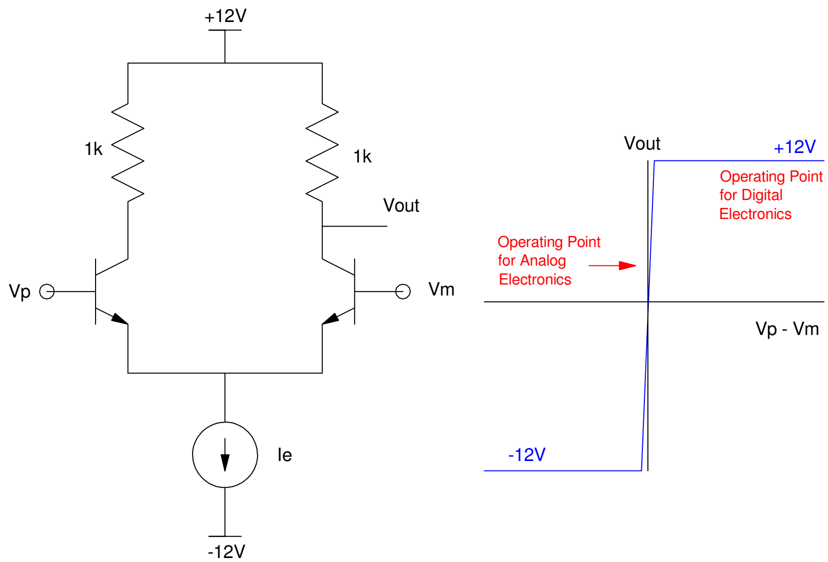
Operational Amplifiers

Operational Amplifiers (Op-Amps) are high gain differential amplifiers. The heart of an op-amp is a pair of transistors implementing emitter-coupled logic:



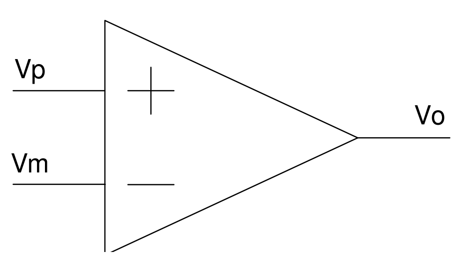
In ECE 320, we wanted binary outputs: a motor is either 100% on or 0% on (i.e. off). Likewise, we operated the transistors at the rails: either +5V for logic 1 or 0V for logic 0. Towards this end, the transistors we used were always off or saturated.

In ECE 321, we want analog outputs: a motor can be 30% on or we could apply a sine wave to a speaker (as opposed to a square wave). To do this, we will be using transistor in the active region and using op-amps in the high-gain region (where Vp - Vm ≈ 0V).

When operating in the high-gain region, an op-amp can be modeled as a 2-input device with

$$V_o \approx k(V^+ - V^-)$$

where k is a large number. For short, the following symbol is used for an differential amplifier:

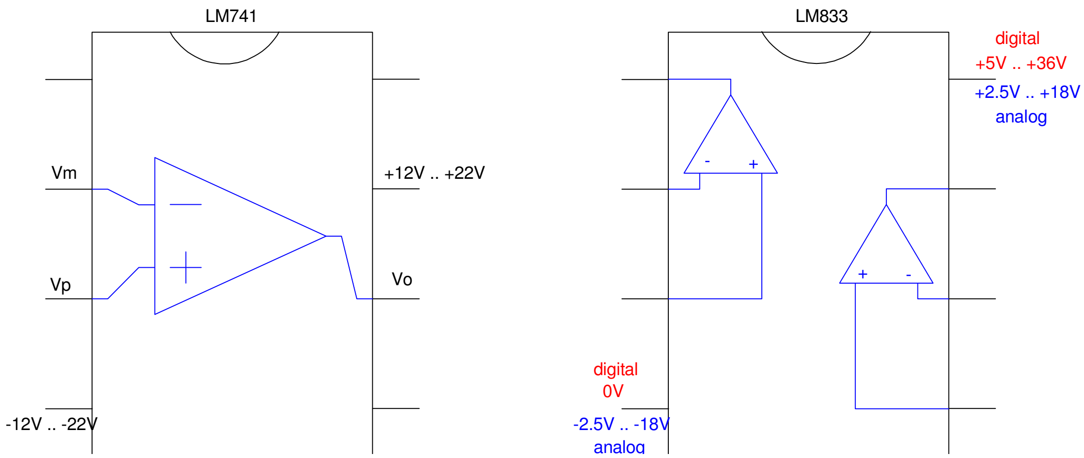


Symbol for an operational amplifier (op-amp)

Operational Amplifier Characteristics

Op-Amps usually come in two packages:

- A single op-amp per package (left), or
- Two op-amps per package



Pin Layout for two common op-amps: LM741 and LM833
The LM833 works for both digital electronics and analog electronics.

If you look up the data sheets for these, you get something like the following:

	LM741	LM833 use this one	Ideal
Input Resistance	2M Ohms	1G Ohm	infinite
Current Out (max)	25mA	50mA	infinite
Operating Voltage	+/- 12V .. +/- 22V	+/- 2.5V .. +/- 15V	any
Differential Mode Gain	200,000	310,000	infinite
Common Mode Rejection Ratio	90dB	100dB	common mode gain = 0
Slew Rate	0.5V/us	7V/us	infinite
Gain Bandwidth Product	1.5MHz	15MHz	infinite
Price (qty 100)	$0.35	$0.52	-

Input resistance: The input of the op-amp does draw some current. If you keep the currents involved much larger (meaning at 1V, resistors are less than 50M Ohm), you can ignore the current into V+ and V-.

Current Out (max): These op-amps can drive 1k Ohms at 10V (10mA), but they can't drive an 8-Ohms speaker (too much current).

Operating Voltage: Options for the power supply.

- LM741 op-amps need +12V and -12V to operate (they don't work for digital electronics)
- LM833 op-amps can operate single sided (0..5V, 0..10V) for digital electronics or dual power supplies (-5V..+5V, or -10V to +10V) for analog electronics.

Differential Mode Gain: The gain from (V+ - V-) to the output

Common Mode Rejection Ratio: The gain from (V+ + V-) is this much less than the differential mode gain. Note that

$$dB = 20 \cdot \log_{10}(gain)$$

Slew Rate: The output can't change from -10V to +10V in zero time. It can only ramp up this fast.

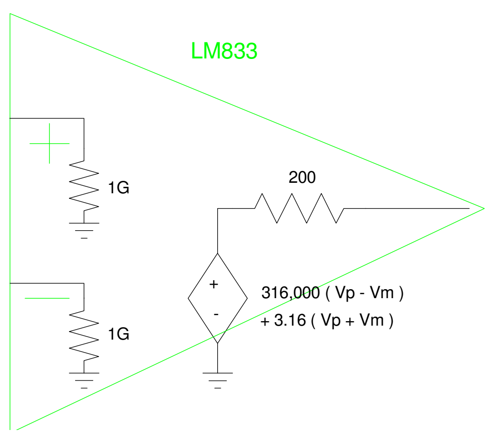
Gain Bandwidth Product: The gain times the bandwidth is this number. For an LM833 (15MHz), you can have a

- Gain of 1.00 out to 15MHz
- Gain of 100.0 out to 150kHz

Note that this means for audio applications (20-20kHz) you need to keep the gain of each op-amp less than 750.

Operational Amplifier Circuit Analysis

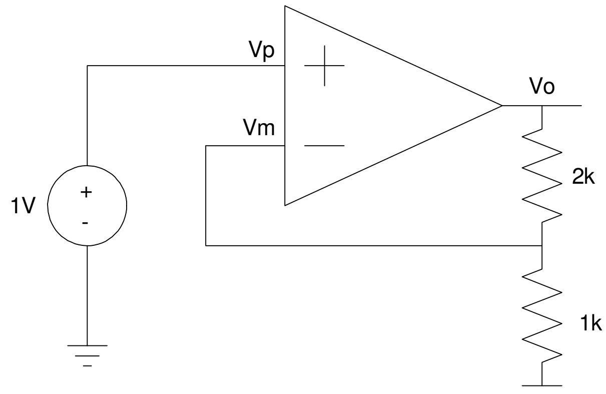
When you have an op-amp in a circuit, you essentially have a voltage controlled voltage source. For an LM833 op-amp, for example, it's circuit model is:



Circuit Model for an LM833.

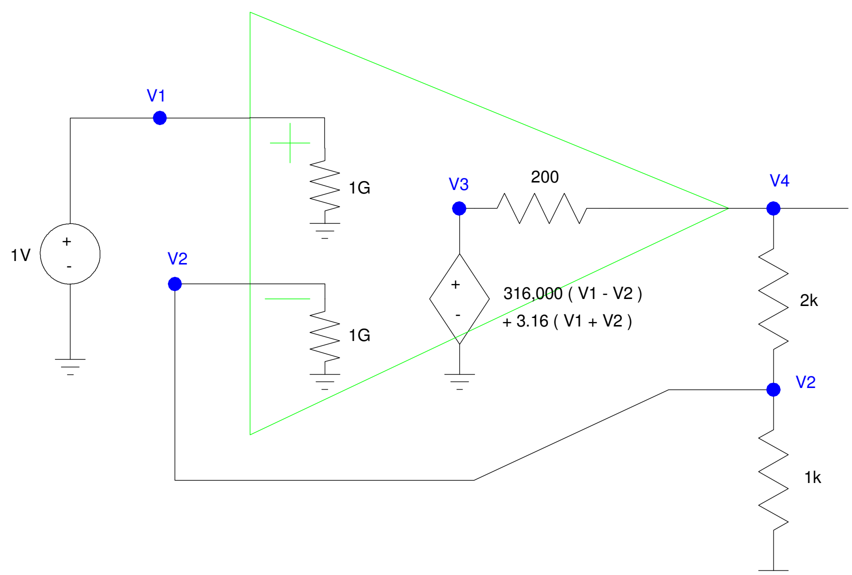
Using this model, you can determine the voltages for an op-amp circuit.

Example: Determine the voltages for the following op-amp circuit.



Case 1: LM833 Op-Amp.

Replace the op-amp with its circuit model



Write the voltage node equations:

$$V_1 = 1$$

$$\left(\frac{V_2}{1G}\right) + \left(\frac{V_2 - V_4}{2k}\right) + \left(\frac{V_2}{1k}\right) = 0$$

$$V_3 = 316,000(V_1 - V_2) + 3.16(V_1 + V_2)$$

$$\left(\frac{V_4 - V_3}{200}\right) + \left(\frac{V_4 - V_2}{2000}\right) = 0$$

Solve in Matlab:

```
A = [1,0,0,0;0,1/1e9+1/2000+1/1000,0,-1/2000 ;
316003.16,-316003.16,-1,0;0,-1/2000,-1/200,1/2000+1/200]

    1.            0.           0.          0.
    0.            0.0015000    0.        - 0.0005
    316003.16   - 316003.16  - 1.          0.
    0.          - 0.0005     - 0.005       0.0055

B = [1;0;0;0]

    1.
    0.
    0.
    0.

V = inv(A)*B

V1    1.
V2    0.9999899
V3    3.1999698
V4    2.9999716
```

Case 2: Ideal Op-Amp

Note that

$$V_o = 316,000(V_p - V_m)$$

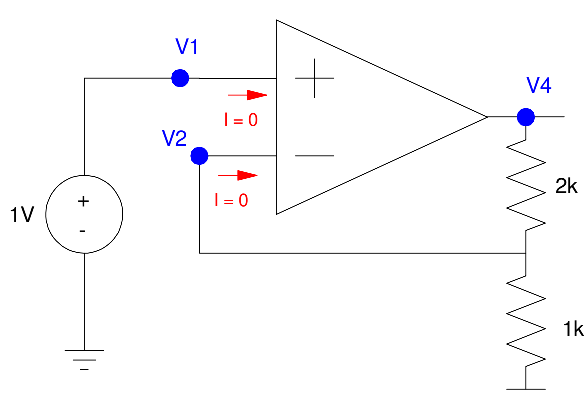
Assuming the output is finite, this means that

$$V_p \approx V_m.$$

In the case where the gain goes to infinity (ideal op-amp), you get

$$V_p = V_m.$$

Also, the 1G Ohm input impedance is so large that the current to the + and - inputs is negligible.



Case 2: Replace the op-amp with an ideal op-amp

Assuming an ideal op-amp, the voltage node equations become:

$$V_1 = 1$$

$$V_1 = V_2$$

$$\left(\frac{V_2}{1k}\right) + \left(\frac{V_2 - V_4}{2k}\right) = 0$$

Solving results in

	Ideal OpAmp	LM833
V1	1.	1.
V2	1.	0.9999899
V3	n/a	3.1999698
V4	3.000000	2.9999716

Note:

- The results are almost identical. You have to go out to the 6th decimal place to see the difference.
- It's a *lot* easier to use the ideal op-amp model.
- If you use a different op-amp, the results will be about the same. The ideal op-amp model is a close approximation for an op-amp under most conditions.

note: "Most" means

- *You keep impedances less than 10M Ohms (so you can ignore the 1G input impedance), and*
- *You keep impedances more than 200 Ohms (so you draw less than 50mA)*

Likewise, from here on, we'll be assuming ideal op-amps. If your results are slightly different from what CircuitLab gives you, it's because CircuitLab uses the model for an LM833 (or whatever op-amp you're using).

Voltage Nodes with Op-Amps

With op-amp circuits, you almost always use voltage nodes. With voltage nodes, you write N equations for N unknowns. The trick with op-amps is the voltage node equation at the output is

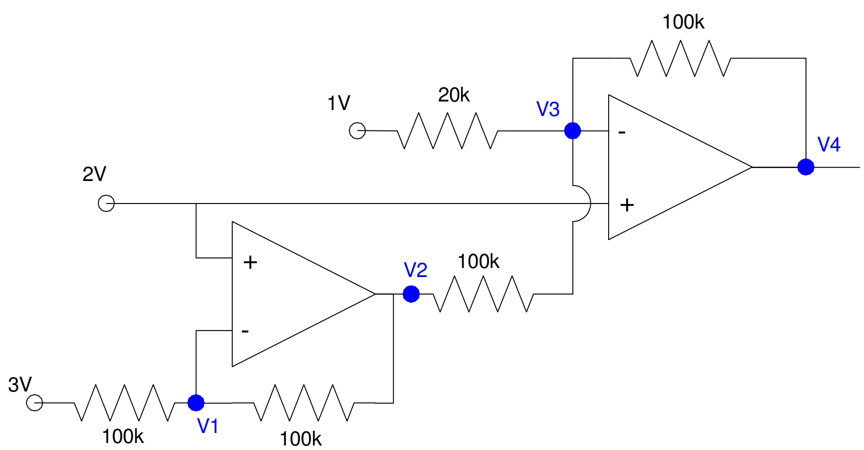
- $V_p = V_m$

You do this for two reasons:

- (1) If you have negative feedback (i.e. are analyzing an amplifier as opposed to a Schmitt trigger), the high gain will force Vm to be close to Vp. Otherwise, the output would rail at the power supply.
- (2) You can't write the voltage node equation at Vo. The op-amp is an active device and will source or sink as much current as needed to force Vm to match Vp. If you try to write the voltage node equation at Vo, the current from the op-amp is "as much as needed." That doesn't help when writing voltage node equations.

Example 2: Assume ideal op-amps

- Write the voltage node equations for the following op-amp circuit
- Find the voltages



Example 2: Find the voltages

There are four unknown voltage nodes. We need to write 4 equations to solve for 4 unknowns. Start with the easy ones. For ideal op-amps with negative feedback

$$V_p = V_m$$

meaning

$$V_1 = 2V \quad (1)$$

$$V_3 = 2V \quad (2)$$

Now write two more equations. It's tempting, but you can't write the node equations at V2 or V4

- Equation (1) and (2) *are* the node equations at the outputs - you've already done that.
- You don't know the current from the op-amp - meaning you can't sum the currents to zero.

Instead, find two mode nodes where you *can* sum the currents to zero: nodes V1 and V3.

$$\left(\frac{V_1-3}{100k}\right)+\left(\frac{V_1-V_2}{100k}\right)=0 \qquad (3) \quad \text{* 100k to clear the denominator}$$

$$\left(\frac{V_3-V_2}{100k}\right)+\left(\frac{V_3-1}{20k}\right)+\left(\frac{V_3-V_4}{100k}\right)=0 \qquad (4) \quad \text{* 100k to clear the denominator}$$

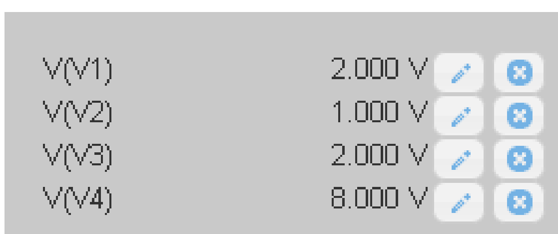
Solving

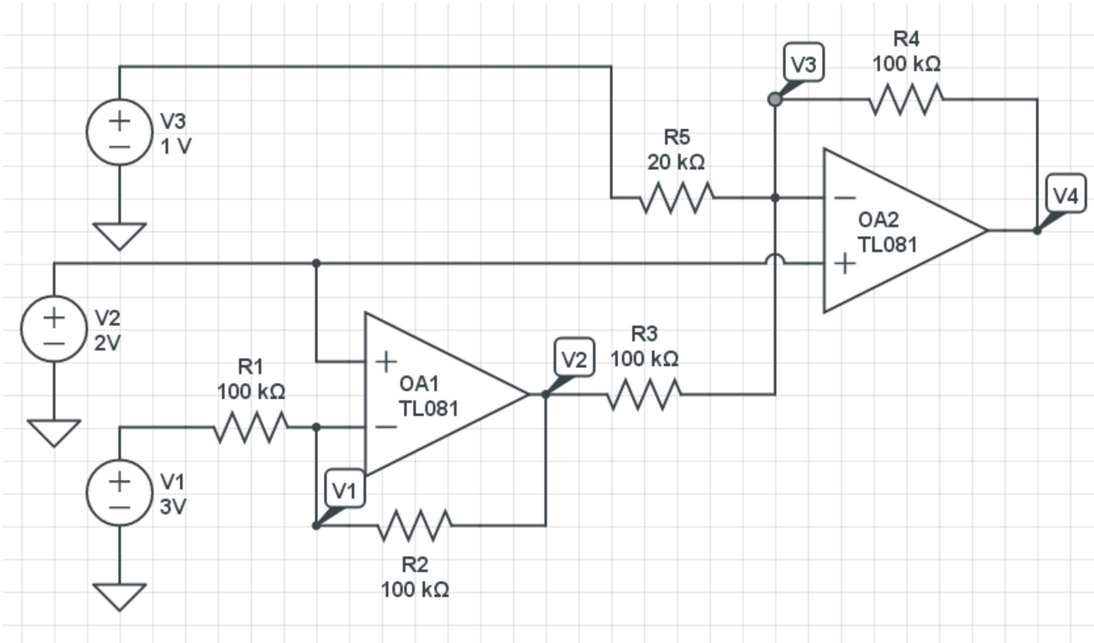
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & -1 & 7 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}=\begin{bmatrix} 2 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

In Matlab:

```
>> A = [1,0,0,0 ; 0,0,1,0 ; 2,-1,0,0 ; 0,-1,7,-1]

      1      0      0      0
      0      0      1      0
      2     -1      0      0
      0     -1      7     -1

>> B = [2;2;3;5]

      2
      2
      3
      5

>> V = inv(A)*B

V1     2
V2     1
V3     2
V4     8
```

This checks with the CircuitLab



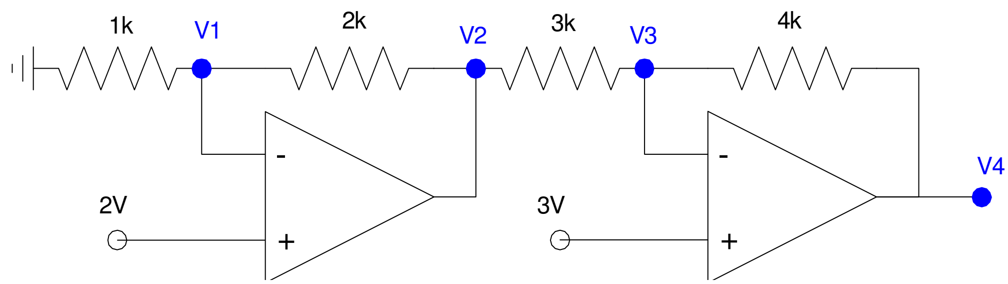


CircuitLab results for example 2: The votlages match our computations.

Note the following:

- You don't need to use the op-amps with the +/- power supplies For analog circuits, the output should be finite (i.e. a voltage in the range of -10V to +10V). As long as the power supply allows this (i.e you're using enough voltage to drive the output), the power supply doesn't matter.
- Using the wrong op-amp (TL081 instead of an LM833) is also OK. They both behave like an ideal op-amp (and likewise have almost identical results).

Example 3: Assume ideal op-amps. Find the node voltages.



There are four unknown voltages, so we need to write 4 equations to solve for 4 unknowns.

Start with the easy ones: at the output of each op-amp, V+ = V-

$$V_1 = 2 \quad (1)$$

$$V_3 = 3 \quad (2)$$

Sum the currents to zero at nodes 1 and 3 for the remaining two equations

$$\left(\frac{V_1}{1k}\right) + \left(\frac{V_1 - V_2}{2k}\right) = 0 \quad (3)$$

$$\left(\frac{V_3 - V_2}{3k}\right) + \left(\frac{V_3 - V_4}{4k}\right) = 0 \quad (4$$

In matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \left(\frac{1}{1k} + \frac{1}{2k}\right) & \left(\frac{-1}{2k}\right) & 0 & 0 \\ 0 & \left(\frac{-1}{3k}\right) & \left(\frac{1}{3k} + \frac{1}{4k}\right) & \left(\frac{-1}{4k}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Solving:

```
>> A = [1,0,0,0 ; 0,0,1,0 ; 1/1000+1/2000, -1/2000, 0, 0 ; 0,-1/3000, 1/3000+1/4000,-1/4000]

    1.0000         0         0         0
         0         0    1.0000         0
    0.0015   -0.0005         0         0
         0   -0.0003    0.0006   -0.0003

>> B = [2;3;0;0];
>> V = inv(A)*B

V1     2.0000
V2     6.0000
V3     3.0000
V4    -1.0000
```

Checking in CircuitLab

V(V1)	2.000 V
V(V2)	4.000 V
V(V3)	3.000 V
V(V4)	2.000 V