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# **Superposition (take 3)**

## **ECE 211 Circuits I**

### **Lecture #32**

Please visit [Bison Academy](#) for corresponding lecture notes, homework sets, and solutions

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# Superposition (take 3)

## Recap: Buck Converter

- $V_1$  wasn't a sine wave
- We changed the problem so that it was a sine wave

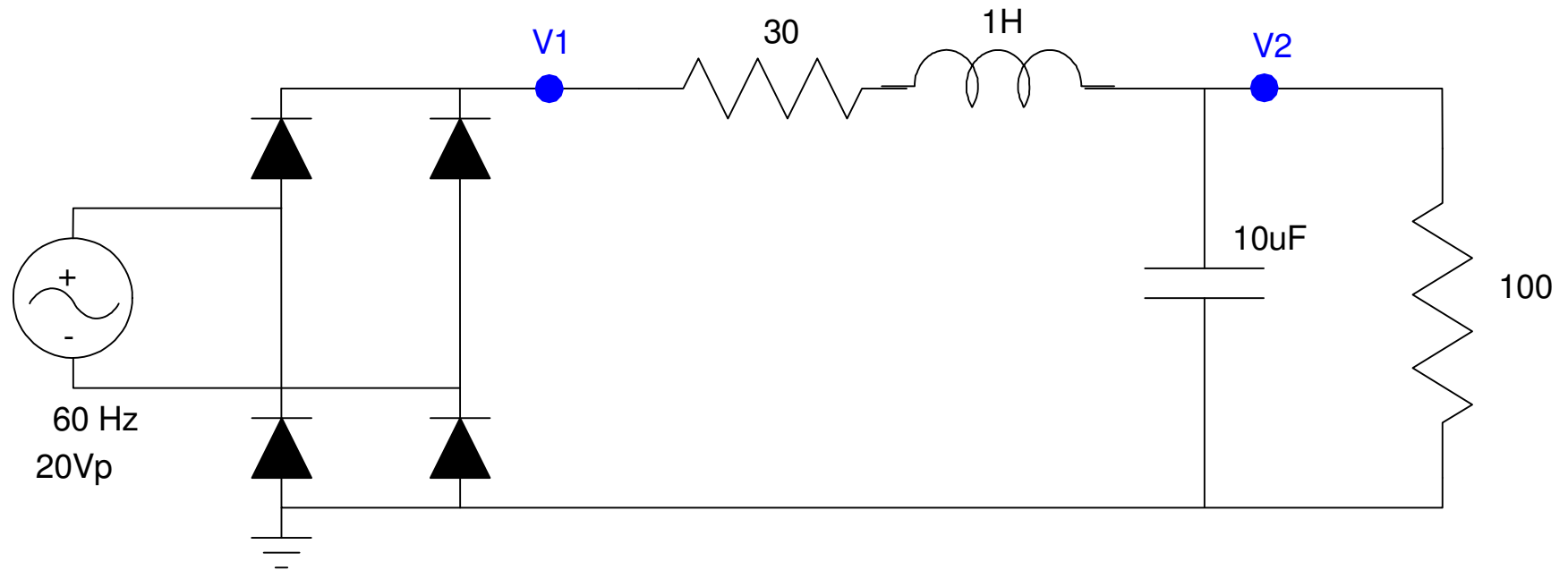
With Fourier Transforms, we don't have to make any approximations

- The results are more accurate
  - But the problem gets a lot harder
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## Example 1: AC to DC Converter

$$V_1 = |19.3 \sin(377t)| - 0.7$$

Determine the voltage at V2



AC to DC Converter covered in ECE 320 Electronics I

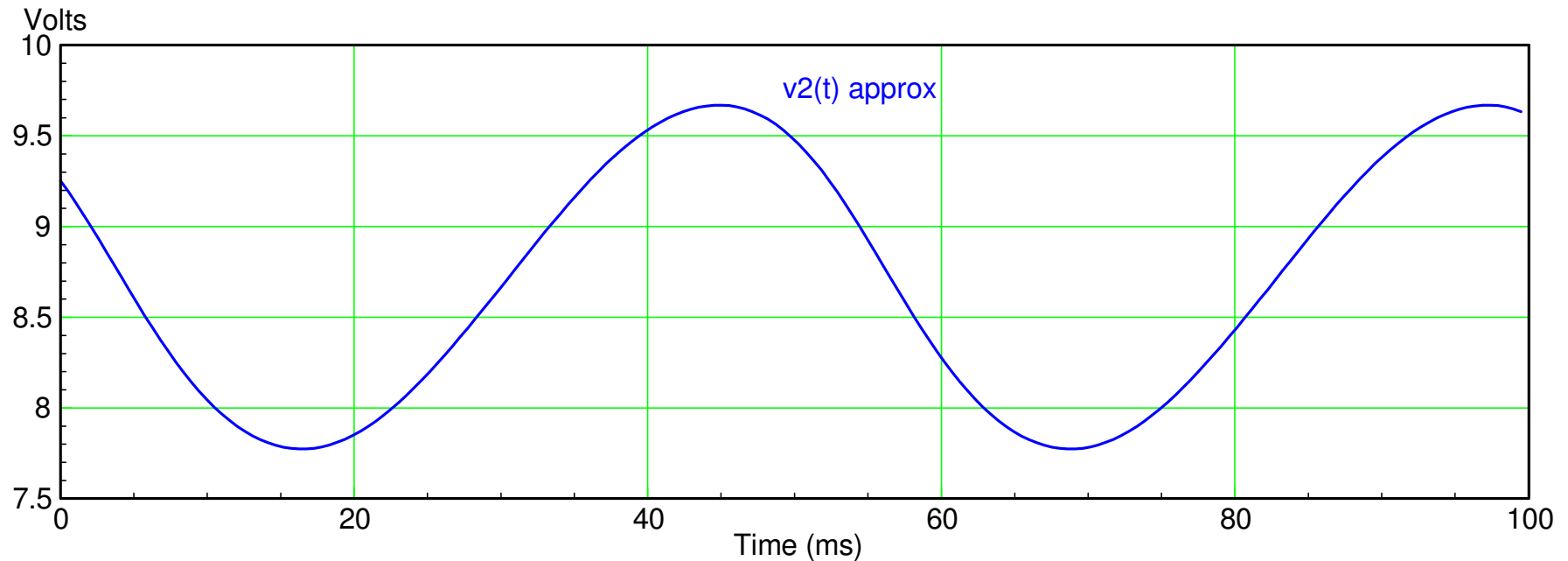
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## ECE 320 Solution (approximate)

$$v_1(t) = |19.3 \sin(377t)| - 0.7$$

$$v_1(t) \approx 11.33 + 9.35 \cos(754t)$$

$$v_2(t) \approx 8.70 + 1.082 \cos(754t - 119^\circ)$$



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## Fourier Transform Solution

$$v_1(t) = |19.3 \sin(377t)| - 0.7$$

$$v_1(t) = 11.331 - 8.488 \cos(754t) - 1.698 \cos(1508t) \\ - 0.728 \cos(2262t) - 0.404 \cos(3016t) + \dots$$

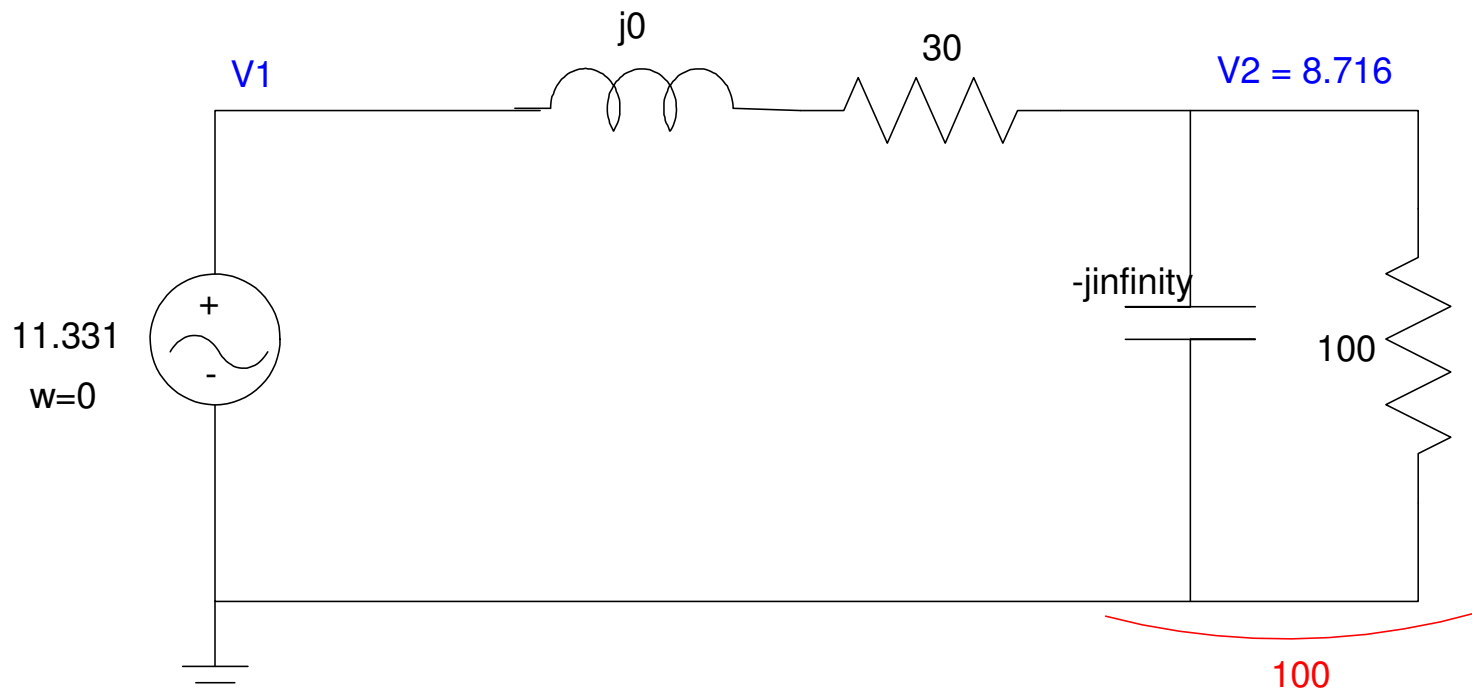
Now solve using superposition

## DC Term:

$$V_1 = 11.331$$

$$V_2 = \left( \frac{100}{100+30} \right) \cdot 11.331$$

$$V_2 = 8.716$$



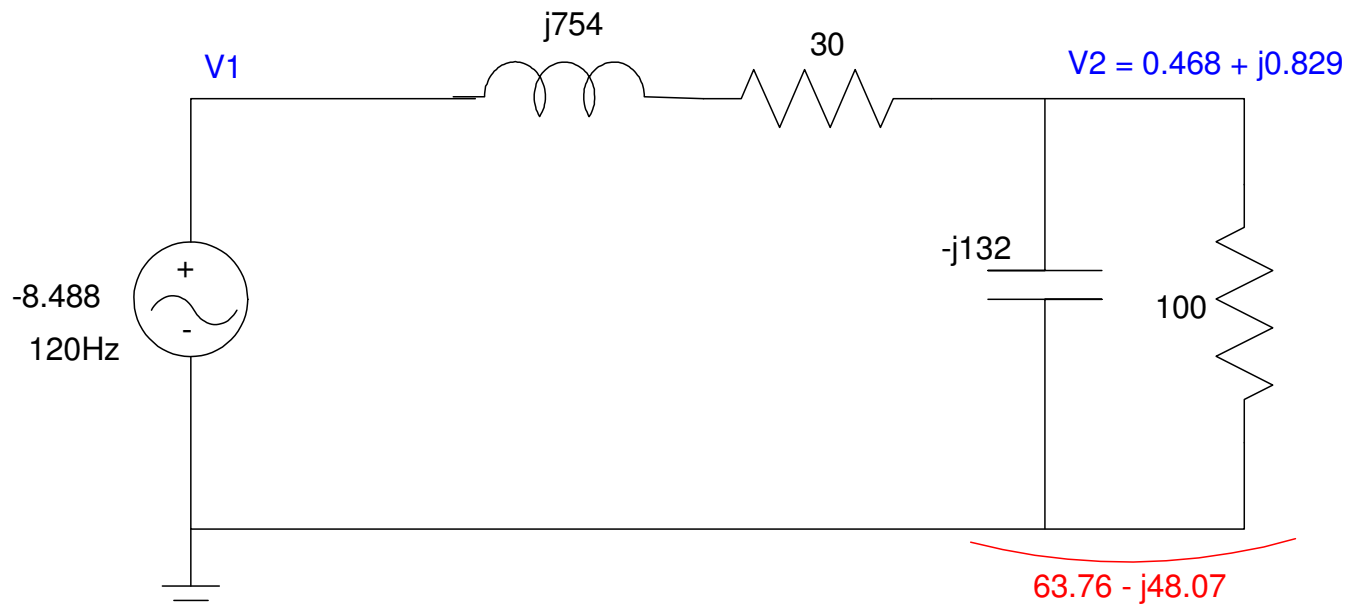
## 1st Harmonic (120Hz)

$$v_1(t) = -8.488 \cos(754t)$$

$$V_2 = \left( \frac{(63.76 - j48.07)}{(63.76 - j48.07) + (30 + j754)} \right) \cdot (-8.488 + j0)$$

$$V_2 = 0.468 + j0.829$$

$$v_2(t) = 0.468 \cos(754t) - 0.829 \sin(754t)$$



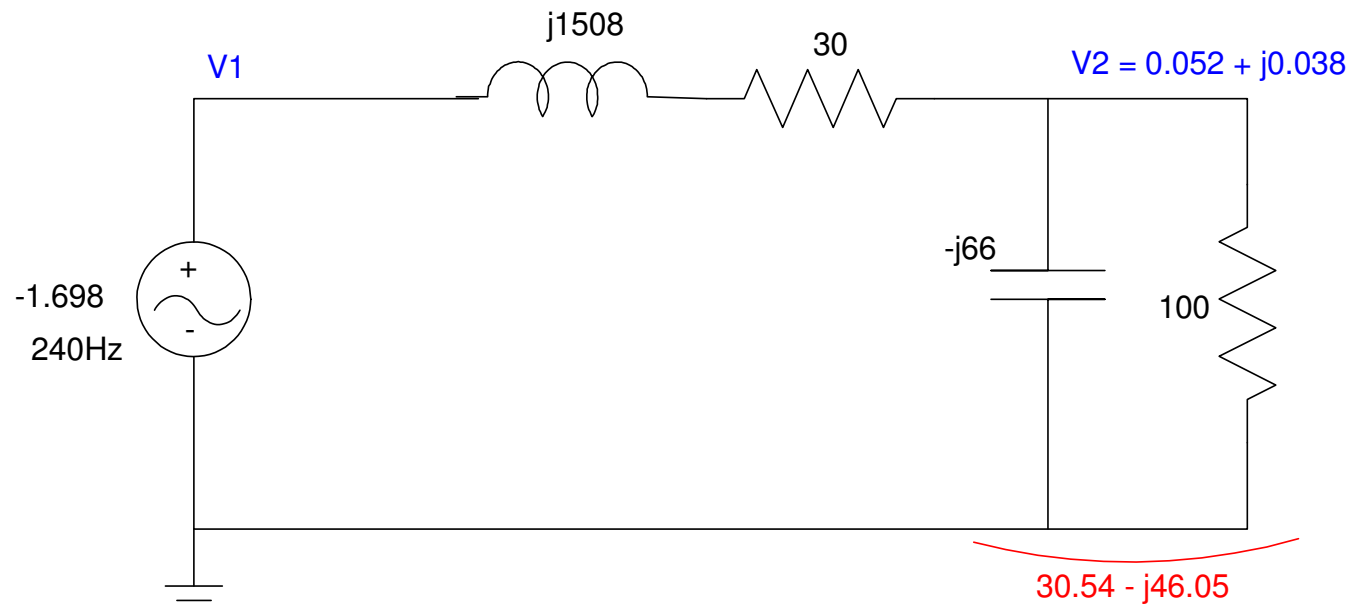
## 2nd Harmonic (240Hz)

$$v_1 = -1.698 \cos(1508t)$$

$$V_2 = \left( \frac{30.54 - j46.05}{(30.54 - j46.05) + (30 + j1508)} \right) \cdot (-1.698 + j0)$$

$$V_2 = 0.052 + j0.038$$

$$v_2(t) = 0.052 \cos(1508t) - 0.038 \sin(1508t)$$





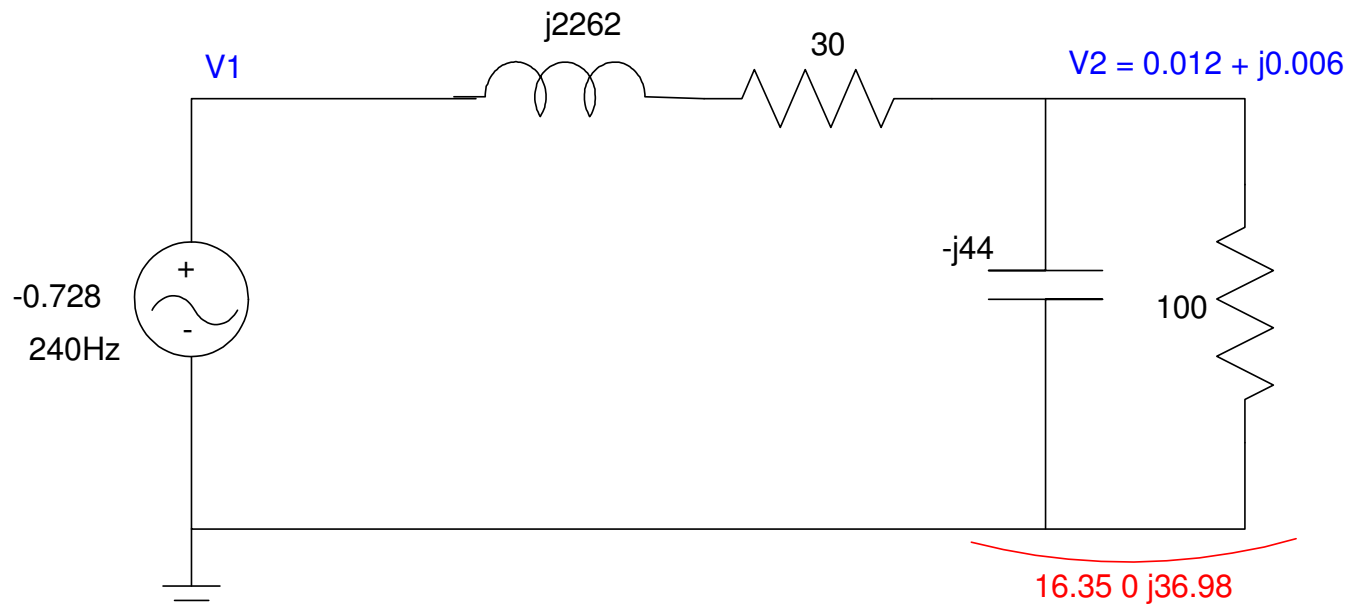
### 3rd Harmonic: (360Hz)

$$v_1(t) = -0.728 \cos(2262t)$$

$$V_2 = \left( \frac{(16.35 - j36.98)}{(16.35 - j36.98) + (30 + j2262)} \right) \cdot (-0.728 + j0)$$

$$V_2 = 0.012 + j0.006$$

$$v_2(t) = 0.012 \cos(2262t) - 0.006 \sin(2262t)$$



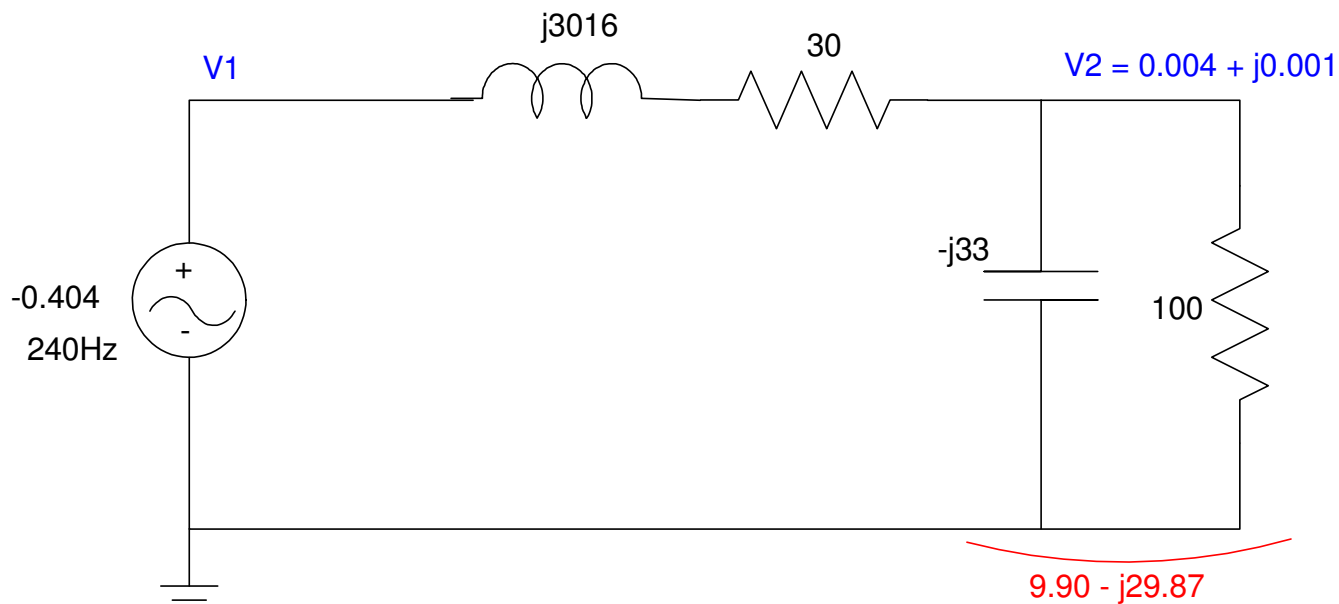
## 4th Harmonic: (480Hz)

$$v_1(t) = -0.404 \cos(3016t)$$

$$V_2 = \left( \frac{(9.90 - j29.87)}{(9.90 - j29.87) + (30 + j3016)} \right) \cdot (-0.404 + j0)$$

$$V_2 = 0.004 + j0.001$$

$$v_2(t) = 0.004 \cos(3016t) - 0.001 \sin(3016t)$$



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The total answer will be the sum of all the terms

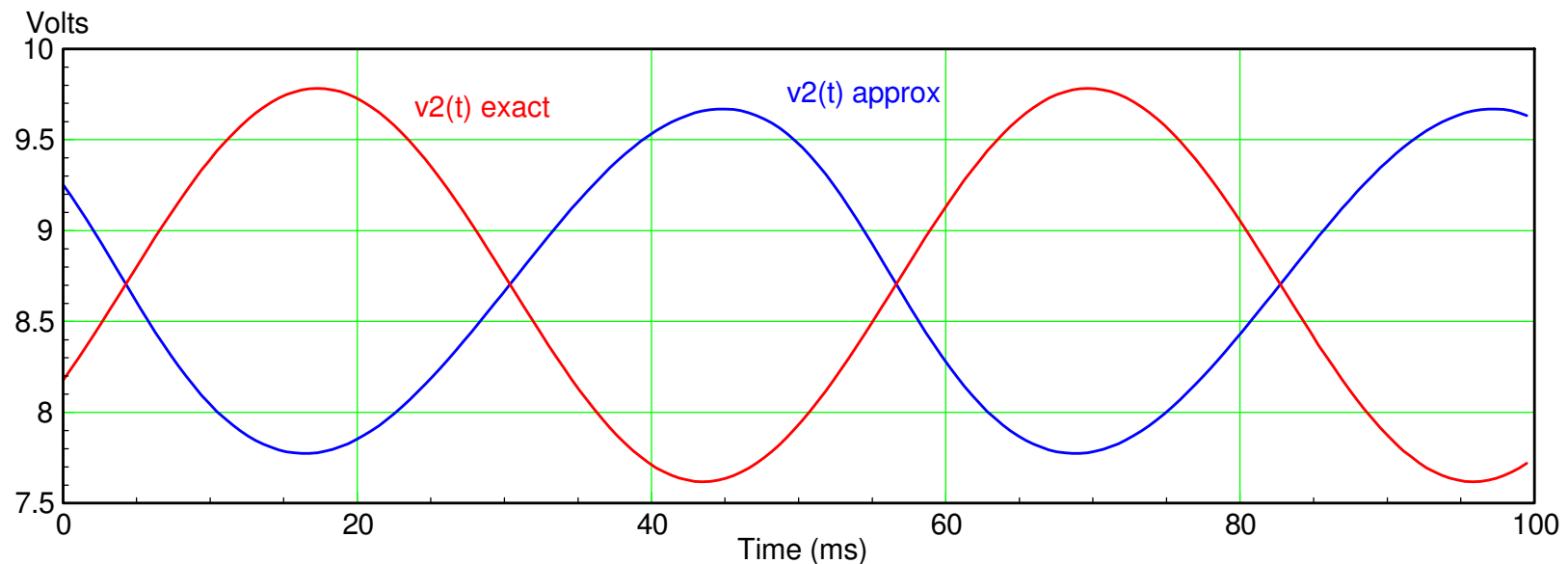
$$v_2(t) = 8.716$$

$$+0.468 \cos(754t) - 0.829 \sin(754t)$$

$$+0.052 \cos(1508t) - 0.038 \sin(1508t)$$

$$+0.012 \cos(2262t) - 0.006 \sin(2262t)$$

$$+0.004 \cos(3016t) - 0.001 \sin(3016t)$$



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## Note the following:

- The results using Fourier Transforms matches CircuitLab
- The reason our previous answer from lecture #30 (column #2) was off is we overestimated the 1st harmonic.
- In theory, you have to take the Fourier Series out to infinity. Actually, you can get a pretty good approximation just using the DC term and the 1st harmonic.