ECE 320 - Homework #7

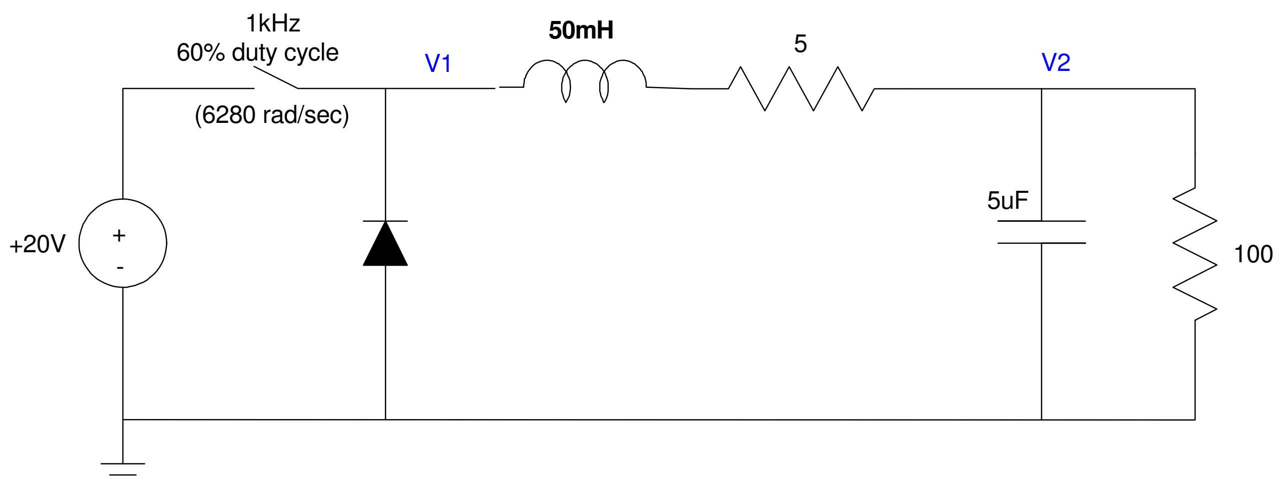
Fourier Transform, DC to AC, SCR. Due March 1st, 2021

Fourier Transform

The voltage V1 is a 60% duty cycle square wave

$$V_1(t) = V_1(t + 1ms) \qquad \textit{V1 is periodic in 1ms - i.e. it's a 1kHz square wave}$$

$$V_1(t) = \begin{cases} +20V & 0 < t < 600\mu s \\ -0.7V & 600\mu s < t < 1000\mu s \end{cases}$$



1) Determine the first five terms for the Fourier transform for V1(t)
   - DC
   - 1kHz, sine and cosine
   - 2kHz, sine and cosine

$$V_1(t) = a_0 + a_1 \cos(\omega_0 t) + b_1 \sin(\omega_0 t) + a_2 \cos(2\omega_0 t) + b_2 \sin(2\omega_0 t)$$

Option 1: Slow Fourier Transform (my prefrence)

```
>> a0 = mean(V1)
a0 =    11.7188

>> a1 = 2*mean(V1 .* cos(2*pi*t))
a1 =    -3.8689

>> b1 = 2*mean(V1 .* sin(2*pi*t))
b1 =    11.9197

>> a2 = 2*mean(V1 .* cos(2*pi*2*t))
a2 =     3.1342

>> b2 = 2*mean(V1 .* sin(2*pi*2*t))
b2 =     2.2743
```

Option #2: Complex Fourier Transform (same result)

```
>> c0 = mean(V1)

c0 =   11.7188

>> c1 = 2*mean(V1 .* exp(-j*2*pi*t))

c1 =  -3.8689 -11.9197i

>> c2 = 2*mean(V1 .* exp(-j*2*pi*2*t))

c2 =   3.1342 - 2.2743i
```

meaning

$$v_1(t) = 11.7188 \qquad \textit{DC term}$$

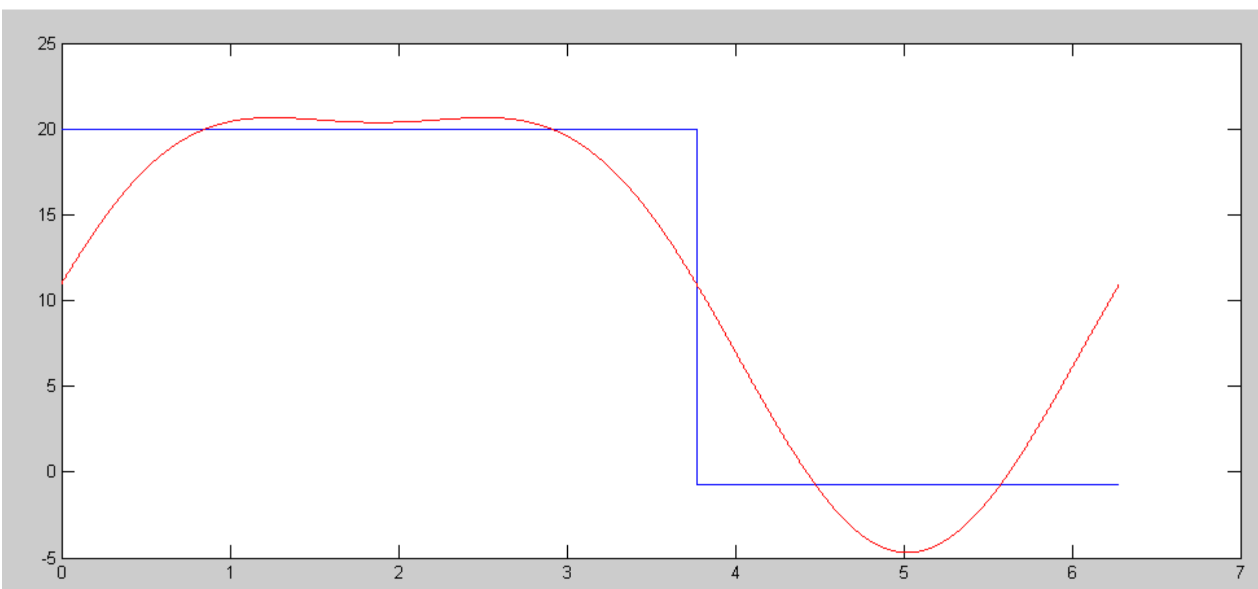
$$-3.8689\cos(\omega_0 t) + 11.9197\sin(\omega_0 t) \qquad \textit{1kHz term}$$

$$+3.1342\cos(2\omega_0 t) + 2.2743\sin(2\omega_0 t) \qquad \textit{2kHz term}$$

$$\vdots$$

Checking in Matlab

```
t = t * 2*pi;
V1f = a0 + a1*cos(t) + b1*sin(t) + a2*cos(2*t) + b2*sin(2*t);
plot(t,V1,'b',t,V1f, 'r');
```



2) Determine V2(t) at each frequency

- DC
- 1kHz
- 2kHz

Treat this as three separate problems...

DC:

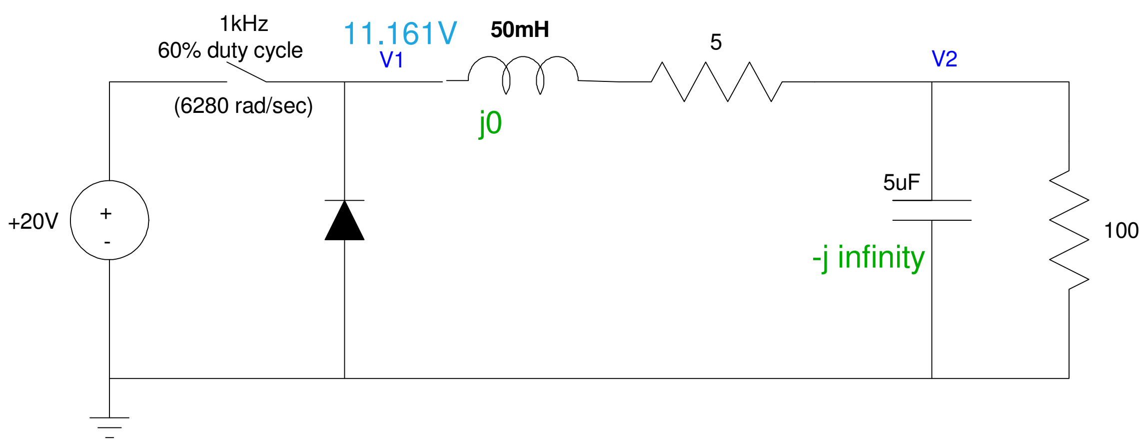
$$\omega = 0$$

$$V_1 = 11.7188$$

$$L \rightarrow j\omega L = 0$$

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

$$V_2 = \left(\frac{100}{100+5}\right)(11.7188) = 11.161$$



1kHz:

$$v_1(t) = -3.8689\cos(\omega_0 t) + 11.9197\sin(\omega_0 t)$$

$$\omega_0 = 2\pi f = 6280$$

$$V_1 = -3.8689 - j11.9197$$

$$L \to j\omega L = j314\Omega$$

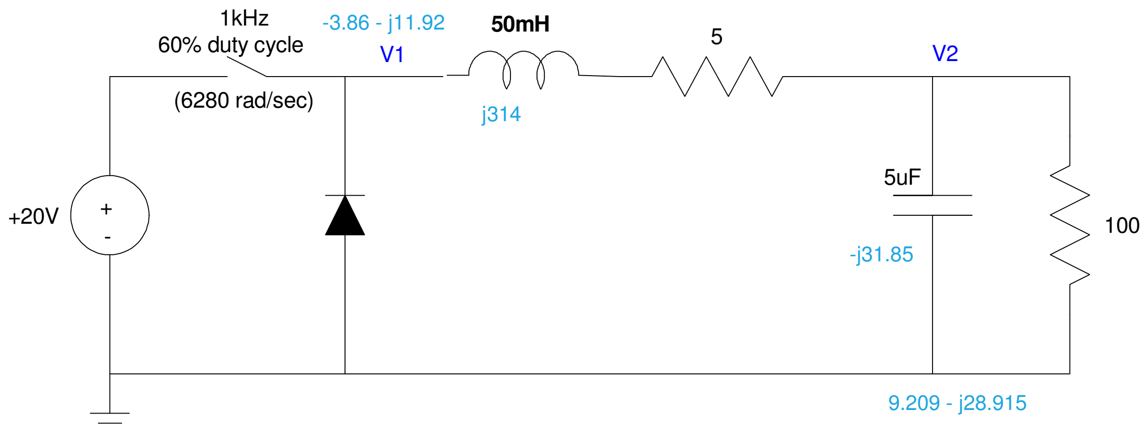
$$C \to \tfrac{1}{j\omega C} = -j31.847\Omega$$

$$100\Omega \,||\, -j31.847\Omega = 9.209 - j28.915$$

$$V_2 = \left(\tfrac{(9.209-j28.915)}{(9.209-j28.915)+(5+j314)}\right)(-3.8689 - j11.9197)$$

$$V_2 = -0.013 + j1.211$$

$$v_2(t) = -0.013\cos(\omega_0 t) - 1.211\sin(\omega_0 t)$$



2kHz:

$$2\omega_0 = 2 \cdot 6280 = 12,560$$

$$v_1(t) = +3.1342\cos(2\omega_0 t) + 2.2743\sin(2\omega_0 t)$$

$$V_1 = 3.1342 - j2.2743$$

$$L \to j\omega L = j628\Omega$$

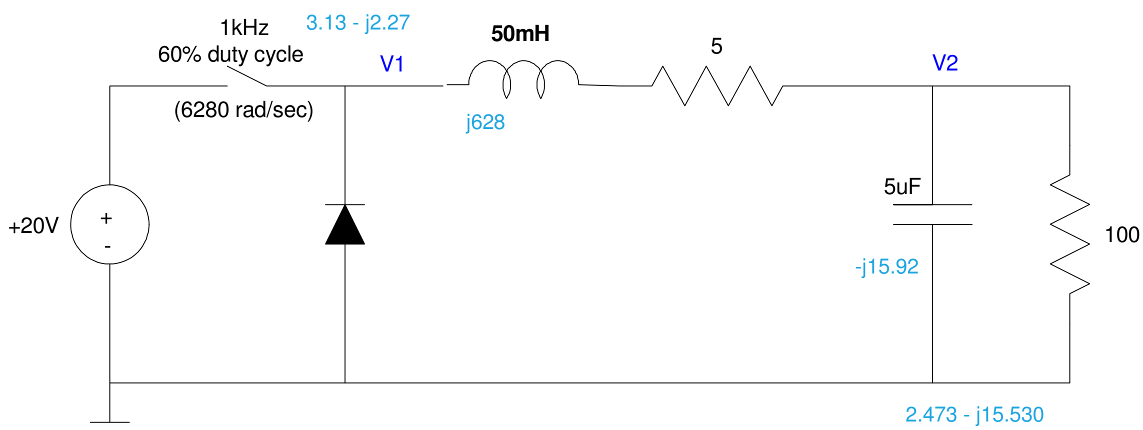
$$C \to \frac{1}{j\omega C} = -j15.924\Omega$$

$$100\Omega \,||\, -j15.924\Omega = 2.473 - j15.530\Omega$$

$$V_2 = \left(\frac{(2.473 - j15.530)}{(2.473 - j15.530) + (5 + j628)}\right)(3.1342 - j2.2743)$$

$$V_2 = -0.089 + j0.044$$

$$v_2(t) = -0.089\cos(2\omega_0 t) - 0.044\sin(2\omega_0 t)$$



So, the total answer is DC + 1kHz + 2kHz:

$$v_2(t) = 11.161$$
$$-0.013\cos(\omega_0 t) - 1.211\sin(\omega_0 t)$$
$$-0.089\cos(2\omega_0 t) - 0.044\sin(2\omega_0 t)$$

3) Compare the results from this homework set vs. homework set #6, problem #4 and #5.

- Does using the Fourier transform for V1(t) give more accurate results in predicting V2(t)?

Yes, using the Fourier Transforms predicts the AC voltage at V2 more accurately. For a lot more work.

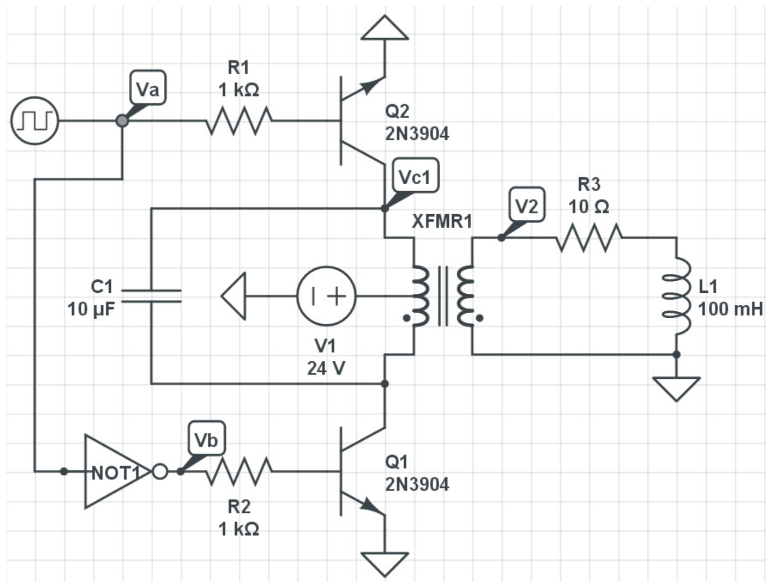
		V2	
	DC	AC: 1kHz	AC (2KHz)
Calculated HW #6	11.16 V	2.200 Vpp	0 Vpp
Simulated HW #6	11.026 V	2.588 Vpp	?
Calculated Using Fourier Transforms HW #7	11.161 V	2.422 Vpp	0.199 Vpp

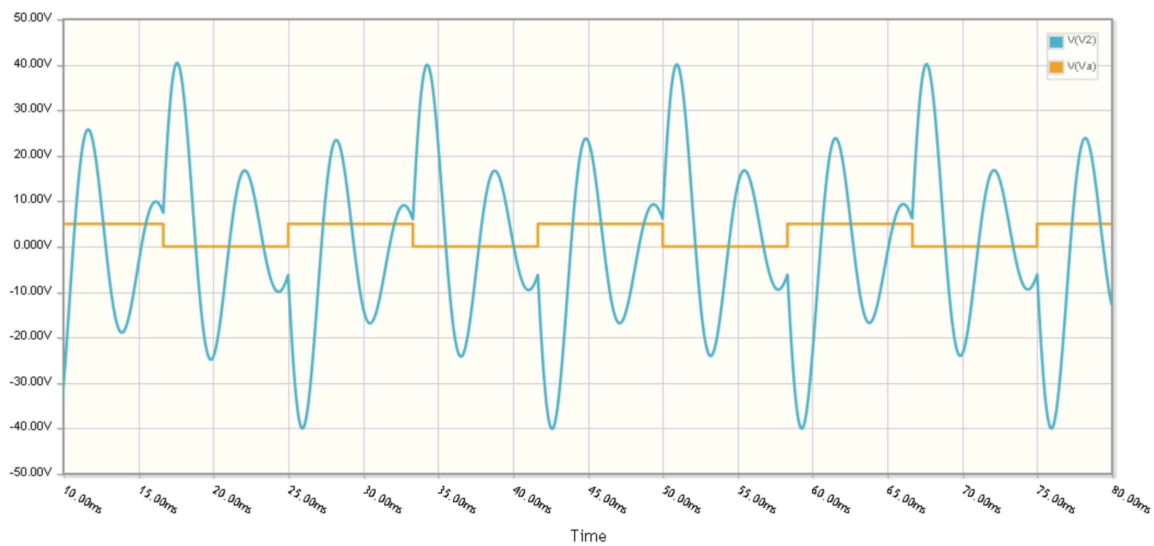
DC to AC

4) Let

- A = 0V / 5V square wave, 60Hz, 0 degree time delay
- B = 0V / 5V square wave, 60Hz, 180 degree time delay
- C1 = 10uF

Determine using CircuitLab the voltage V2 (i.e. the votlage across a DC motor, modeled as a 10 Ohm & 200mH load)





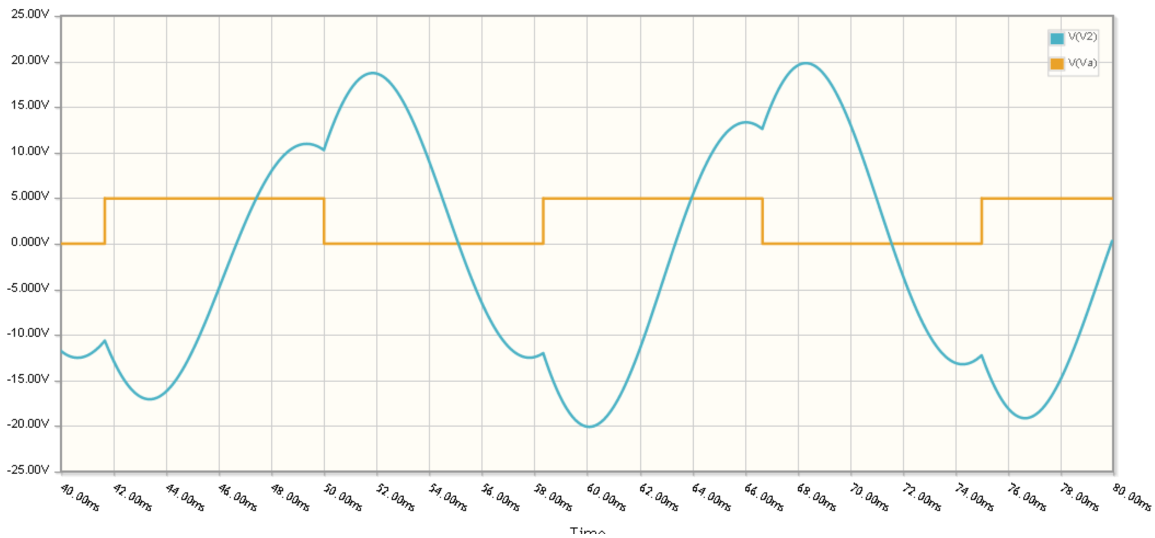
5) Adjust C so that the voltage across the motor is as close to a sine wave as possible (trial and error)

Part of the problem is the LC isn't resonant at 60Hz. To do this, let the LC tank resonate at 60Hz (377 rad/sec)

$$Ls + \frac{1}{Cs} = \left(s^2 + \frac{1}{LC}\right)L^2$$

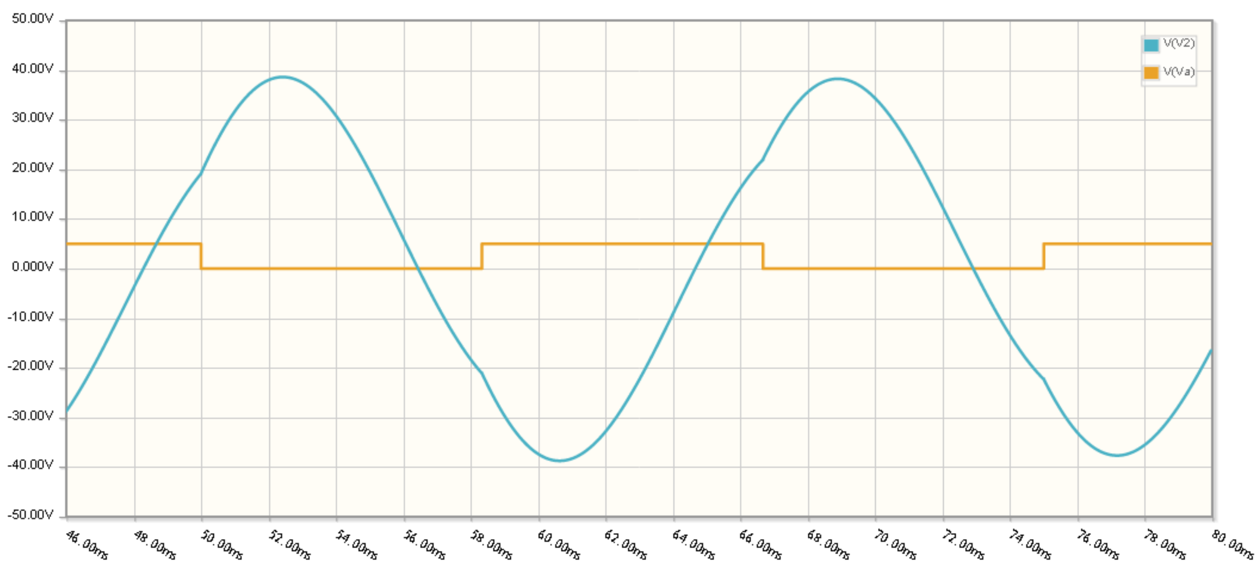
$$\frac{1}{LC} = 377^2$$

L = 100mH, C = 70.36uF



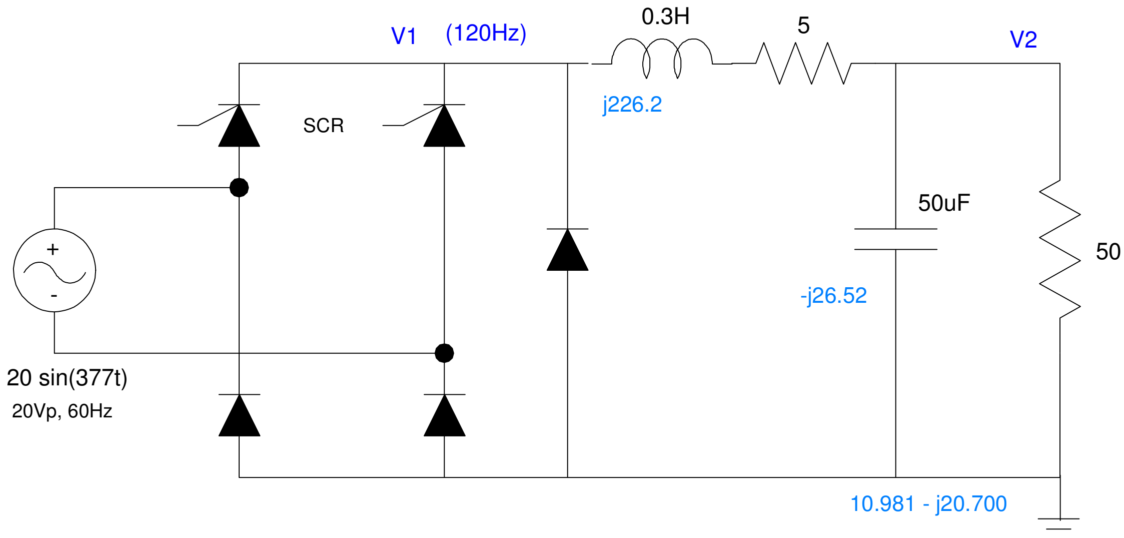
v2(t) with C = 70.36uF

Increasing C to 110 uF works slightly better (closer to a 60Hz sine wave at V2)



SCR

6) Assume a firing angle of 25 degrees. Determine the voltage at V1 and V2 (both DC and AC).



$$V_1(DC) = \left(\frac{V_p+0.7}{\pi}\right)(1+\cos\theta)-0.7$$

$$V_1(DC) = \left(\frac{18.6+0.7}{\pi}\right)(1+\cos(25^0))-0.7$$

$$V_1(DC) = 11.011V$$

$$V_2(DC) = \left(\frac{50}{50+5}\right)11.011V = 10.010V$$

$$V_1(AC) = 18.6V-(-0.7V) = 19.3V_{pp}$$

$$V_2(AC) = \left(\frac{(10.981-j20.700)}{(10.981-j20.700)+(5+j226.2)}\right)\cdot 19.3V_{pp}$$

$$V_2(AC) = 2.194V_{pp}$$

7) Change this circuit so that

- The voltge at V2 is 8.00V (DC)
- With a ripple of 500mVpp

DC Analysis

$$V_1(DC) = \left(\frac{50+5}{50}\right) 8.00V = 8.80V$$

$$V_1(DC) = \left(\frac{V_p+0.7}{\pi}\right)(1 + \cos\theta) - 0.7$$

$$8.80V = \left(\frac{18.6+0.7}{\pi}\right)(1 + \cos\theta) - 0.7$$

$$\theta = 56.88^0$$

AC Analysis

$$V_1(AC) \approx 19.3V_{pp}$$

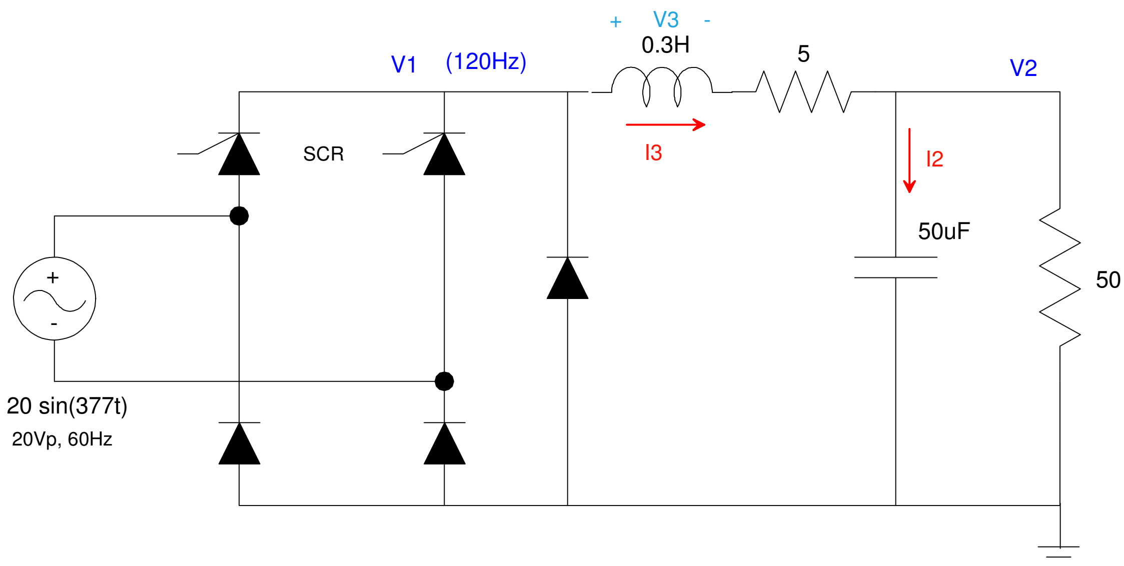
When C = 50uF, V2(AC) = 2.194Vpp

If C is 2x larger, the ripple is 2x smaller

$$C \to \left(\frac{2.194V_{pp}}{0.5V_{pp}}\right) 50\mu F = 219.4\mu F$$

8) Simulate this circuit in Matlab by

- Writing the differential eqautions which describe this circuit (state variables: IL and Vc)
- Specify V1(t) as a full-wave rectified sine wave, clipped at X degrees (from problem #4)
- Use numerical integration to find V2(t)



$$V_3 = 0.3\frac{dI_3}{dt} = V_1 - 5I_3 - V_2$$

$$I_2 = 219.4\mu F\,\frac{dV_2}{dt} = I_3 - \frac{V_2}{50}$$

The coupled differential equations that descibe this circuit is thus

$$\frac{dV_2}{dt} = 4557.9I_3 - 91.158V_2$$

$$\frac{dI_3}{dt} = 3.333V_1 - 16.667I_3 - 3.3333V_2$$

Solving in Matlab using numerical integration

Matlab Script:

```
t = 0;
dt = 1e-6;
I3 = 0;
V2 = 0;
V = [];

while(t < 10/120)

    phase = mod(377*t,pi);

    if(phase < 56.88*pi/180)
        V1 = -0.7;
    else
        V1 = 20*sin(phase) - 1.4;
    end
    V1 = max(V1, -0.7);

    dV2 = 4557.9*I3 - 91.158*V2;
    dI3 = 3.333*V1 - 16.667*I3 - 3.3333*V2;

    V2 = V2 + dV2*dt;
    I3 = I3 + dI3*dt;
    t = t + dt;

    if(t > 8/120)
       V = [V ; V1, V2];
    end
end

t = [1:length(V)]' * dt;
plot(t,V(:,1),'b',t,V(:,2), 'r');
```

Result:

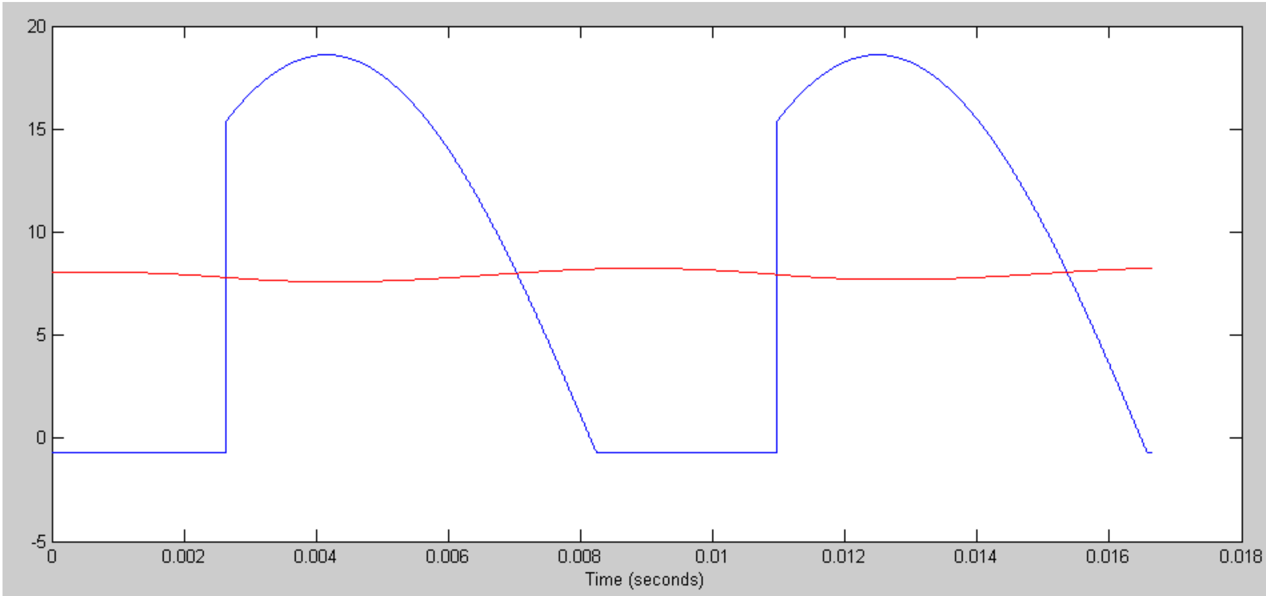
```
ans =

    V1(DC)     V2(DC)
    8.6687    7.9246

>> max(V) - min(V)

ans =
   V1pp         V2pp
   19.3000    0.6623
```

The AC term is slightly off due to approximating the 1st harmonic as max-min rather than the 1st term in the Fourier series expansion.



Resulting Signals at V1 (blue) and V2 (red)