Current Loops with Phasors

ECE 211 Circuits I

Lecture #26

Please visit Bison Academy for corresponding lecture notes, homework sets, and solutions

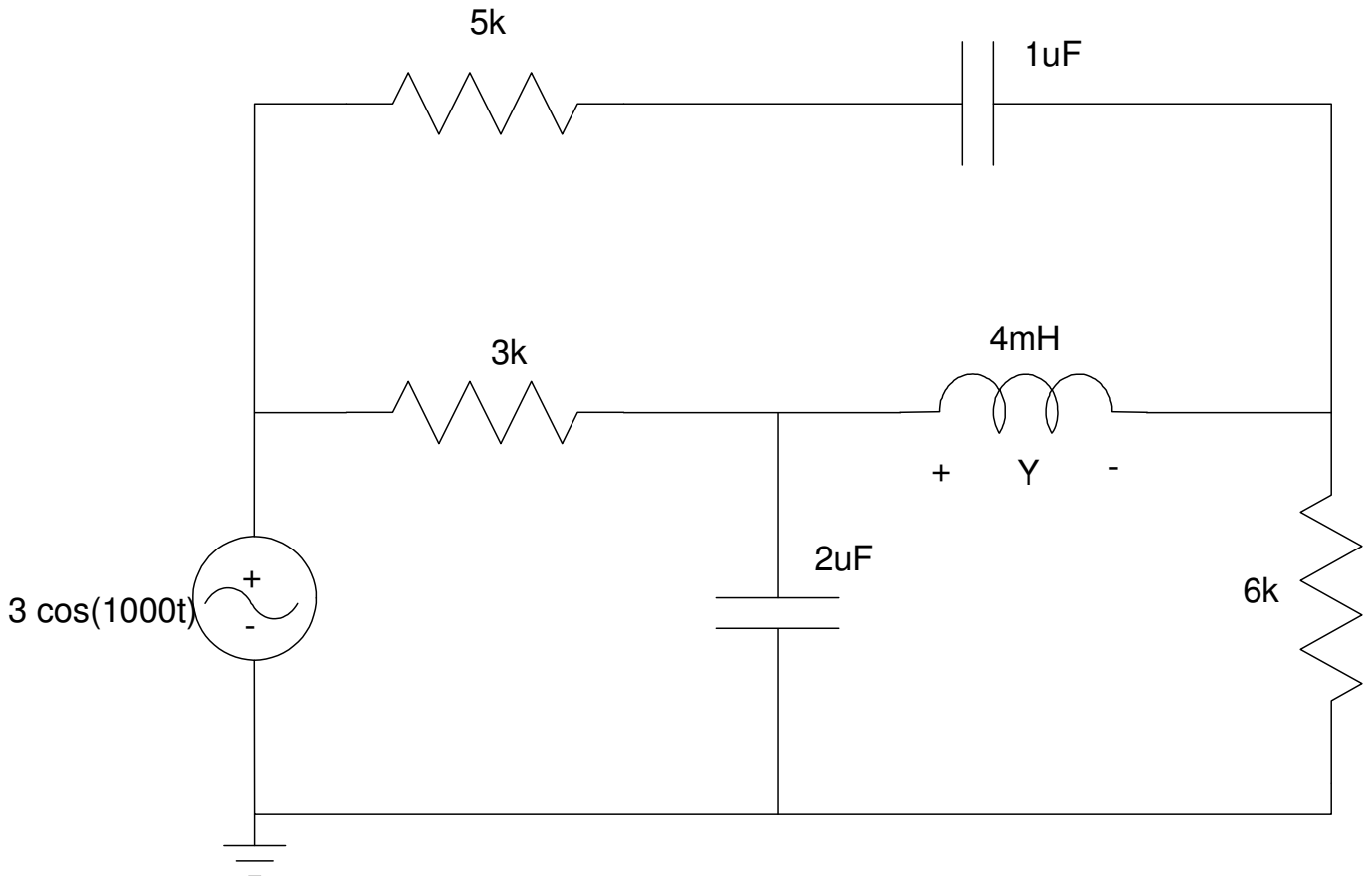
Current Loops with Phasors

Current loops also works with phasors

	VI relationship	Phasor Notation
Voltage	$v(t) = a\cos(\omega t) + b\sin(\omega t)$	$V = a - jb$
Resistor	$v = iR$	$Z_R = R$
Inductor	$v = L\frac{di}{dt}$	$Z_L = j\omega L$
Capacitor	$i = C\frac{dv}{dt}$	$Z_C = \frac{1}{j\omega C}$

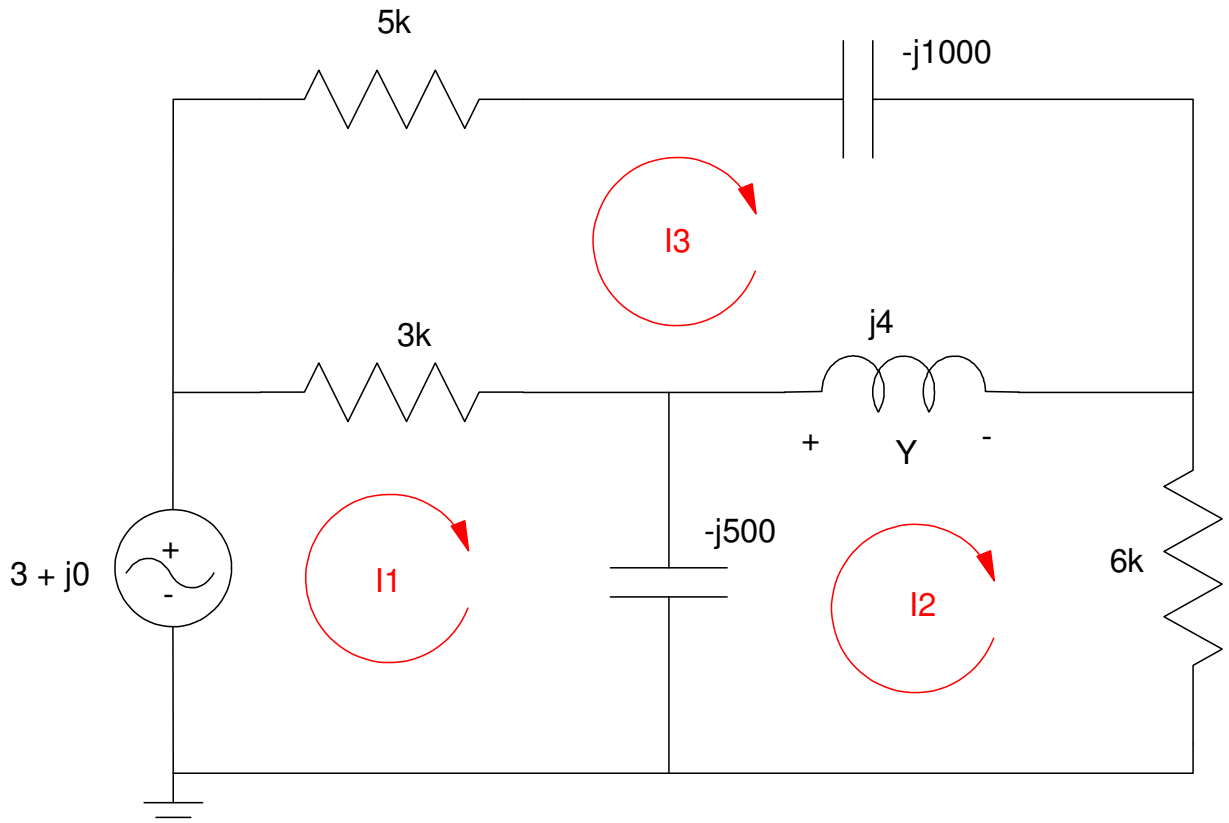
Example 1:

Determine the currents in the following circuit



Step 1:

- Define the currents
- Convert to Phasors



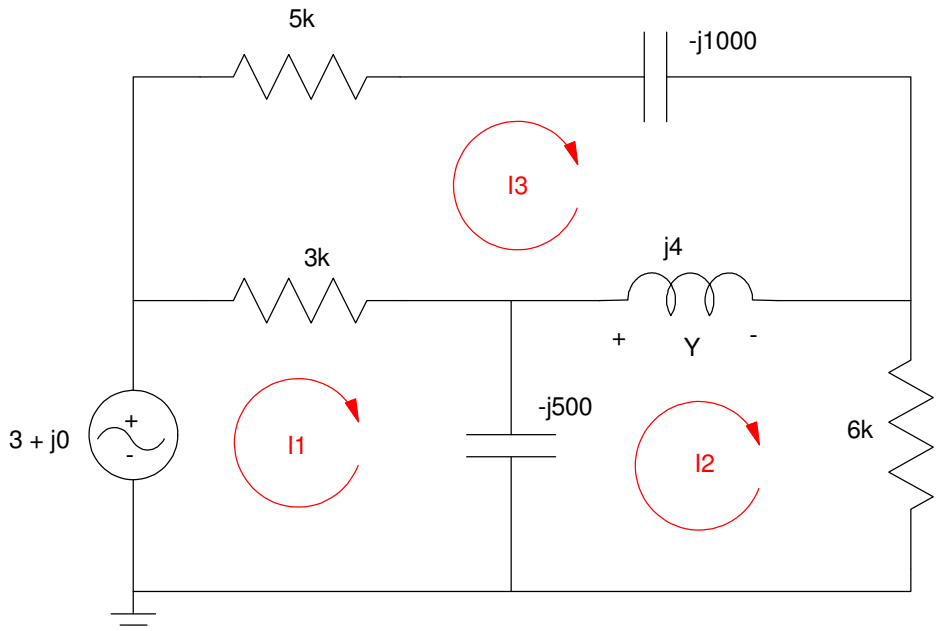
Step 2: Write the current loop equations

3 equations for 3 unknowns

$$-3 + 3000(I_1 - I_3) - j500(I_1 - I_2) = 0$$

$$-j500(I_2 - I_1) + j4(I_2 - I_3) + 6000I_2 = 0$$

$$-j1000(I_3) + 5000(I_3) + j4(I_3 - I_2) + 3000(I_3 - I_1) = 0$$



Step 3: Solve

Group terms:

$$(3000 - j500)I_1 + (j500)I_2 + (-3000)I_3 = 3$$

$$(j500)I_1 + (6000 - j496)I_2 + (-j4)I_3 = 0$$

$$(-3000)I_1 + (-j4)I_2 + (8000 - j996)I_3 = 0$$

```
A = [3000-j*500, j*500, -3000 ; j*500,6000-j*496 , -j*4;-3000,
-j*4, 8000-j*996]

    3000. - 500.i    500.i           - 3000.
    500.i            6000. - 496.i   - 4.i
  - 3000.          - 4.i               8000. - 996.i

I = inv(A) * [3;0;0]

I1:    0.0014008 + 0.0004598i
I2:    0.0000475 - 0.0001125i
I3:    0.0004962 + 0.0002342i
```

Step 4: Convert back to time

In phasor form:

```
I1:     0.0014008 + 0.0004598i
I2:     0.0000475 - 0.0001125i
I3:     0.0004962 + 0.0002342i
```

In the time-domain

$$i_1(t) = 1.4008\cos(1000t) - 0.4598\sin(1000t) \quad \text{mA}$$

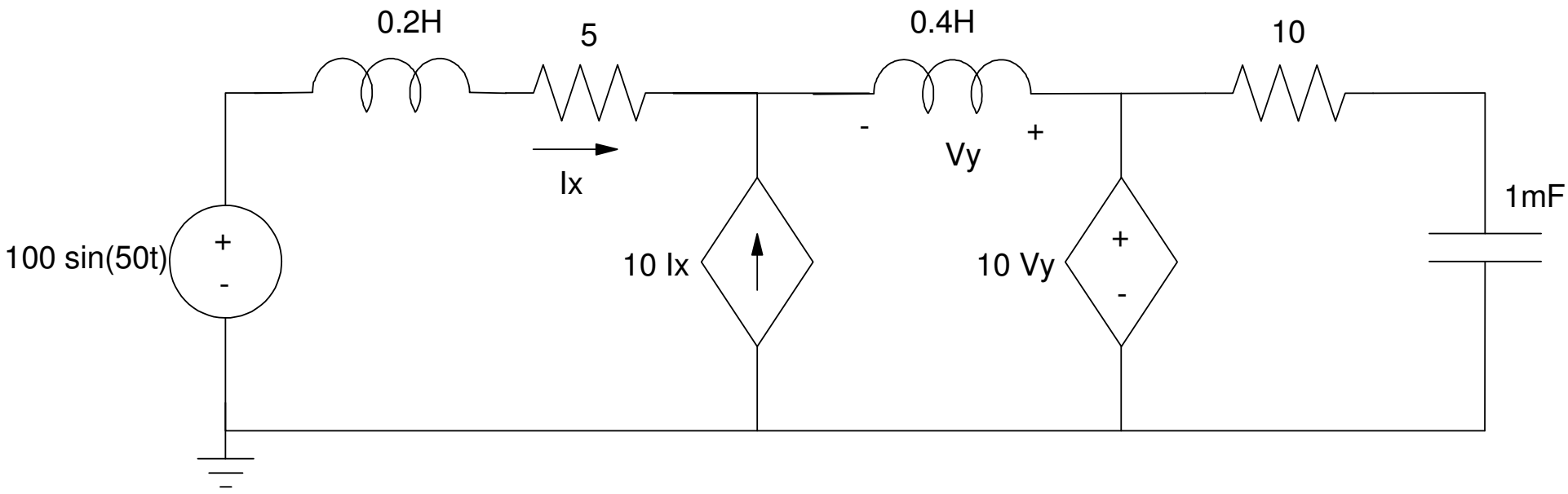
$$i_2(t) = 0.0475\cos(1000t) + 0.1125\sin(1000t) \quad \text{mA}$$

$$i_3(t) = 0.4962\cos(1000t) - 0.2342\sin(1000t) \quad \text{mA}$$

Current Loops with Dependent Sources

Not surprisingly, it also works with dependent sources

Example: Determine the currents in the following circuit



Step 1:

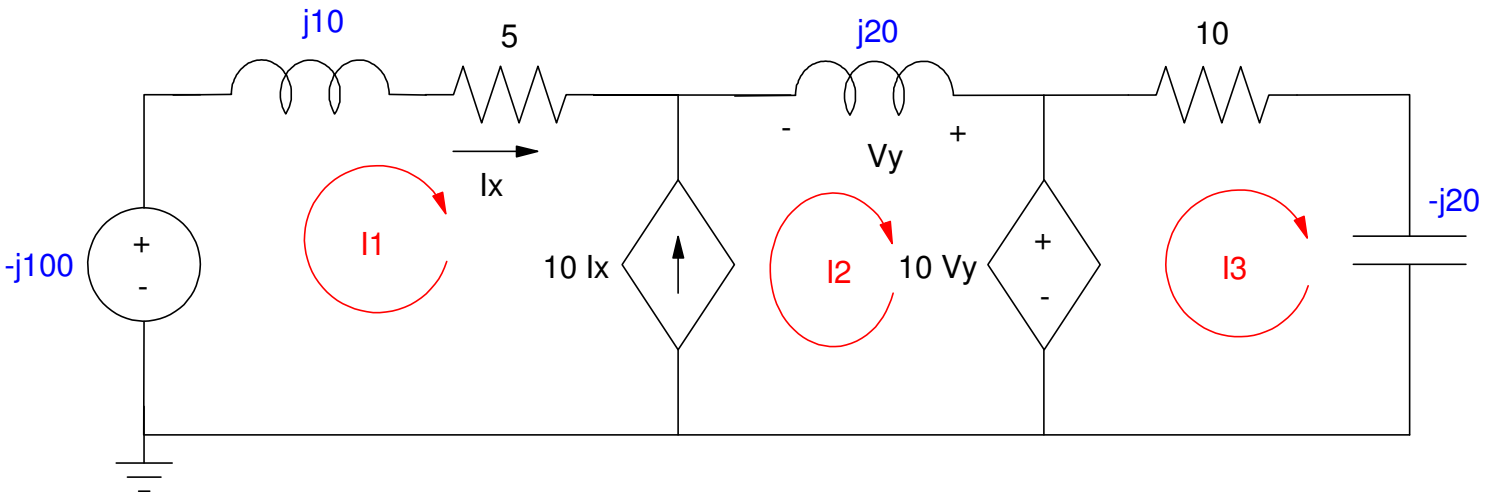
Define the loop currents

Convert to the phasor domain: (shown in blue)

$$a\cos(\omega t) + b\sin(\omega t) \rightarrow a - jb$$

$$L \rightarrow j\omega L$$

$$C \rightarrow \frac{1}{j\omega C}$$



Step 2: Write N equations for N uknowns

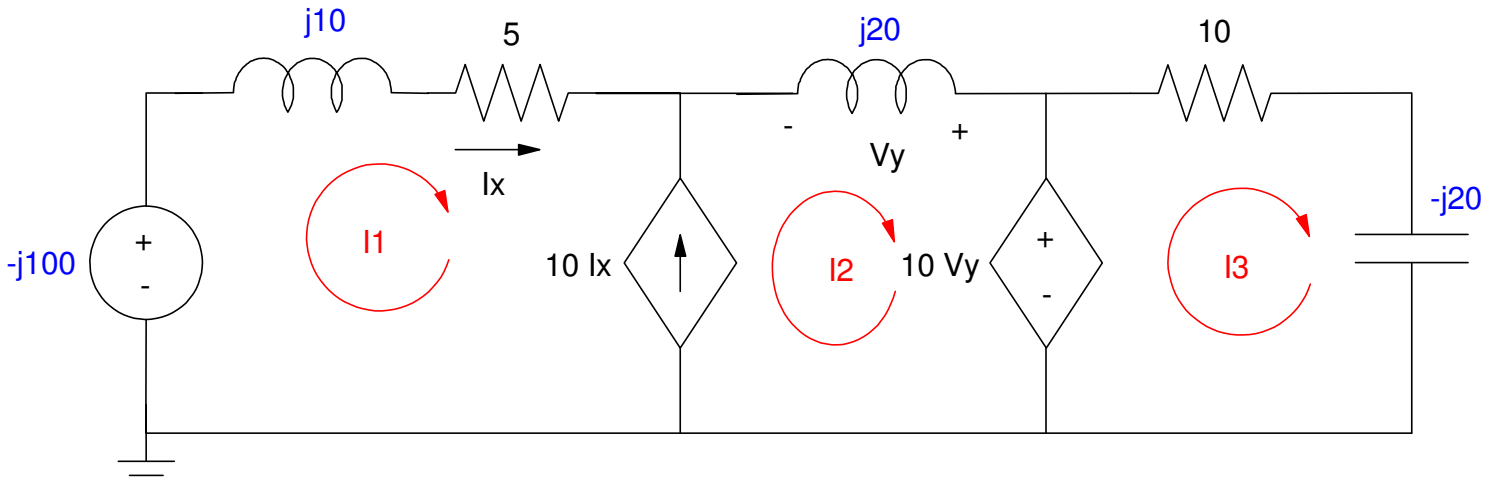
$$I_x = I_1$$

$$V_y = j20 \cdot I_2$$

$$10I_x = I_1 - I_2$$

$$-10V_y + 10I_3 - j20I_3 = 0$$

$$-(0 - j100) + (5 + j10)I_1 + j20 \cdot I_2 + (10 - j20)I_3 = 0 \quad \textit{super loop}$$



Step 3: Solve

Group terms

$$I_x - I_1 = 0$$

$$V_y - j20 \cdot I_2 = 0$$

$$10I_x - I_1 + I_2 = 0$$

$$-10V_y + 10I_3 - j20I_3 = 0$$

$$(5 + j10)I_1 + j20 \cdot I_2 + (10 - j20)I_3 = -j100$$

Place in matrix form

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 \\ 0 & -j20 & 0 & 0 & 1 \\ -1 & 1 & 0 & 10 & 0 \\ 0 & 0 & 10 - j20 & 0 & -10 \\ (5 + j10) & (j20) & (10 - j20) & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_x \\ V_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -j100 \end{bmatrix}$$

Solve in Matlab

```
A = [-1,0,0,1,0 ; 0,-j*20,0,0,1];
A = [A; -1,1,0,10,0 ; 0,0,10-j*20,0,-10];
A = [A ; 5+j*10,j*20,10-j*20,0,0]
B = [0;0;0;0;-j*100]
I = inv(A)*B
```

```
I1    0.0507611  -0.0001288i
I2   -0.4568498  +0.0011595i
I3    3.6501607  -1.8366755i
Ix    0.0507611  -0.0001288i
Vy   -0.0231903  -9.136997i
```

Step 4: Convert back to the time domain

Phasor domain

```
I1     0.0507611  -0.0001288i
I2    -0.4568498  +0.0011595i
I3     3.6501607  -1.8366755i
Ix     0.0507611  -0.0001288i
Vy    -0.0231903  -9.136997i
```

Time Domain

$$i_1(t) = 0.0507\cos(50t) + 0.0001288\sin(50t)$$

$$i_2(t) = -0.4568\cos(50t) - 0.0011\sin(50t)$$

$$i_3(t) = -0.0232\cos(50t) + 1.8367\sin(50t)$$