
Op-Amp Circuits with Phasors

ECE 211 Circuits I

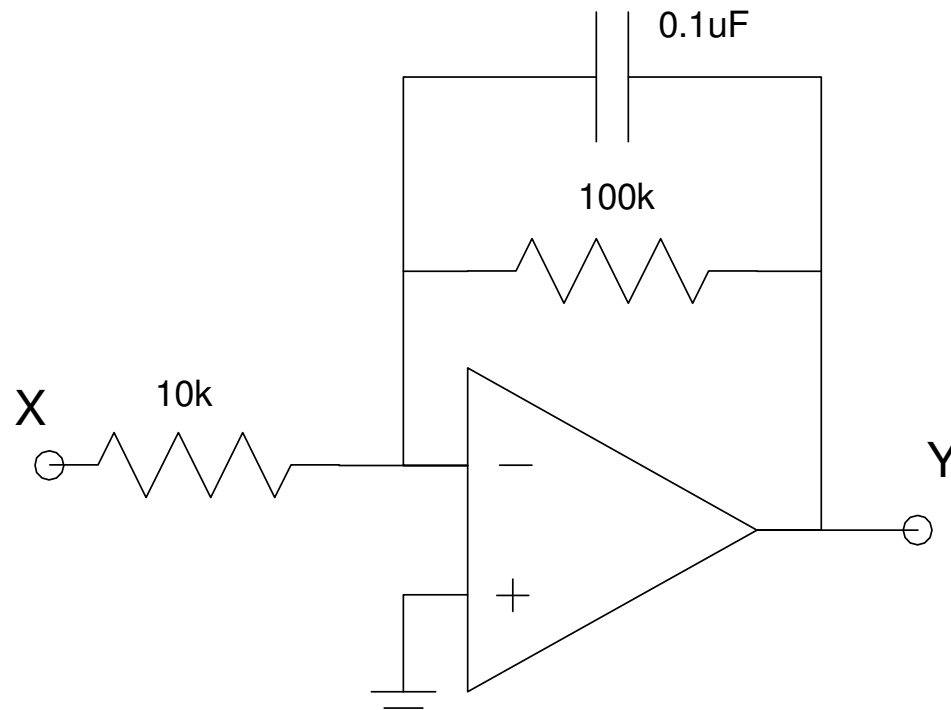
Lecture #28

Please visit [Bison Academy](#) for corresponding
lecture notes, homework sets, and solutions

Single-Pole Low-Pass Filter

Find the voltage, $y(t)$, for

$$x(t) = 3 \sin(50t)$$

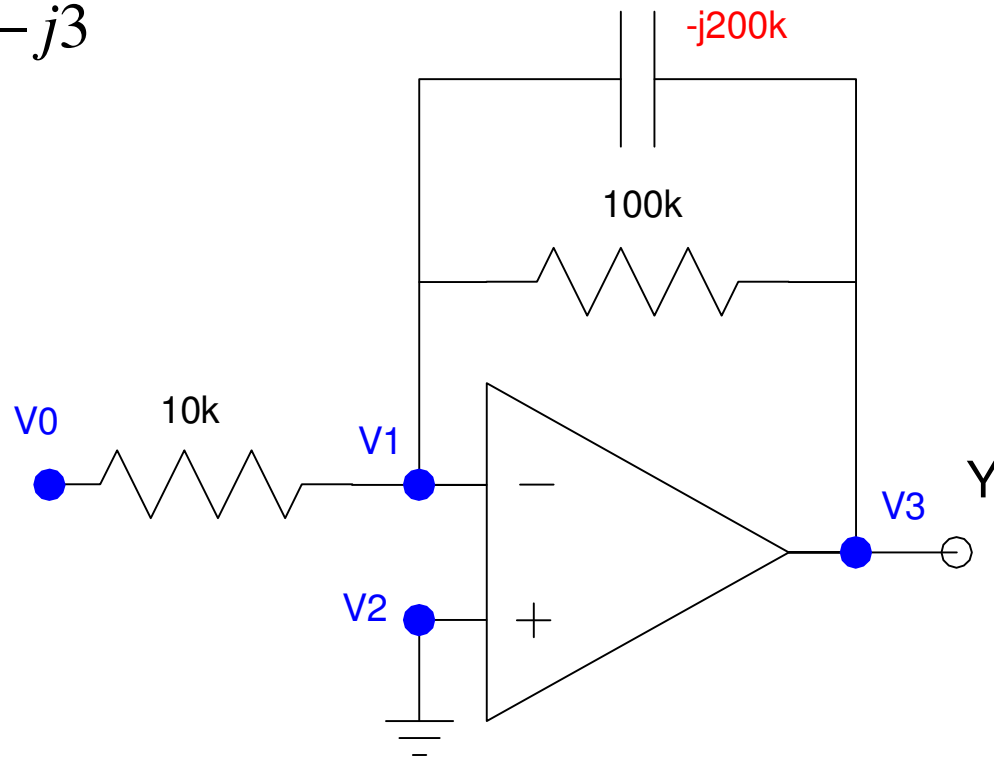


Step 1:

- Define ground (already done)
- Define the voltage nodes
- Convert to the phasor domain

$$x(t) = 3 \sin(50t) \rightarrow X = 0 - j3$$

$$0.1\mu F \rightarrow \frac{1}{j\omega C} = -j200k$$



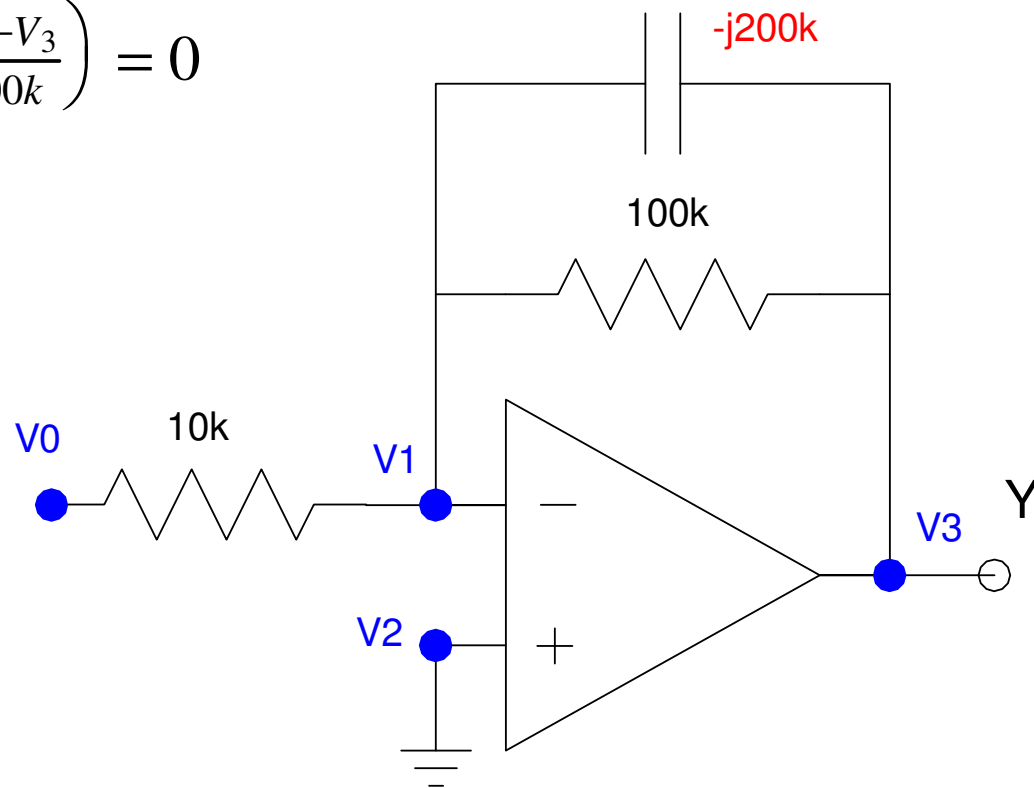
Step 2: Write the voltage node equations

$$V_0 = -j3$$

$$V_1 = V_2$$

$$V_2 = 0$$

$$\left(\frac{V_1 - V_0}{10k} \right) + \left(\frac{V_1 - V_3}{-j200k} \right) + \left(\frac{V_1 - V_3}{100k} \right) = 0$$



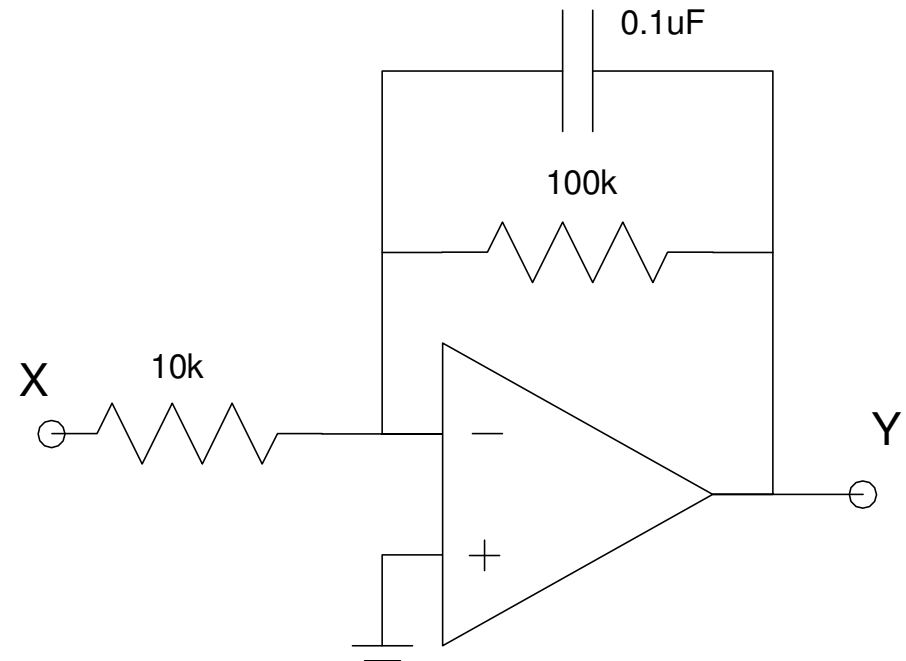
Solve

$$V_3 = 12 + j24 = 26.8\angle 63^\circ$$

meaning

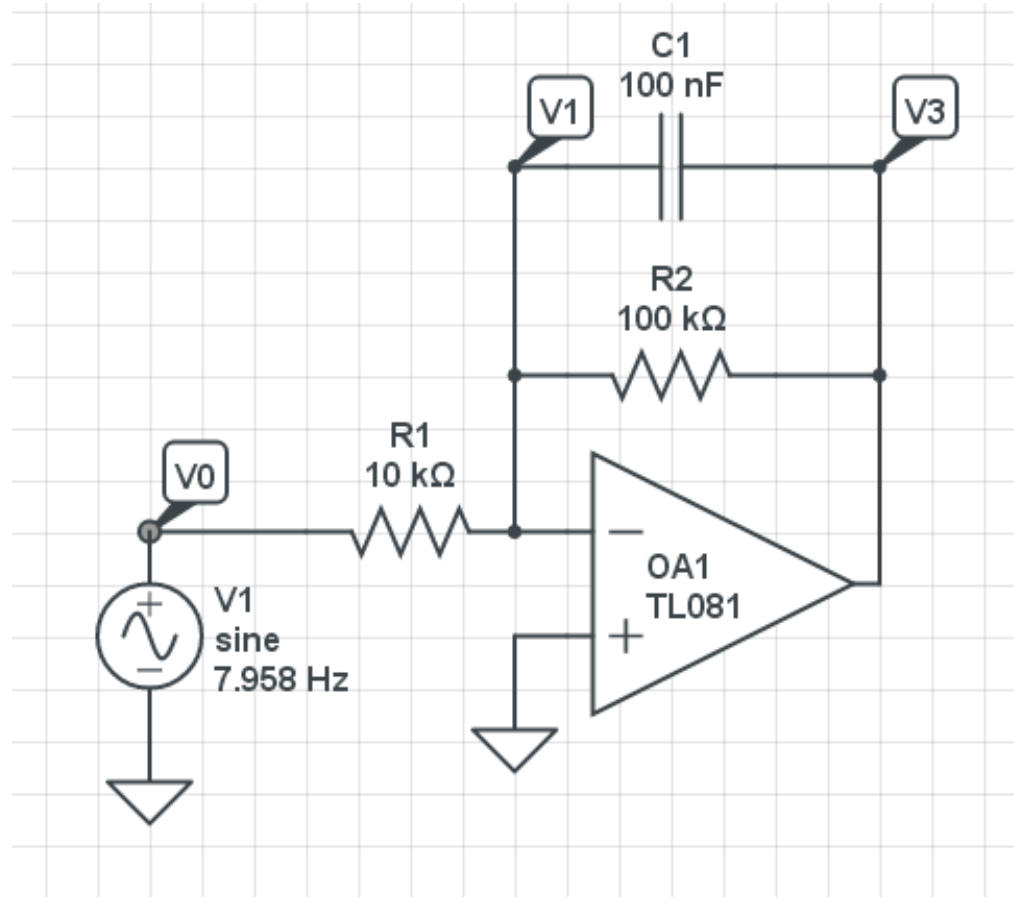
$$v_3(t) = 12 \cos(50t) - 24 \sin(50t)$$

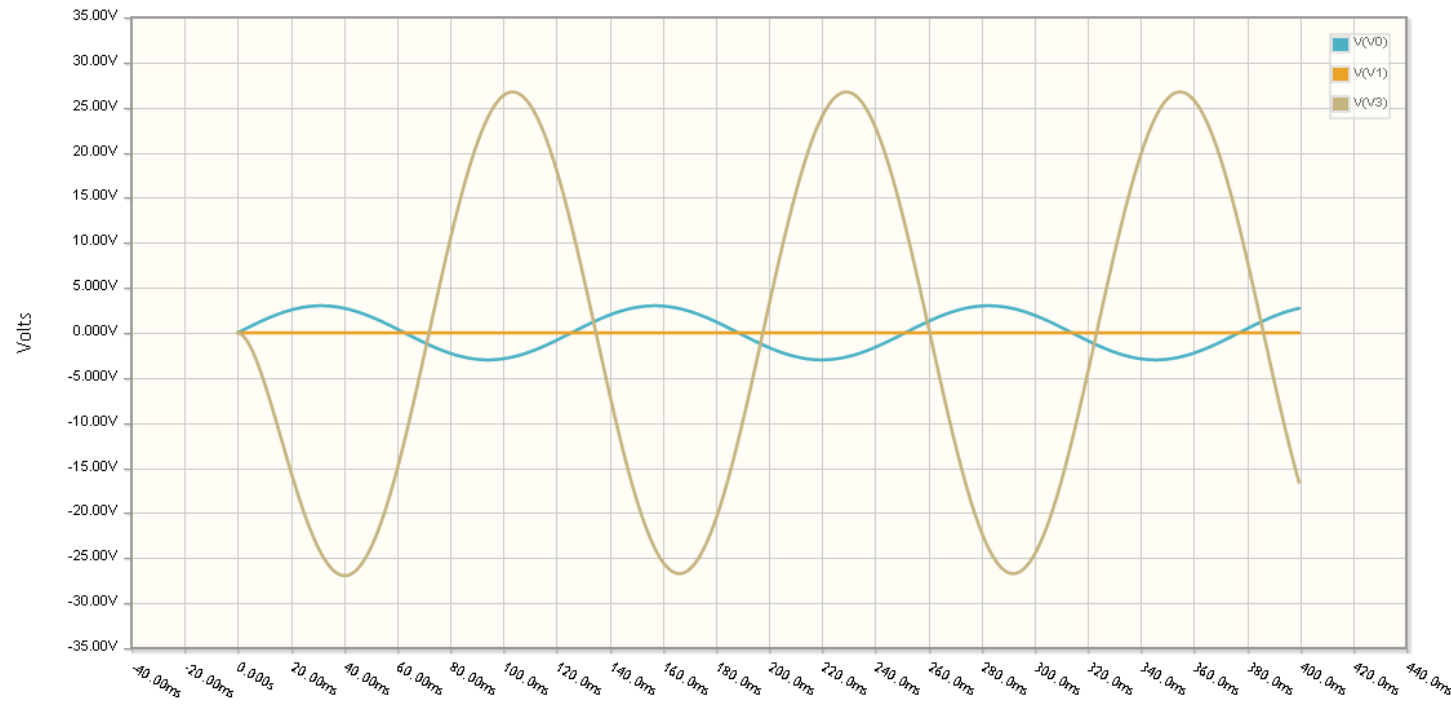
$$v_3(t) = 26.8 \cos(50t + 63^\circ)$$



Checking in CircuitLab: The input is

$$Hz = \frac{\omega}{2\pi} = \frac{50 \text{ rad/sec}}{2\pi} = 7.958 Hz$$





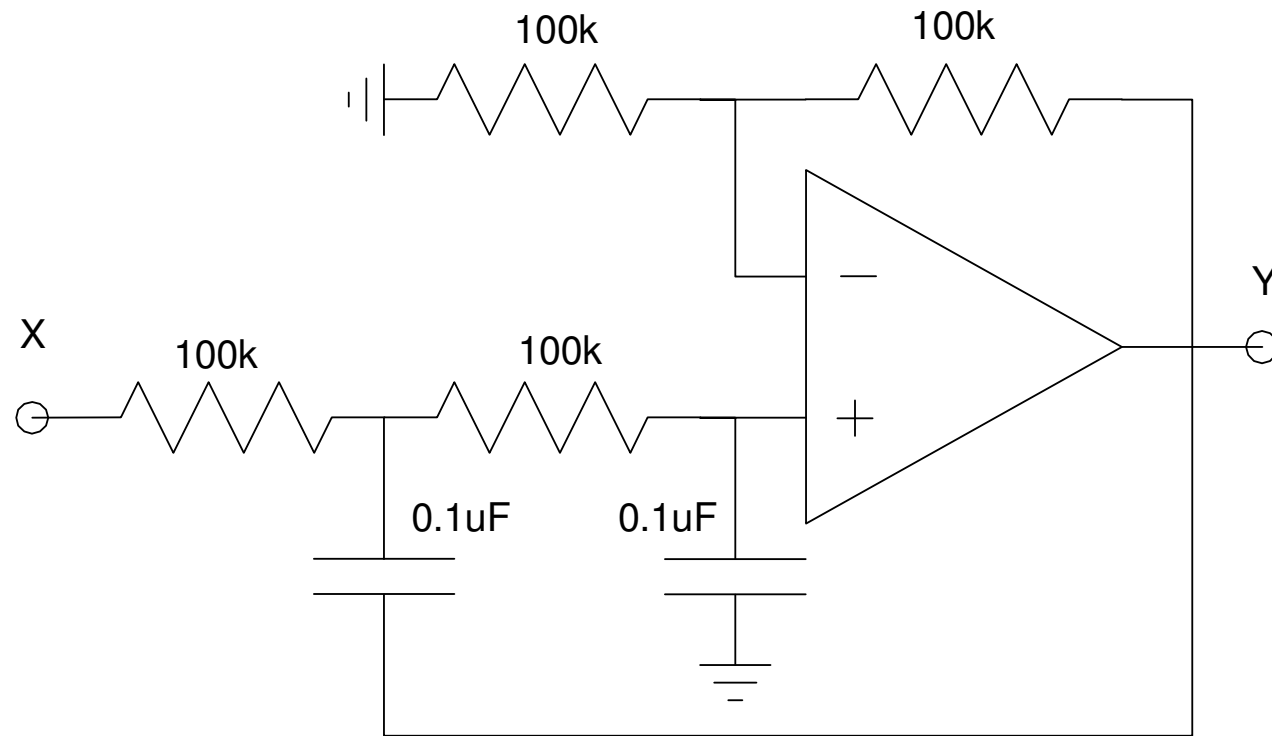
- The period is 126ms ($7.958\text{Hz} = 50\text{ rad/sec}$)
- The peak is 26.825V (vs. 26.8V calculated)
- The peak of V3 is at $t = 104.6\text{ms}$ (delay = -297 degrees)

This matches our calculations

Example 2: Two-Pole Op-Amp Circuit

Find $y(t)$ for

$$x(t) = 3 \cos(40t)$$

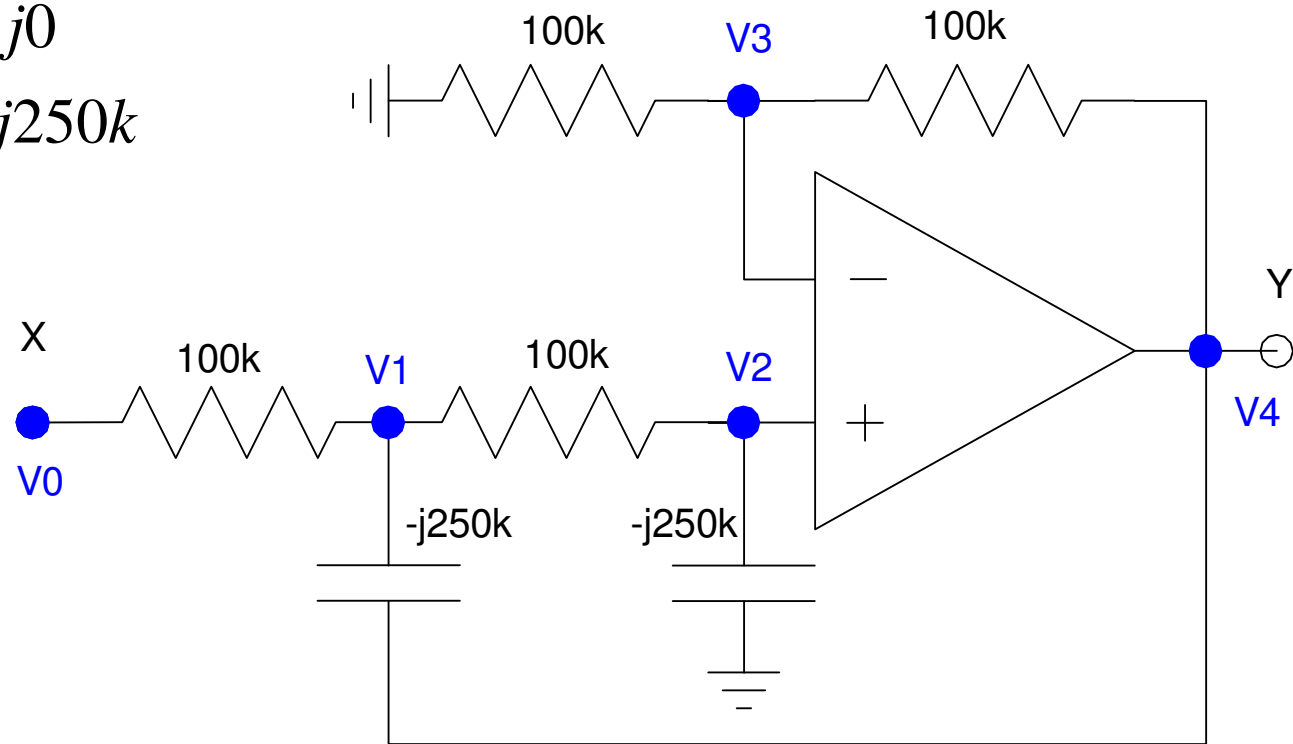


Step 1:

- Define circuit ground (already done)
- Define the voltage nodes
- Convert to phasors

$$3 \cos(40t) \rightarrow 3 + j0$$

$$0.1 \mu F \rightarrow \frac{1}{j\omega C} = -j250k$$



Step 2: Write the voltage node equations.

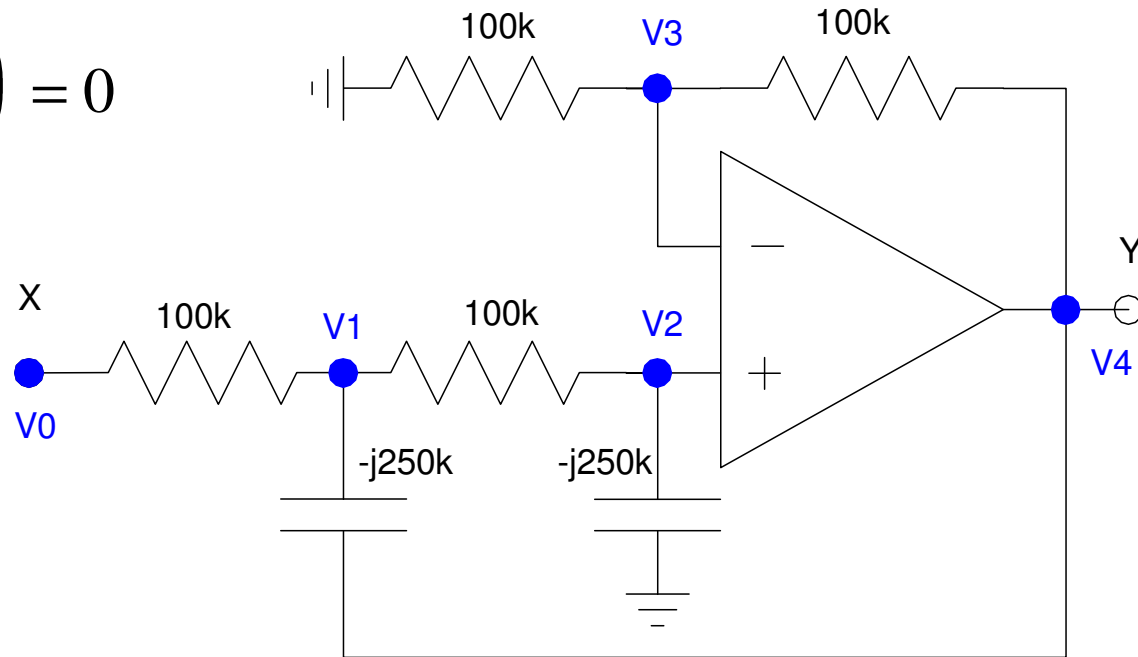
$$V_0 = 3 + j0$$

$$V_2 = V_3$$

$$\left(\frac{V_1 - V_0}{100k} \right) + \left(\frac{V_1 - V_4}{-j250k} \right) + \left(\frac{V_1 - V_2}{100k} \right) = 0$$

$$\left(\frac{V_2 - V_1}{100k} \right) + \left(\frac{V_2 - 0}{-j250k} \right) = 0$$

$$\left(\frac{V_3 - 0}{100k} \right) + \left(\frac{V_3 - V_4}{100k} \right) = 0$$



Step 3: Solve

Group terms

$$V_0 = 3$$

$$V_2 - V_3 = 0$$

$$\left(\frac{-1}{100k}\right)V_0 + \left(\frac{1}{100k} + \frac{1}{-j250k} + \frac{1}{100k}\right)V_1 + \left(\frac{-1}{100k}\right)V_2 + \left(\frac{-1}{100k}\right)V_4 = 0$$

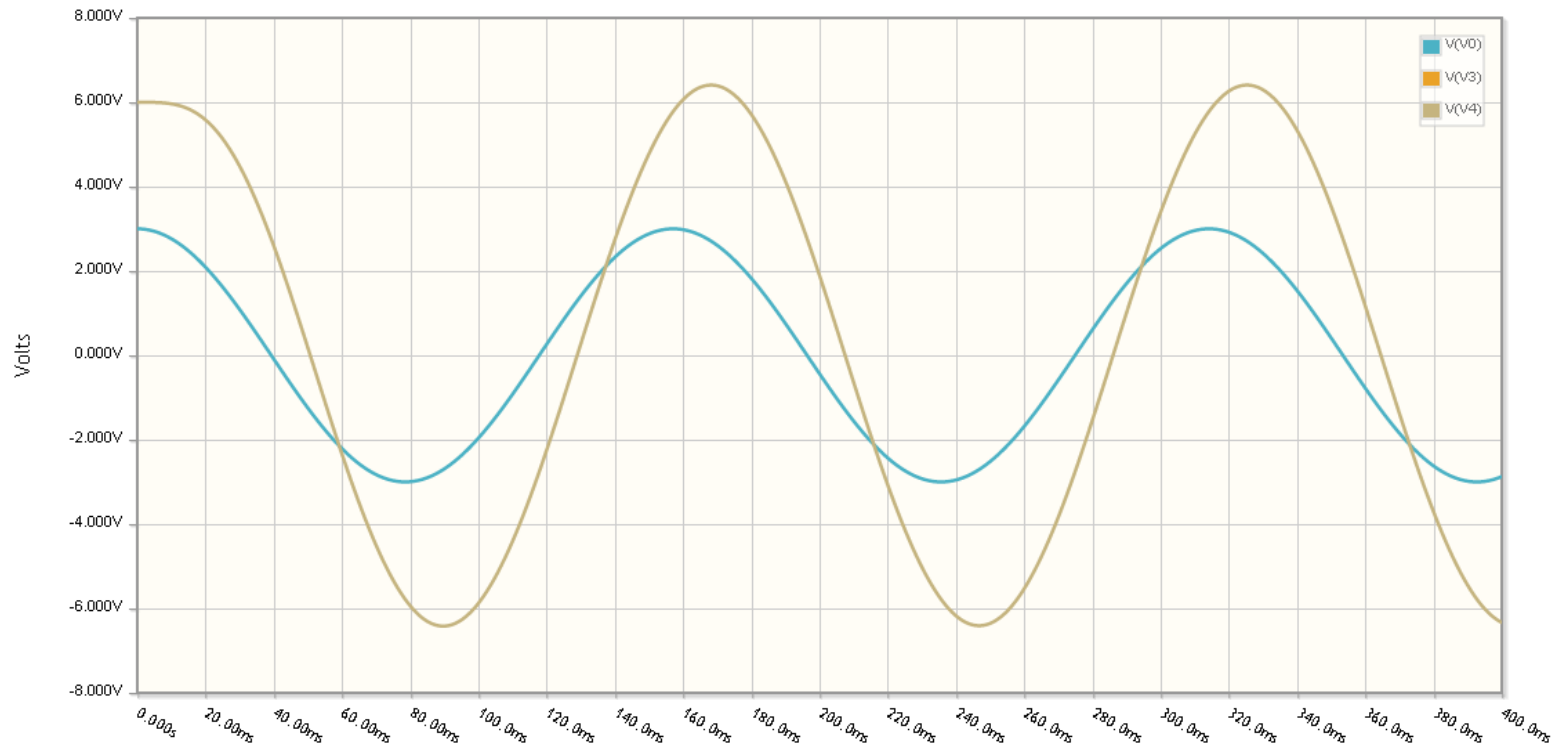
$$\left(\frac{-1}{100k}\right)V_1 + \left(\frac{1}{100k} + \frac{1}{-j250k}\right)V_2 = 0$$

$$\left(\frac{1}{100k} + \frac{1}{100k}\right)V_3 + \left(\frac{-1}{100k}\right)V_4 = 0$$

V1	3.4658041	-	0.2218115i	
V2	2.9112754	-	1.3863216i	
V3	2.9112754	-	1.3863216i	
V4	5.8225508	-	2.7726433i	= Y

$$v_4(t) = 5.822 \cos(40t) + 2.77 \sin(40t)$$

Checking in CircuitLab:



The output peak is 6.447V (vs. 6.447V computed)

$$\phi = \left(\frac{9.9\text{ms delay}}{157\text{ms period}} \right) \cdot 360^\circ = 22.7^\circ \text{ delay (vs. 25 degrees computed)}$$