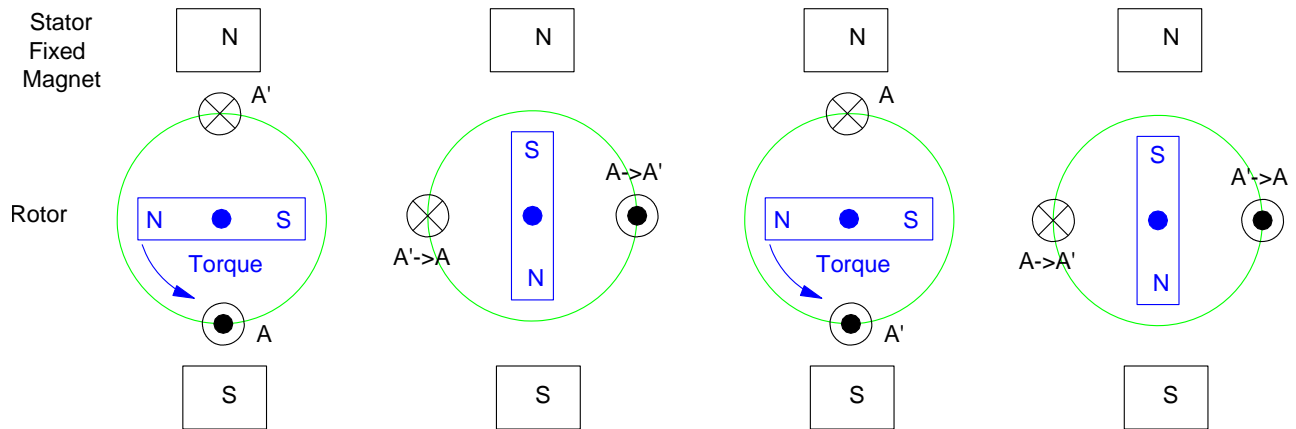


Permanent Magnet DC Motors

A DC motor has

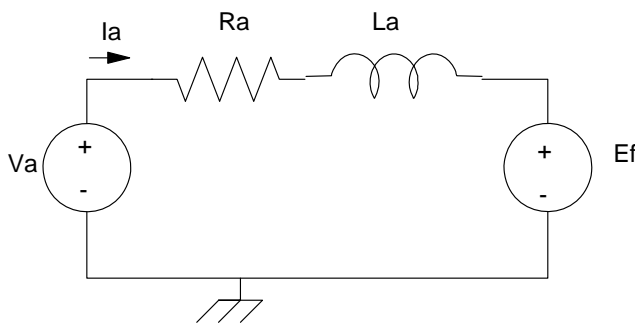
- A constant (DC) magnetic field for the stator, and
- A constant (DC) magnetic field in the rotor,
- That switches as the motor rotates.

This switching results in a constant torque in one direction.



For small motors, the stator is made from permanent magnets. This is useful since it takes no energy to maintain a magnetic field in a permanent magnet. For larger motors, you can get stronger magnets using electromagnets, resulting in higher torques.

For a permanent magnet DC motor, the model is as follows where V_t is the armature voltage, R_a and L_a are the armature resistance and inductance, and E_f is the back EMF. (Motors are also generators. As you spin a motor, you produce a voltage, E_f .) Since we're using DC analysis, the inductor doesn't matter.



$$T = K_t I_a$$

$$E_f = K_t \omega$$

Example: Parameters for a DC permanent magnet motor (<http://www.motiontek.ca/>)

<p>Price \$178 CA With 500 CPR Encoder and latching cable 1'</p> <p>Add To Cart</p> <p>Price \$98 CA With No Encoder</p> <p>Add To Cart</p>	<p>SM23240 DC Servo Motor Specification Download SM23240.pdf</p>																		
		<table border="1"> <tr><td>Frame Size</td><td>Nema23</td></tr> <tr><td>Constant Torque</td><td>50 oz/in – 0.35 N.M</td></tr> <tr><td>Peak Torque</td><td>240 oz/in – 1.7 N.M</td></tr> <tr><td>Continuous Current</td><td>3.4 Amp</td></tr> <tr><td>Peak Current</td><td>15 Amp</td></tr> <tr><td>Maximum Speed</td><td>6500 RPM ±10 % at 60V</td></tr> <tr><td>Resistance</td><td>2.739 ohm</td></tr> <tr><td>Inductance</td><td>2.72 mh</td></tr> <tr><td>Terminal Voltage</td><td>60 VDC</td></tr> </table>	Frame Size	Nema23	Constant Torque	50 oz/in – 0.35 N.M	Peak Torque	240 oz/in – 1.7 N.M	Continuous Current	3.4 Amp	Peak Current	15 Amp	Maximum Speed	6500 RPM ±10 % at 60V	Resistance	2.739 ohm	Inductance	2.72 mh	Terminal Voltage
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		<p>Encoder Optional Encoder Specification</p>																	
		Encoder Type Optical, 5 Pins Single-Ended, 500 CPR, Latching connector & optional latching cable 5 pins.																	

Specs:

$$R_a = 2.739\Omega$$

$$K_t = \frac{1.7Nm}{15A} = 0.1133 \frac{Nm}{A} = 0.1133 \frac{V}{rad/sec}$$

Power

$$P = E_f I_a = (K_t \omega) \left(\frac{T}{K_t} \right) = T \omega$$

Problem: Plot the current and power vs. motor speed for $V_a = \{20V, 40V, 60V\}$.

$$I_a = \left(\frac{V_t - E_f}{R_a} \right) = \left(\frac{V_t}{R_a} \right) - \left(\frac{K_t}{R_a} \right) \omega$$

$$P = E_f I_a = (K_t \omega) \left(\left(\frac{V_t}{R_a} \right) - \left(\frac{K_t}{R_a} \right) \omega \right)$$

$$P = \left(\frac{K_t^2}{R_a} \right) \left(\frac{V_t}{K_t} - \omega \right) \omega$$

$$P = 0.004687(529.6 - \omega)\omega$$

-->Kt = 0.1133;

-->Ra = 2.739;

-->Va = 20;

-->Wmax = Va/Kt

Wmax =

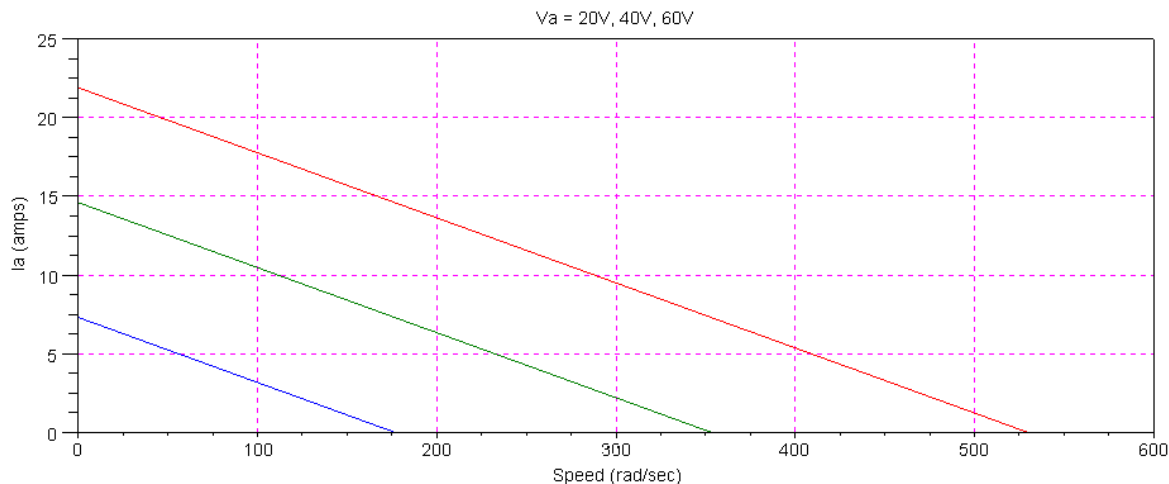
176.52251

-->w20 = [0:0.001:1]' * Wmax;

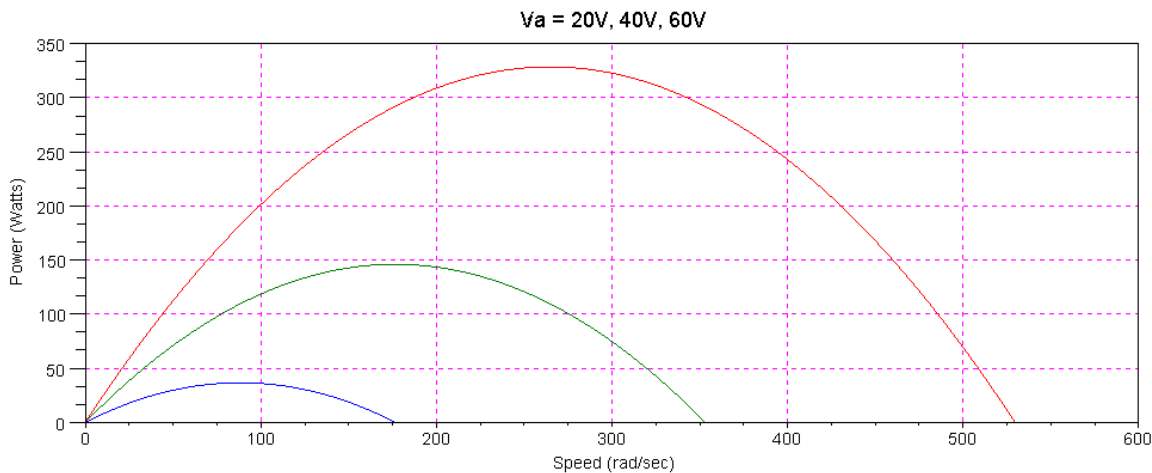
-->Ef20 = Kt*w20;

--> $I_{a20} = (V_a - E_{f20}) / R_a;$
 --> $P_{20} = E_{f20} \cdot I_{a20};$

Ditto for $V_a = 40V$ and $60V$.



Current vs. Motor Speed. Note that torque is proportional to current. DC motors have good start-up current - which corresponds to large inrush currents.



Power vs. Motor Speed. Note that there is a sweet-spot where the motor wants to run. This depends upon the voltage available (V_a). Power is also has square-law relationship with voltage ($1/2$ the voltage produces $1/4$ th the power).

Gearing and DC Motors:

Note that the DC motor has a 'sweet spot:' You get maximum power out at speeds close to midrange.

This is what gears and transmissions are for: Try to keep the motor's speed constant at midband regardless of the car's speed.

Problem: An RC car is to use the previous motor to go from 0 to 22.5 m/s (0 to 50mph). Assume a 60V power supply and that the wheels of the car are 3cm diameter.

At 22.5 m/s, the wheels are rotating

$$(22.5 \frac{m}{s}) \left(\frac{\text{rotation}}{0.03\pi m} \right) \left(\frac{2\pi \text{ rad}}{\text{rotation}} \right) = 1504 \text{ rad/sec}$$

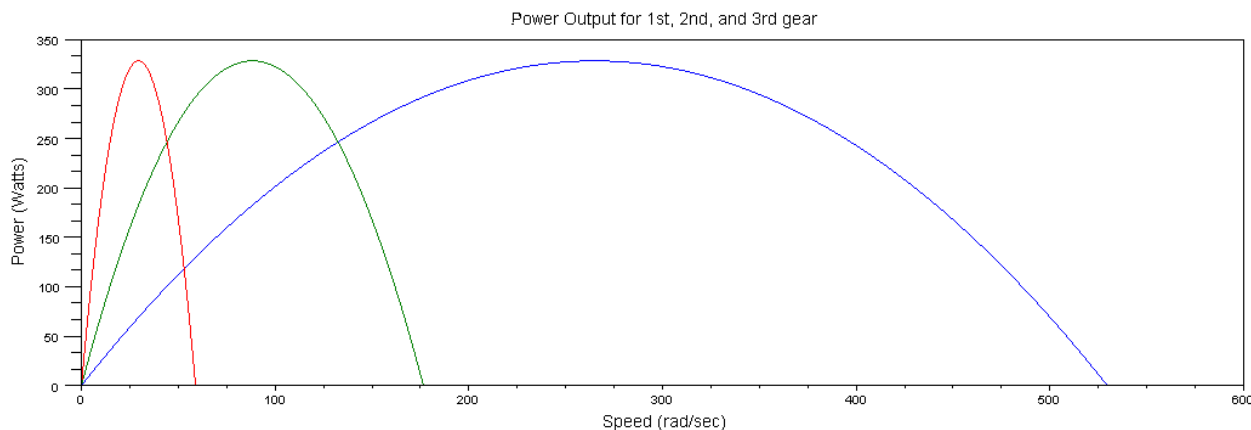
The motor won't spin that fast, so you need to use a gear box. Assume the motor is spinning at 400 rad/sec at max speed. The gear ratio then needs to be 1:5 (the motor spins once as the wheels spin 5 times).

$$N_{\text{high gear}} = \left(\frac{400}{1504} \right) = 0.266 \approx 1 : 3.76$$

A 3.76 gear reduction reduces the starting torque by a factor of 3.76, which is going to hurt the acceleration. To compensate for this, let's use a transmission with three gears:

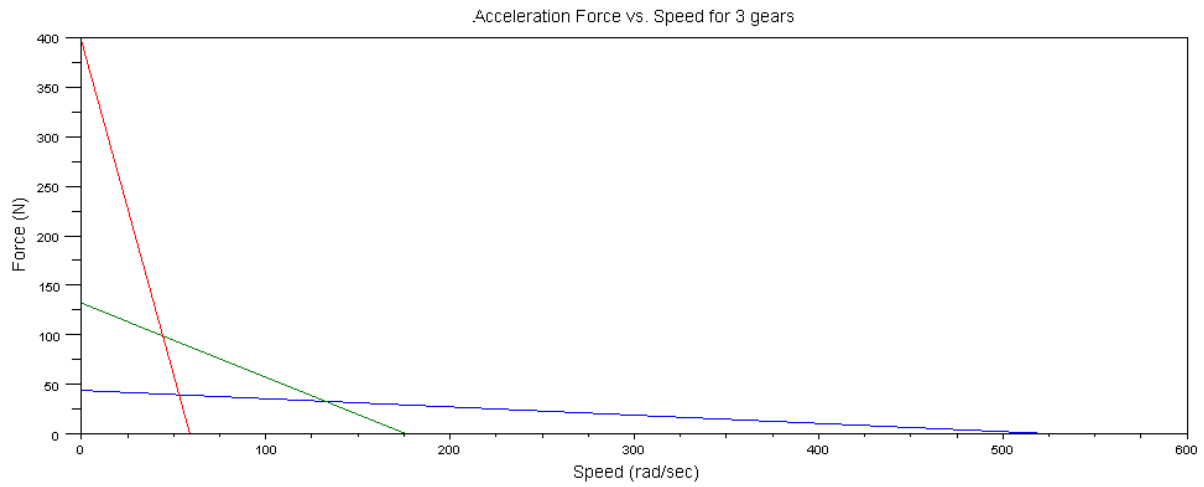
RC Car's speed		Gear Reduction	Motor Speed rad/sec
m/s	rad/sec		
7.5 .. 22.5	501 .. 1504	1 : 3.76	133 .. 400
2.5 .. 7.5	167 .. 501	3 : 3.76	133 .. 400
0 .. 2.5	0 .. 167	9 : 3.76	0 .. 400

This improves the power output. The power with a single gear is shown below with the blue curve. With three gears, the red and green increase the power output at low speeds.



Power delivered to the wheels for 1st (red), 2nd (green) and 3rd (blue) curves.

The acceleration also improves with a transmission. With a single gear, the force on the car would be the blue curve below. By using a lower gear at lower speeds, more torque is sent to the wheels, providing greater force (and acceleration) while starting.



Acceleration force for 1st (red), 2nd (green) and 3rd (blue) gears. (Force * radius of wheels = torque)