Voltage Nodes with Phasors

ECE 211 Circuits I

Lecture #25

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

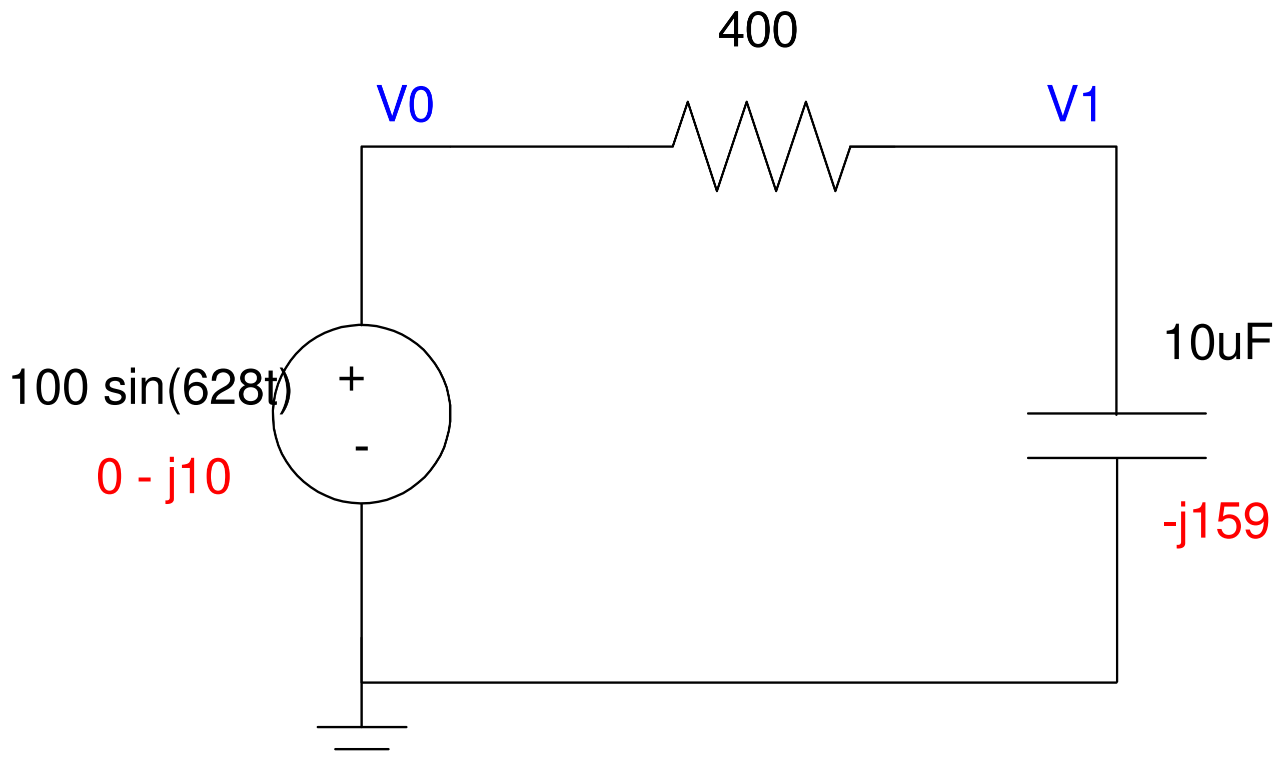
Voltage Nodes:

- The sum of the current from a node must sum to zero
- For AC, both the real part and complex part must sum to zero

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

Example 1: RC Circuit

Determine the node voltages for the following circuit:



Step 1: Replace the elements with their phasor values (shown in red)

Step 2: Write the voltage node equations

- Same as DC

$$V_0 = 0 - j100$$

$$\left(\frac{V_1 - V_0}{400}\right) + \left(\frac{V_1}{-j159}\right) = 0$$

Solve:

$$V_1 = \left(\frac{-j159}{-j159+400}\right)(0 - j100)$$

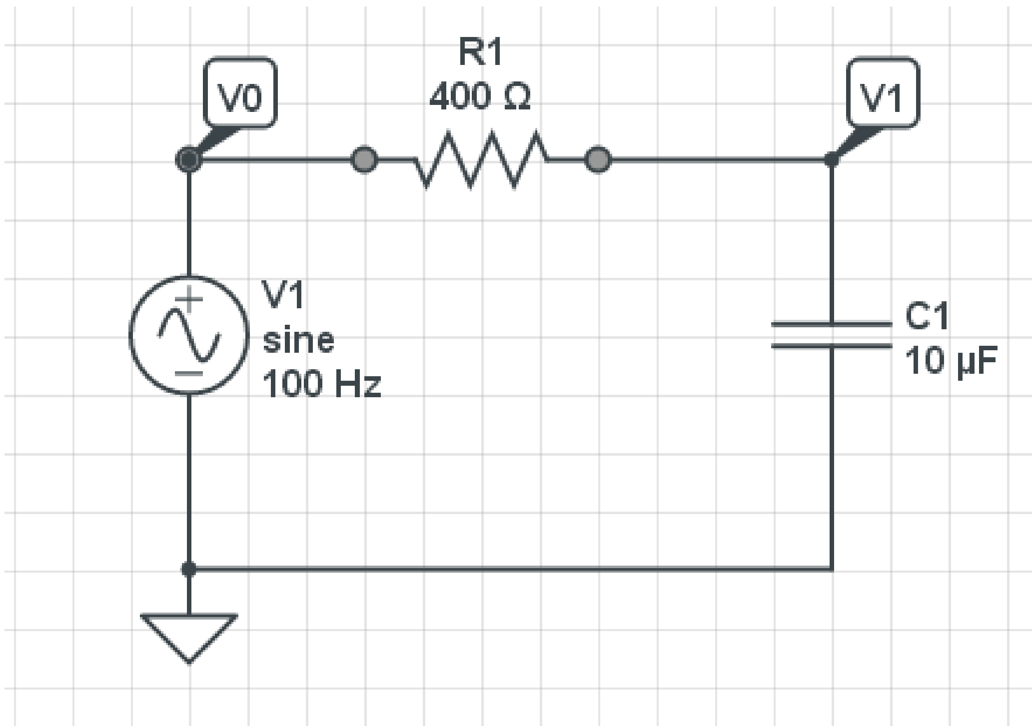
$$V_1 = 34.326 - j13.645$$

$$v_1(t) = 34.326\cos(628t) + 13.645\sin(628t)$$

Verify with CircuitLab

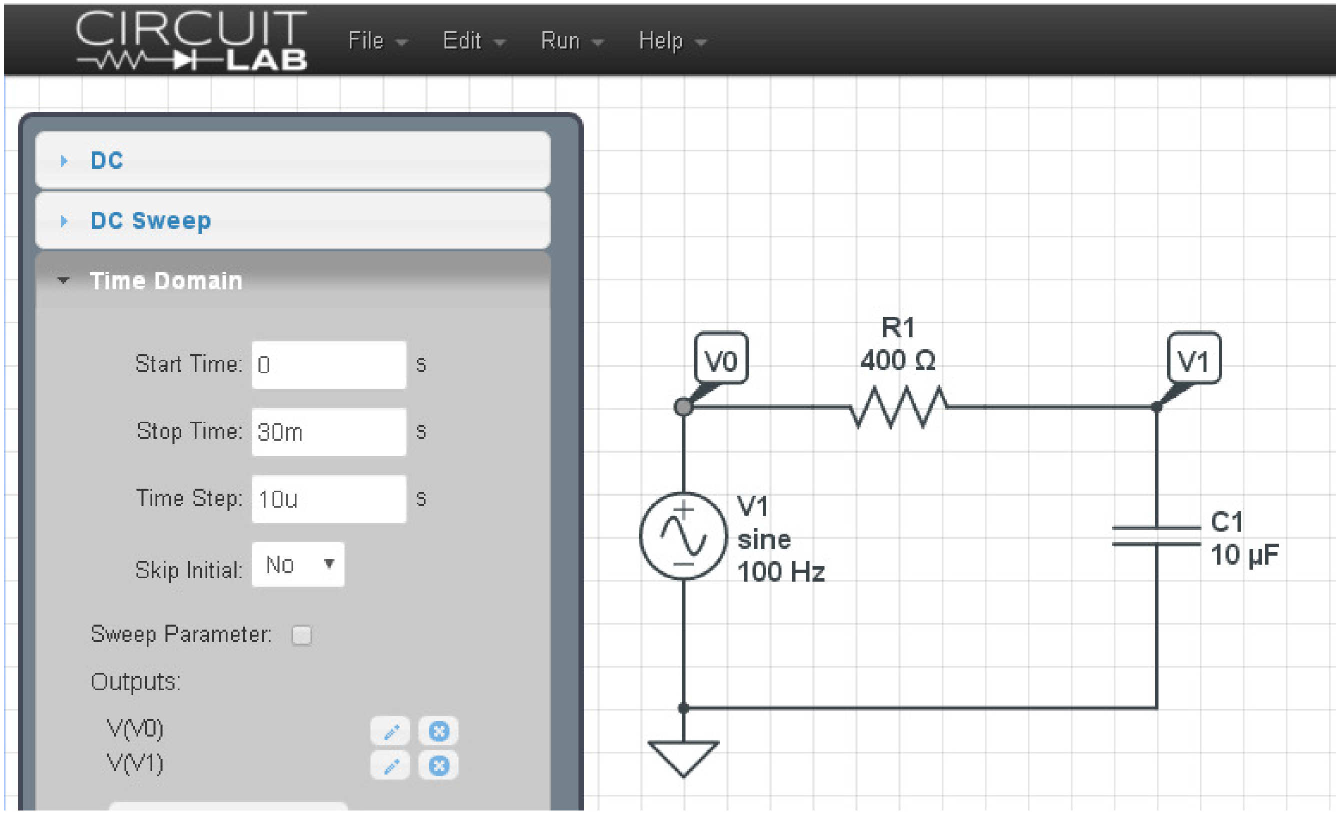
Make the input a sine wave with

- no DC offset
- 100V amplitude
- 100Hz   (628 rad/sec)

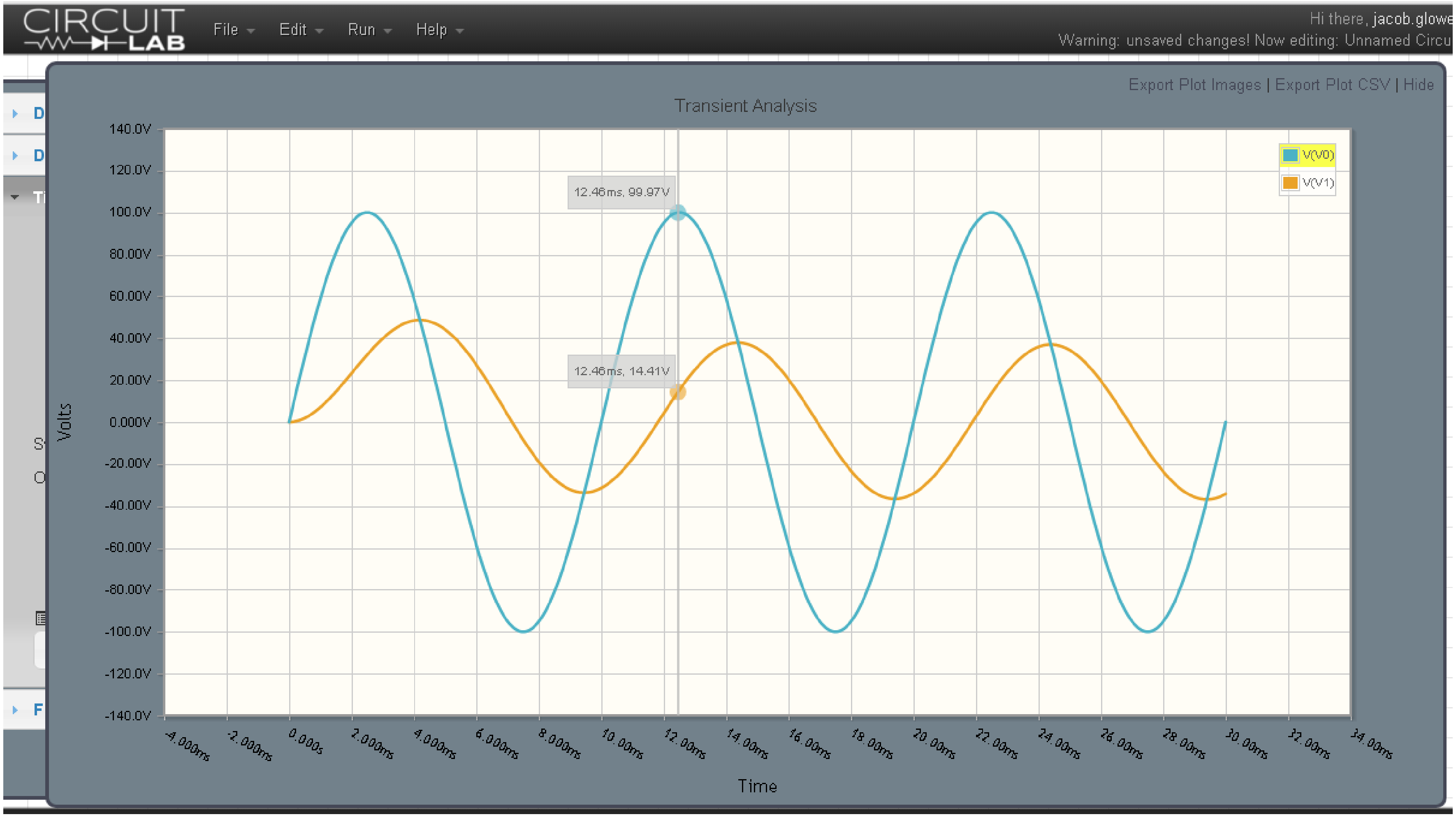


Run a transient response for

- 30ms (3 cycles)
- 10us step size



This results in a simulated input and output waveform:



The peak tells you the amplitude (Vp)

- Vp = 37.87V
- Delayed by 4.39ms from the zero crossing of V0

The zero crossing is used as a reference since V0 = 100 sin(628t) (V0 is zero at t=0). The phase shift is thus

$$\theta_1 = -\left(\frac{4.39ms\text{ delay}}{10\text{ms period}}\right)360^0$$

$$\theta_1 = -158^0$$

hence

$$V_1 = 37.87\angle -158^0$$

This matches our calculations

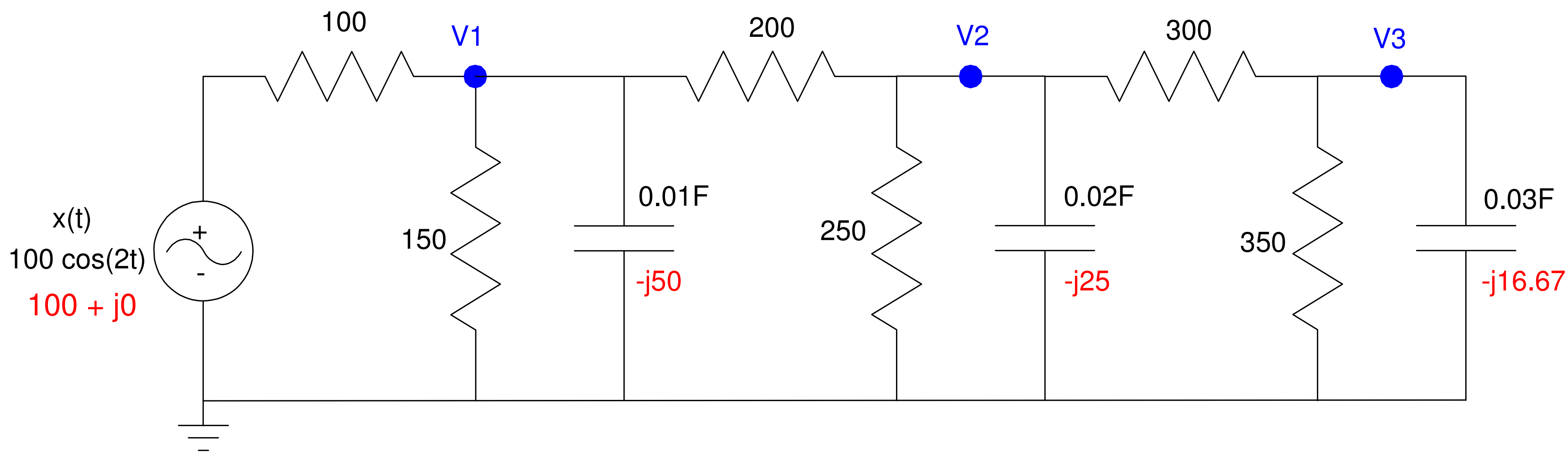
$$V_1 = 34.326 - j13.645$$

$$V_1 = 36.939\angle -158.3^0$$

Example 2: 3-Stage RC Circuit

Find the voltages for the following circuit when the input is

$$x(t) = 100 \cos(2t)$$



Step 1: Express in Phasor form

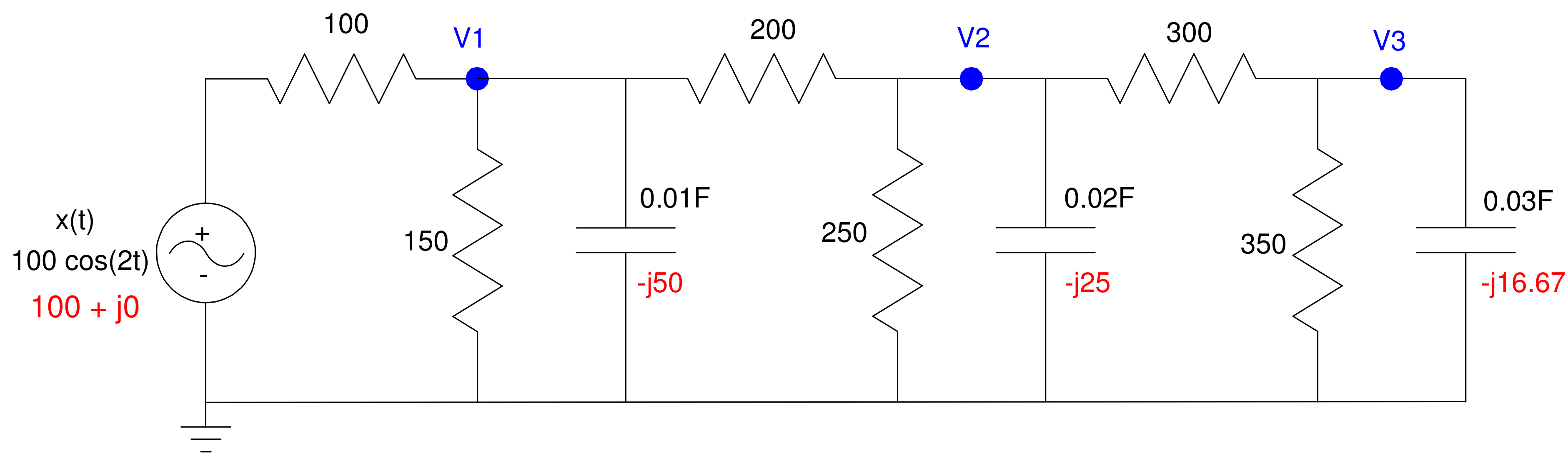
- (shown in red)

Step 2: Write N equations for N unknowns

$$\left(\frac{V_1 - X}{100}\right) + \left(\frac{V_1}{150}\right) + \left(\frac{V_1}{-j50}\right) + \left(\frac{V_1 - V_2}{200}\right) = 0$$

$$\left(\frac{V_2 - V_1}{200}\right) + \left(\frac{V_2}{250}\right) + \left(\frac{V_2}{-j25}\right) + \left(\frac{V_2 - V_3}{300}\right) = 0$$

$$\left(\frac{V_3 - V_2}{300}\right) + \left(\frac{V_3}{350}\right) + \left(\frac{V_3}{-j16.67}\right) = 0$$



Step 3: Solve.

First, group terms

$$\left(\frac{1}{100}+\frac{1}{150}+\frac{1}{-j50}+\frac{1}{200}\right)V_1+\left(\frac{-1}{200}\right)V_2=\left(\frac{1}{100}\right)X$$

$$\left(\frac{-1}{200}\right)V_1+\left(\frac{1}{200}+\frac{1}{250}+\frac{1}{-j25}+\frac{1}{300}\right)V_2+\left(\frac{-1}{300}\right)V_3=0$$

$$\left(\frac{-1}{300}\right)V_2+\left(\frac{1}{300}+\frac{1}{350}+\frac{1}{-j16.67}\right)V_3=0$$

Place in matrix form

$$\begin{bmatrix}\left(\frac{1}{100}+\frac{1}{150}+\frac{1}{-j50}+\frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0\\ \left(\frac{-1}{200}\right) & \left(\frac{1}{200}+\frac{1}{250}+\frac{1}{-j25}+\frac{1}{300}\right) & \left(\frac{-1}{300}\right)\\ 0 & \left(\frac{-1}{300}\right) & \left(\frac{1}{300}+\frac{1}{350}+\frac{1}{-j16.67}\right)\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\end{bmatrix}=\begin{bmatrix}\left(\frac{1}{100}\right)\\0\\0\end{bmatrix}X$$

Solve in Matlab

```
V = inv(A)*B*X

   24.2853 -23.2416i
   -1.7971 - 3.5726i
   -0.2066 + 0.0785i
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$$v_1(t) = 24.28\cos(2t) + 23.24\sin(2t)$$

$$v_2(t) = -1.79\cos(2t) + 3.57\sin(2t)$$

$$v_1(t) = 0.21\cos(2t) + 0.08\sin(2t)$$

$$v_1(t) = 33.61\cos(2t - 43.7^0)$$

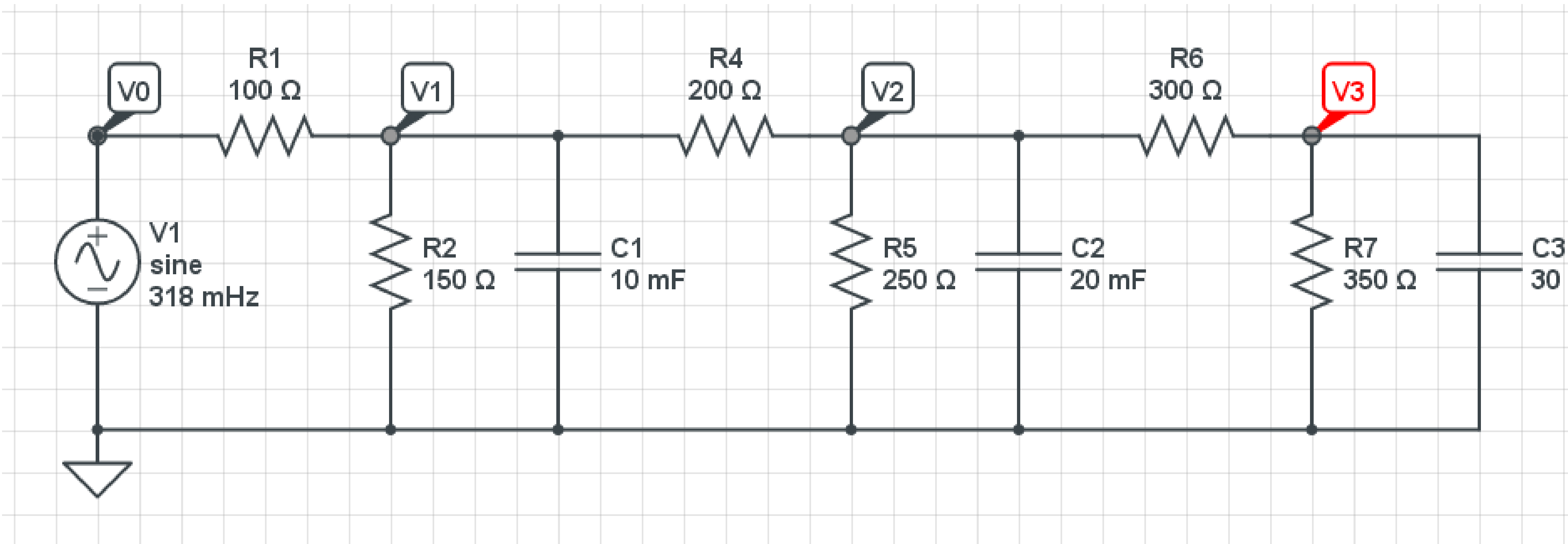
$$v_2(t) = 3.999\cos(2t - 116.7^0)$$

$$v_3(t) = 0.22\cos(2t + 159.2^0)$$

CircuitLab Simulation

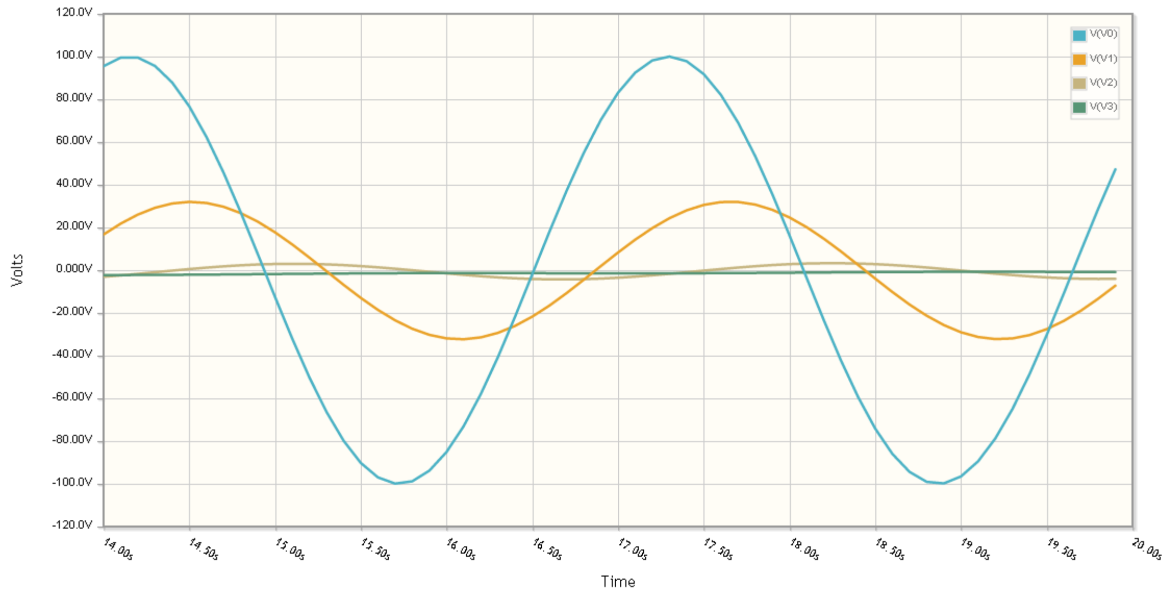
Build the circuit

- Note: 2 rad/sec = 0.318 Hz



Run a transient simulation

- max time = 20 seconds (6 cycles)
- time step = 20ms (1000 points on the plot)



This matches our calculations:

$$v_1(t) = 33.61 \cos(2t - 43.7^0)$$

$$v_2(t) = 3.999 \cos(2t - 116.7^0)$$