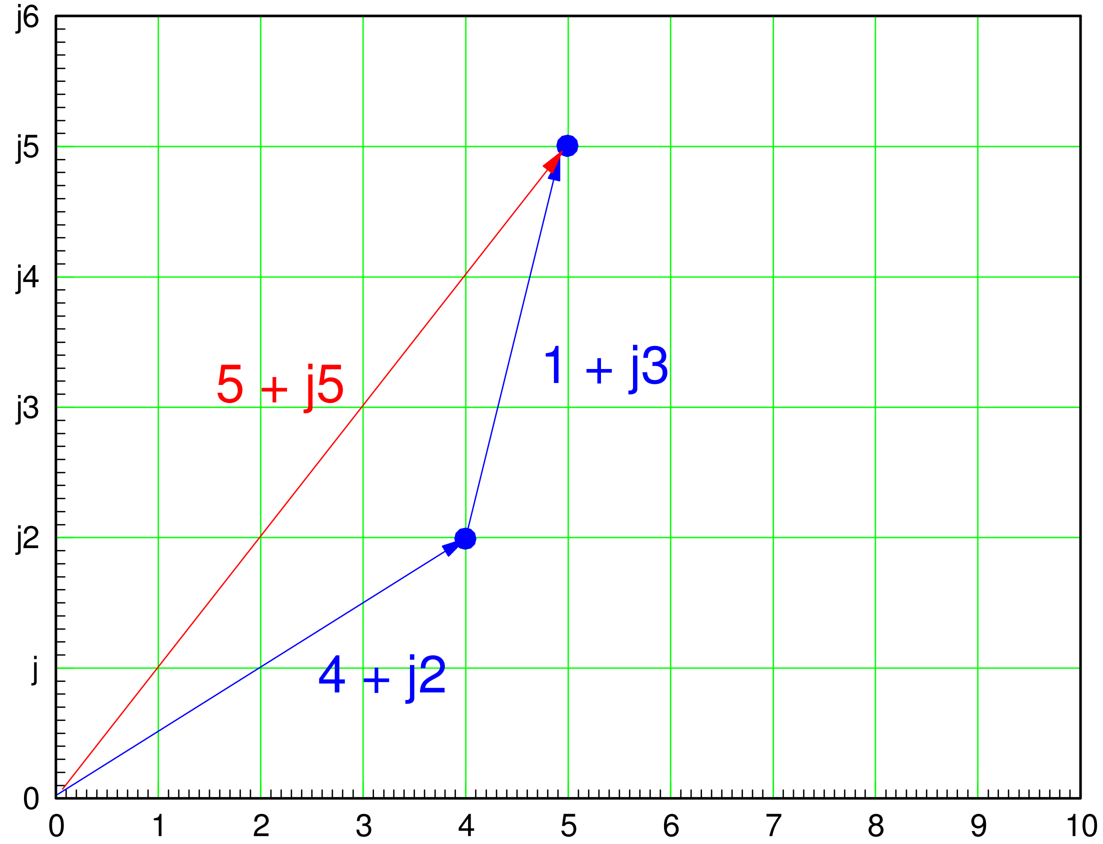
$$y = r \cdot \sin(\theta)$$



Addition

Add real to real, complex to complex

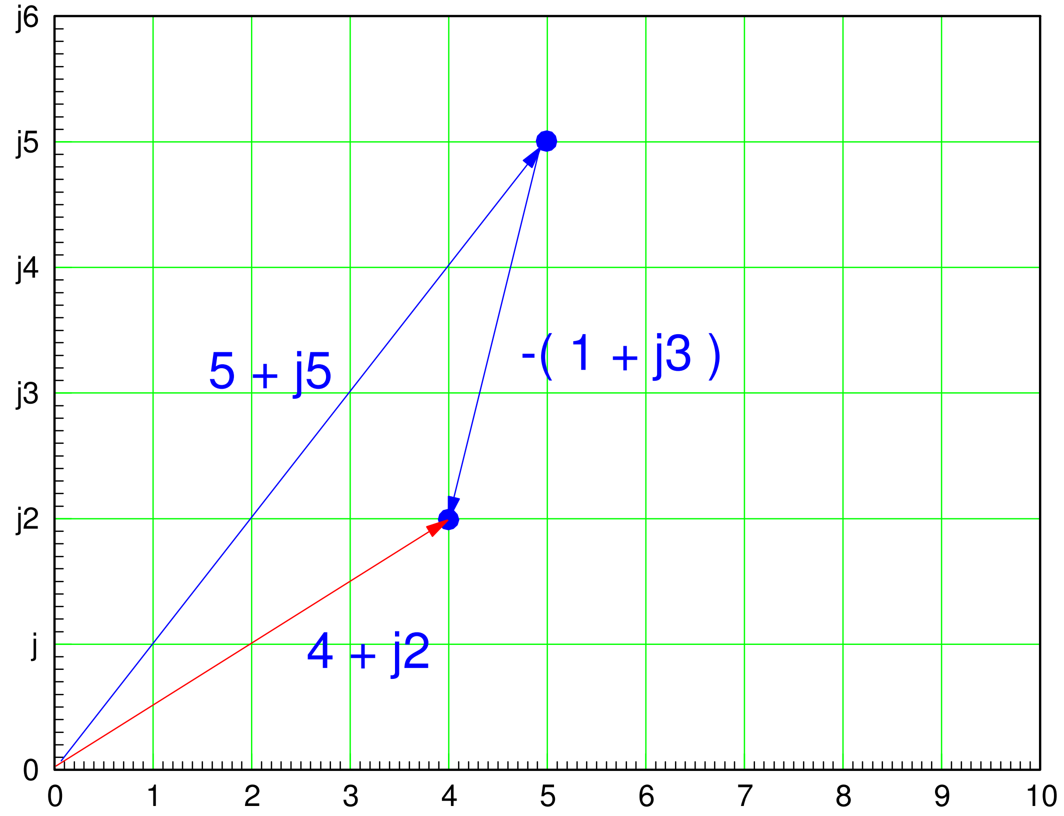
$$
\begin{array}{rr}
 & 4 + j2 \\
+ & 1 + j3 \\
= & 5 + j5
\end{array}
$$



Subtraction:

Subtract real from real, complex from complex

$$
\begin{array}{ll}
 & 5+j5 \\
- & 1+j3 \\
= & 4+j2
\end{array}
$$



Multiplication

Rectangular Form:

$$(2 + j3)(4 + j5) = (2 \cdot 4) + (2 \cdot j5) + (j3 \cdot 4) + (j3 \cdot j5)$$

$$= (8) + (j10) + (j12) + (j^2 15)$$

Note that j2 = -1:

$$= (8 - 15) + j(10 + 12)$$

$$= -7 + j22$$

Multiplication is easier in polar form:

$$(a\angle\theta)(b\angle\phi) = ab\angle(\theta + \phi)$$

$$(a \cdot e^{j\theta})(b \cdot e^{j\phi}) = ab \cdot e^{j(\theta+\phi)}$$

Complex Conjugates:

The complex conjugate (symbol *) is

$$(x+jy)^* = x - jy$$

A number multiplied by its complex conjugate is

- The real squared, plus
- The imaginary squared

$$(x+jy)(x-jy) = (x^2 + jxy - jxy - j^2 y)$$
$$= x^2 + y^2$$

Division

Polar Form

$$\left(\frac{a\angle\theta}{b\angle\phi}\right) = \left(\frac{a}{b}\right)\angle(\theta - \phi)$$

Rectangular Form

$$\left(\frac{a+jb}{c+jd}\right) = \left(\frac{a+jb}{c+jd}\right)\left(\frac{c-jd}{c-jd}\right)$$

$$= \left(\frac{ac-bd+jbc-jad}{c^2+d^2}\right)$$

$$= \left(\frac{ac-bd}{c^2+d^2}\right) + j\left(\frac{bc-ad}{c^2+d^2}\right)$$

HP Calculators

I ***strongly*** recommend getting an HP calculator

- HP35s: $52 on Amazon
- Free42: free app for the HP42 (great calculator)

You will be using complex numbers extensively in Electrical and Computer engineering.

Get a calculator that does complex numbers

I've found that HP calculators are worth about 10 points on midterms (they breeze through complex math)