Kirchoff's Laws (review)

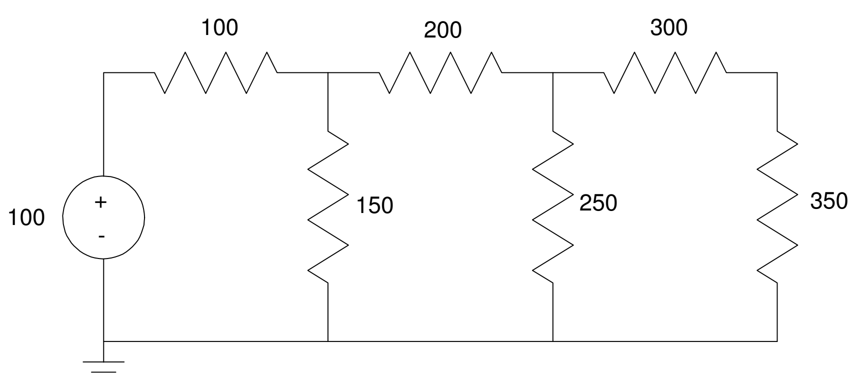
Two of the main techniques used in Circuits I to solve a circuit are *voltage nodes* (KVN) and *current loops* (KCL). These are based upon conservation of current and conservation of voltage:

KVN: The sum of the current from any voltage node must be zero

KCL: The sum of the voltages around any closed path must sum to zero

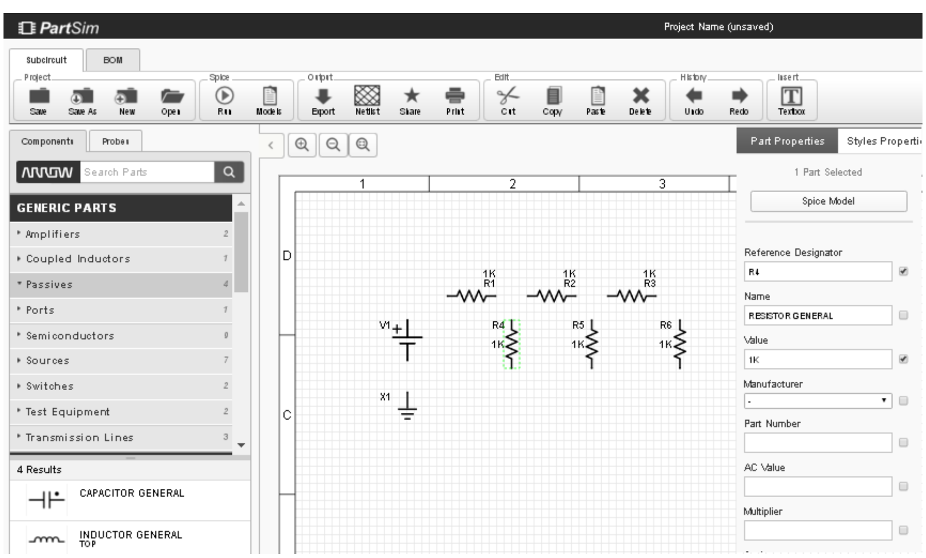
To solve a circuit, you write N equations to solve for N unknowns.

Example 1: Find the voltages for the following circuit;



Solution 1: PartSim. One way to solve is to throw this circuit into a circuit simulator such as PartSim.

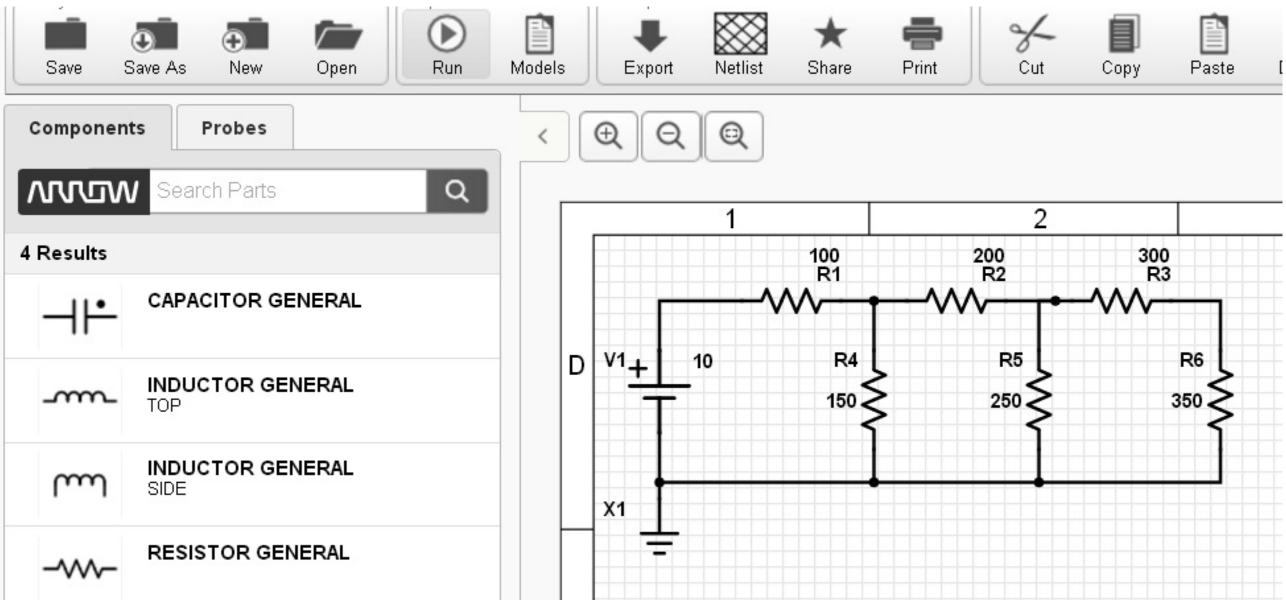
In PartSim, go the menu items on the left and add the voltage source and resistors. The ground is under the heading 'port'



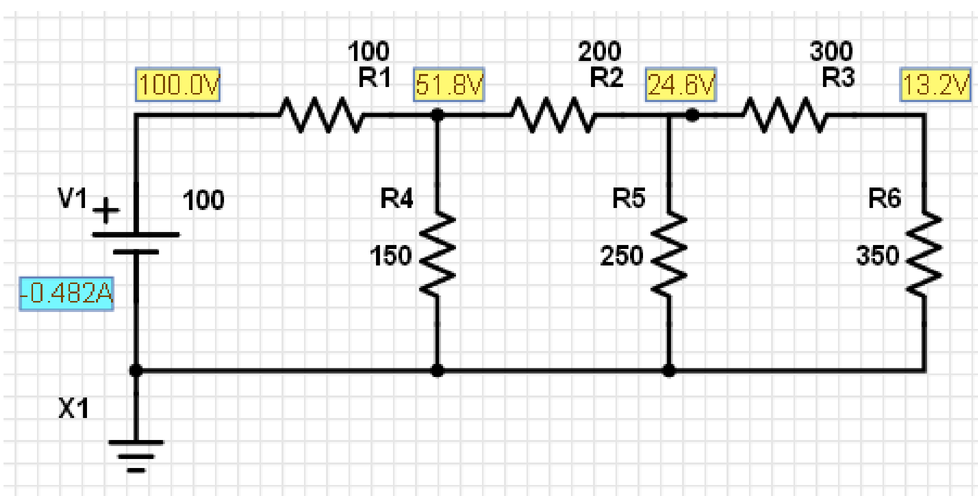
Connect the components by clicking on one end of an element and dragging the wire to another element. Once done, change the values to match the previous circuit

Once connected, you can solve for the voltages by clicking

- Run
- DC Bias



This results in the circuit simulator finding the node voltages:



This is also what you should measure in lab if you built this circuit. To solve for these voltages by hand, two tools are commonly used:

- Current Loops
- Voltage Nodes

Current Loops:

The sum of the voltages around any closed loop must be zero. To use this method

Step 1) Draw the circuit so that there are N distinct "windows". Define the current in each window.

Step 2) Sum the voltages around each loop to zero to create N equations for the N unknown currents.

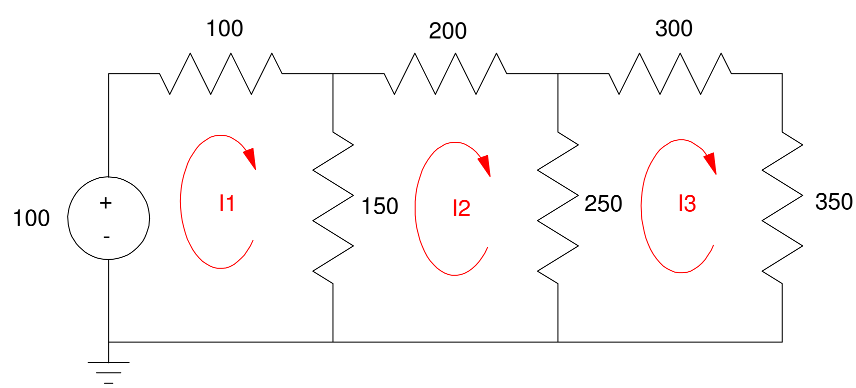
- If you pass through a voltage source, add if you hit the + terminal first, subtract if you hit the - terminal first.
- If you pass

Special Case: If there is a current source in the circuit

- The current source defines one of the currents (one equation)
- The other loops cannot cross the current source since you don't know what voltage it applies. (Current sources supply whatever voltage it takes to maintain current.)

As you go around the loop,

Example 1: Write N equations for N unknowns using KCL:



Example 1: Solve for the currents in the circuit using KCL:

Step 1: There are three windows. Define three currents (show in red)

Step 2: Sum the voltages around each loop to write N equations for N unknowns.

I1: $-100 + 100(I_1) + 150(I_1 - I_2) = 0$

I2: $150(I_2 - I_1) + 200(I_2) + 250(I_2 - I_3) = 0$

I3: $250(I_3 - I_2) + 300(I_3) + 350(I_3) = 0$

Note: When you go around loop I1, the coefficients of I1 are all positive, all other coefficients are negative.

Step 3: Solve. MATLAB helps here - especially if you use matrix algebra.

An nxm matrix has

- n rows
- m columns

When you multiply matrices, the inner coefficient must match

$$A_{2x3} \cdot B_{3x1} = C_{2x1}$$

element cij is

$$c_{ij} = \Pi(a_{ik} b_{kj})$$

or for a 2x3 multiplied by a 3x1

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \cdot \begin{vmatrix} b_{11} \\ b_{21} \\ b_{31} \end{vmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix}$$

The identity matrix is analogous to the number one: it leaves a matrix unchanged

$$IA = A$$

A matrix inverse is the matrix which produces the identity matrix:

$$A^{-1}A = I$$

Only square matrices (NxN) can be inverted.

If you can write N equations with N unknowns as

$$AX = B$$

then you can solve for X as

$$X = A^{-1}B$$

Example:

```
-->A = rand(3,3)

    0.2113249    0.3303271    0.8497452
    0.7560439    0.6653811    0.6857310
    0.0002211    0.6283918    0.8782165

-->inv(A)*A

    1.    0.    0.
    0.    1.    0.
    0.    0.    1.
```

Going back to the circuit, rewrite the 3 equations with 3 unknowns as

I1: $250I_1 - 150I_2 = 100$

I2: $-150 I_1 + 600 I_2 - 250 I_3 = 0$

I3: $-250 I_2 + 900 I_3 = 0$

Place this in matrix form

$$\begin{bmatrix} 250 & -150 & 0 \\ -150 & 600 & -250 \\ 0 & -250 & 900 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$AX = B$$

Solve in MATLAB

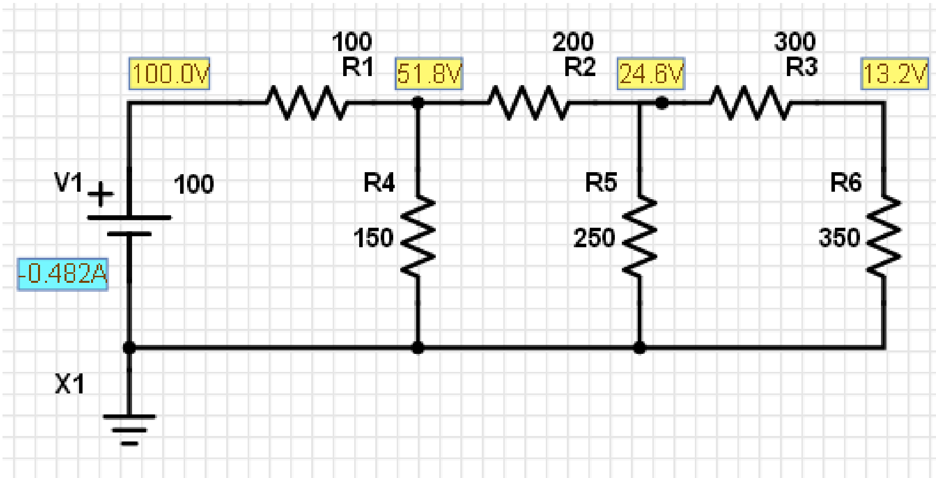
```
-->A = [250,-150,0; -150,600,-250;0,-250,900]

    250.   - 150.     0.
  - 150.     600.   - 250.
    0.     - 250.     900.

-->B = [100;0;0]

    100.
    0.
    0.

-->I = inv(A)*B
```

```
I1:     0.4817150         vs. 0.482A from the PartSim simulation
I2:     0.1361917
I3:     0.0378310
```

Once you know the current you can compute the voltages.

```
-->V1 = 150*(I(1) - I(2))

    51.828499

-->V2 = 250*(I(2) - I(3))

    24.590164

-->V3 = 350*I(3)

    13.240858
```

These agree with the PartSim simulation



Voltage Nodes :

A second way to solve this circuit is to sum the currents to zero at each node.

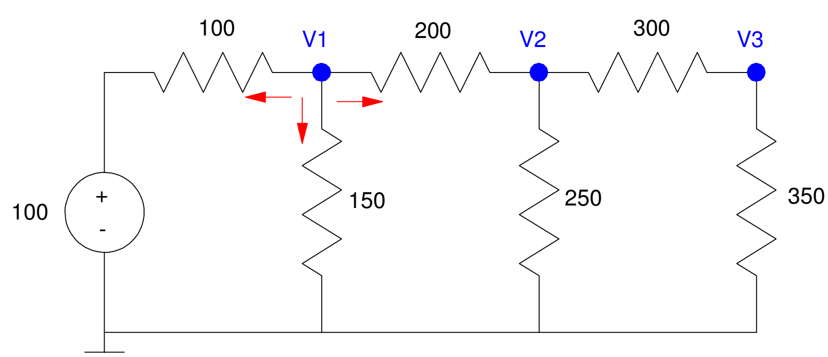
Step 1: Define a ground node. Voltage means nothing without a ground reference

Step 2: Define the voltage at the other N nodes

Step 3: Write N equations for N unknowns by summing the current from each node to zero.

Step 4: Solve N equations for N unknowns.

Example: Find the voltages at each node:



Example: Vind the voltage at each node using KVN

Step 1: Define ground. Already done.

Step 2: Define N voltages for N voltage nodes. Done in blue

Step 3: Write three equations for the three unknown voltages.

At node V1, the current from the load (left, down, right) must add to zero

$$\left(\frac{V_1-100}{100}\right)+\left(\frac{V_1-0}{150}\right)+\left(\frac{V_1-V_2}{200}\right)=0$$

At node V2:

$$\left(\frac{V_2-V_1}{200}\right)+\left(\frac{V_2-0}{250}\right)+\left(\frac{V_2-V_3}{300}\right)=0$$

At node V3:

$$\left(\frac{V_3-V_2}{300}\right)+\left(\frac{V_3-0}{350}\right)=0$$

Step 4: Solve. First, group terms

V1: $\left(\frac{1}{100}+\frac{1}{150}+\frac{1}{200}\right)V_1-\left(\frac{1}{200}\right)V_2=\left(\frac{1}{100}\right)100$

V2: $\left(\frac{-1}{200}\right)V_1+\left(\frac{1}{200}+\frac{1}{250}+\frac{1}{300}\right)V_2-\left(\frac{1}{300}\right)V_3=0$

V3: $\left(\frac{-1}{300}\right)V_2+\left(\frac{1}{300}+\frac{1}{350}\right)V_3=0$

Note that when writing the equations for V1, all terms times V1 are positive while all other terms are negative. This holds for all nodes.

Write in matrix form

$$\begin{bmatrix}\left(\frac{1}{100}+\frac{1}{150}+\frac{1}{200}\right) & \left(\frac{-1}{200}\right) & 0\\ \left(\frac{-1}{200}\right) & \left(\frac{1}{200}+\frac{1}{250}+\frac{1}{300}\right) & \left(\frac{-1}{300}\right)\\ 0 & \left(\frac{-1}{300}\right) & \left(\frac{1}{300}+\frac{1}{350}\right)\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

Solve

```
-->A = [1/100+1/150+1/200,-1/200,0]
 A  =

    0.0216667  - 0.005    0.

-->A = [A;-1/200,1/200+1/250+1/300,-1/300]
 A  =

    0.0216667  - 0.005        0.
  - 0.005        0.0123333  - 0.0033333

-->A = [A;0,-1/300,1/300+1/350]
 A  =

    0.0216667  - 0.005        0.
  - 0.005        0.0123333  - 0.0033333
    0.         - 0.0033333    0.0061905
```

```
-->B = [1;0;0]
 B  =

    1.
    0.
    0.

-->V = inv(A)*B
 V  =

    51.828499
    24.590164
    13.240858
```

Note that this is the same result we got using KCL and with PartSim