ECE 320 - Quiz #7 - Name _____

Fourier Transforms, DC to AC, SCR

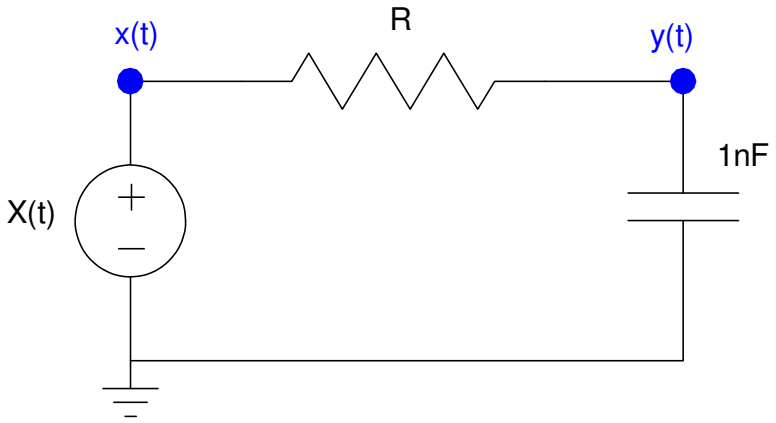
Fourier Transforms

1) Assume the Fourier transform for X(t) is

$$x(t) = 10 + 11\sin(t) + 12\cos(2t)$$

Find y(t). Let R be 1000 + 100*(your birth month) + (your birth day). March 14th would give R = 1514 Ohms.

R 1000 + 100*Month + Day	y(t)
1514	**10 + 11 sin(t) + 12 cos(2t)**



DC:

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

$$Y = X = 10$$

1 rad/sec

$$X = 0 - j11$$

$$C \rightarrow \frac{1}{\omega C} = -j1G\Omega$$

$$Y = \left(\frac{-j1G\Omega}{1514 - j1G\Omega}\right)(0 - j11) = 0 - j11$$

2 rad/sec

$$X = 12 + j0$$

$$C \rightarrow \frac{1}{\omega C} = -j500M\Omega$$

$$Y = \left(\frac{-j500M\Omega}{1514 - j500M\Omega}\right)(12 + j0) = 12 + j0$$

Problem #1 (change C to 1000uF)

$$x(t) = 10 + 11\sin(t) + 12\cos(2t)$$

DC

$$Y = X = 10$$

1 rad/sec

$$C \rightarrow \frac{1}{j\omega C} = -j1000\Omega$$

$$X = 0 - j11$$

$$Y = \left(\frac{-j1000}{1514 - j1000}\right)(0 - j11) = -5.059 - j3.341$$

$$y(t) = -5.059\cos(t) + 3.341\sin(t)$$

2 rad/sec

$$C \rightarrow \frac{1}{j\omega C} = -j500\Omega$$

$$X = 12 + j0$$

$$Y = \left(\frac{-j500}{1514 - j500}\right)(12 + j0) = 1.180 - j3.573$$

$$y(t) = 1.180\cos(2t) + 3.573\sin(2t)$$

y(t)

$$y(t) = 10 - 5.059\cos(t) + 3.341\sin(t) + 1.180\cos(2t) + 3.573\sin(2t)$$

Fourier Transforms

2) Assume a 1 rad/sec parabolic sine wave (of Ninja Turtles fame).

$$x(t) = \begin{cases} t(\pi - t) & 0 < t < \pi \\ t(\pi + t) & -\pi < t < 0 \end{cases} \qquad x(t + \pi) = x(t)$$

Determine the magnitude of the 3rd harmonic of the Fourier Transform for x(t):

$$a_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cdot \cos(3t) \cdot dt = 0 \qquad \textit{it's an odd funciton}$$

$$b_3 = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cdot \sin(3t) \cdot dt \qquad \textit{solve for b3}$$

note:

- Hand calculations, Matlab, calculators, etc. are allowed...

b3	Method / Resource used to solve for b3
0.0943	**Matlab & numerical integration**

In Matlab

```
>> t = [-1:0.001:1]' * pi;
>> x = t.*(pi-t).*(t>0) + t.*(pi+t).*(t<0);
>> a0 = mean(x)
a0 =  1.2135e-017

>> a1 = 2*mean(x .* cos(t))
a1 = -1.3804e-016

>> b1 = 2*mean(x .* sin(t))
b1 =    2.5452

>> a2 = 2*mean(x .* cos(2*t))
a2 = -8.9705e-017

>> b2 = 2*mean(x .* sin(2*t))
b2 = -1.4294e-016

>> a3 = 2*mean(x .* cos(3*t))
a3 =  1.7176e-017

>> b3 = 2*mean(x .* sin(3*t))
b3 =    0.0943


>>
```

DC to AC Converter

3) Assume the Fourier transform for the output of a DC to AC converter driving a 1 Ohms reisistor is as follows:

- note: units are Vp (peak voltage)

Harmonic	0 (DC)	1	2	3	4	5
an (cosine)	0	18.3	0	2.6	0	0.7
bn (sine)	0	2.7	0	0	1.4	0
Energy (W) $(a_n^2 + b_n^2)/2$	**0**	**171.09**	**0**	**3.38**	**0.98**	**0.24**

Determine the following:

Total Energy in the signal	Energy in the 1st harmonic	Efficiency % of energy in the 1st harmonic
175.69W	**171.09W**	**97.4%**

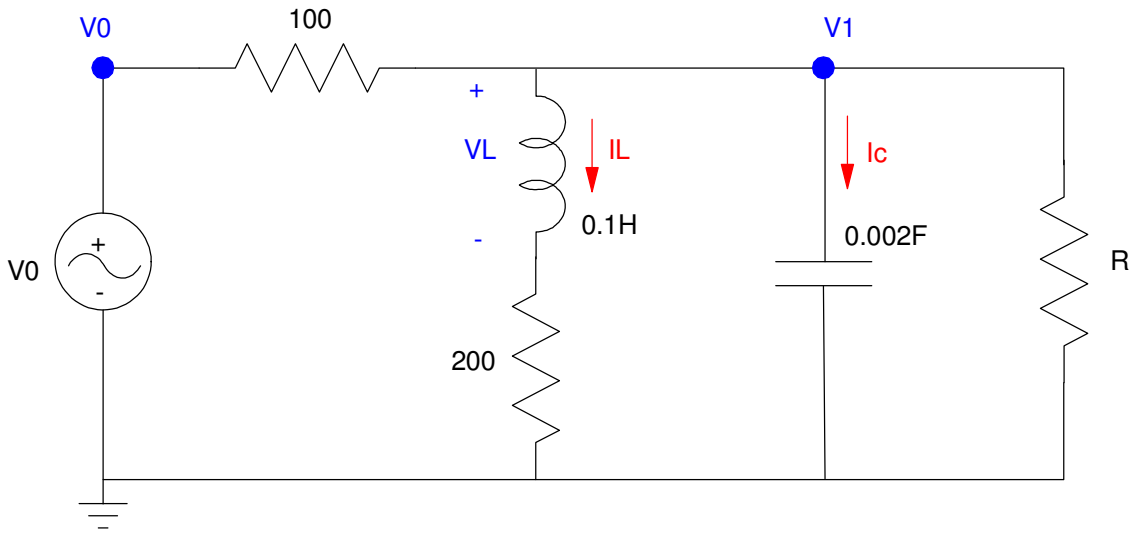
DC to AC Converter: Differential equations for a Circuit

4) Determine the differential equations which describe the following circuit. Note

- $V_L = L\frac{dI_L}{di}$
- $I_c = C\frac{dV_1}{dt}$

Assume R = 1000 + 100*(Birth Month) + (Birth day). For May 15th, for example, R = 1514 Ohms.

$\frac{dI_L}{dt} = 10V_1 - 2000I_L$
$\frac{dV_1}{dt} = 5V_0 - 5.33V_1 - 500I_L$



$$V_L = 0.1\frac{dI_L}{dt} = V_1 - 200I_L$$

$$\frac{dI_L}{dt} = 10V_1 - 2000I_L$$

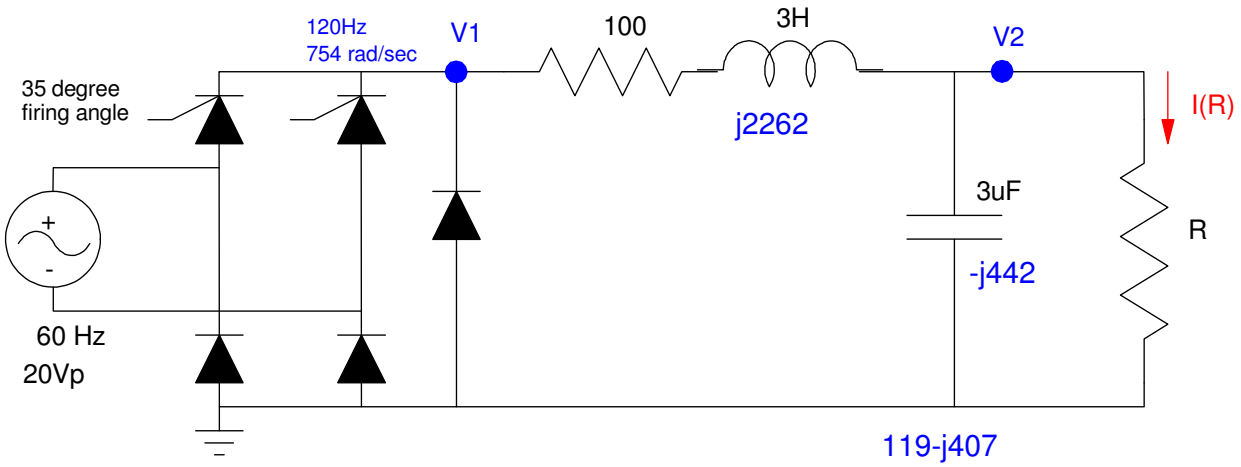
$$0.002\ \frac{dV_1}{dt} = \left(\frac{V_0 - V_1}{100}\right) - I_L - \frac{V_1}{1514}$$

$$\frac{dV_1}{dt} = 5V_0 - 5V_1 - 500I_L - 0.33V_1$$

SCR (5 diode version)

5) SCR: Analysis. Determine the votlages at V1 and V2 (both DC). Assume a firing angle of 35 degrees.

R 1000 + 100*Mo + Day	V1		V2	
	DC	AC (V1pp)	DC	AC (V2pp)
1514	**10.48**	**19.3Vpp**	**9.83**	**4.39Vpp**



35 degrees

$$V_1(DC) = \left(\frac{18.6V+0.7V}{\pi}\right)(1+\cos(35^0)) - 0.7 = 10.476V$$

$$V_1(AC) = 19.3V_{pp}$$

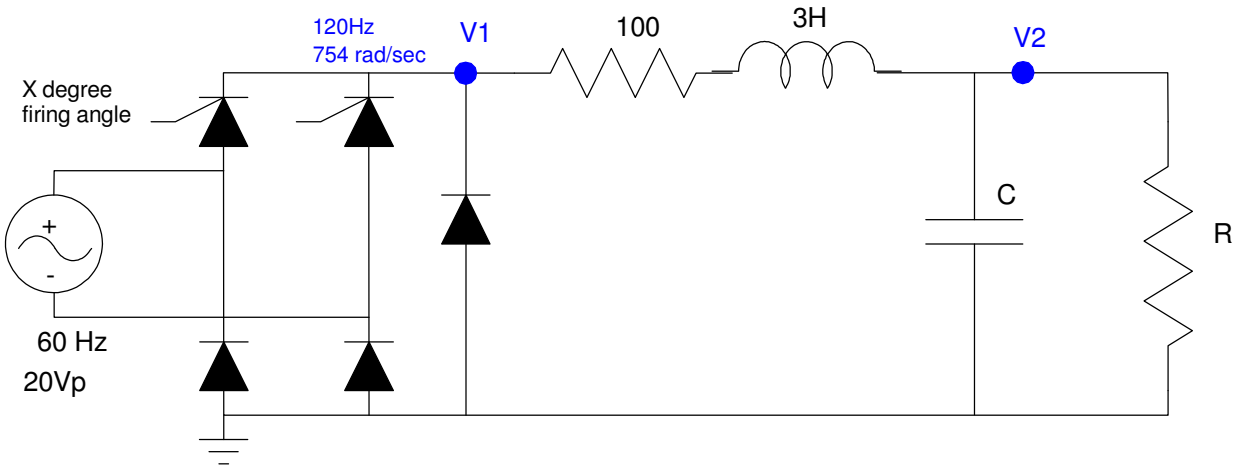
$$V_2(DC) = \left(\frac{1514}{1514+100}\right)10.476V = 9.827V$$

$$V_2(AC) = \left(\frac{119-j407}{(119-j407)+(100+j2262)}\right)19.3V_{pp} = 4.386V_{pp}$$

6) SCR Design. Determine the firing angle and C so that

- V2(DC) = 4.5V
- V2(AC) = 500mVpp
- R = 1000 + 100*(Birth Month) + (Birth Day). May 14th would give R = 1514 Ohms.

V1(DC)	Firing Angle	C	R 1000 + 100*Mo + Day
4.797V	**96.03 deg**	**18.1uF**	**1514**



$$V_2 = \left(\frac{1514}{1514+11}\right) V_1 = 4.50V$$

$$V_1 = 4.797V$$

$$V_1 = 4.797V = \left(\frac{18.6V+0.7V}{\pi}\right)(1+\cos(\theta)) - 0.7$$

$$\theta = 96.037^0$$

$$V_1(AC) = 18.489V - (-0.7V) = 19.189V_{pp}$$

If C = 0

$$V_2(AC) = \left(\frac{1514}{1514+(100+j2262)}\right) 19.189V_{pp} = 10.373V$$

to bring the ripple down to 0.5Vpp

$$\left|\frac{1}{j\omega C}\right| = \left(\frac{0.5Vpp}{10.373Vpp}\right) 1514\Omega = 72.98\Omega$$

$$C = 18.1\mu F$$