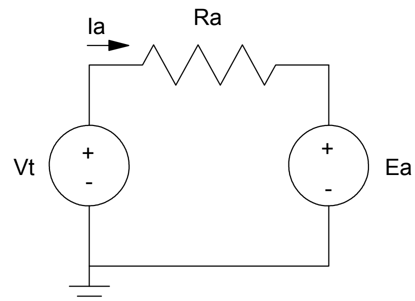
DC Permanent Magenet Motor Example

Objective: Simulate the speed vs. time for a DC permanent magnet motor driving an RC car

Background: Assuming the inductance of the motor is small (it usually is, with a RL time constant on the order of milliseconds), you can ignore the inductance. The model for a permanent magnet DC motor then simplifies to the following



with the equations:

$$T = K_t I_a \qquad \text{T = motor torque (Nm)}$$

$$E_a = K_t \omega \qquad \omega = \text{motor speed (rad/sec)}$$

$$I_a = \left(\frac{V_t - E_a}{I_a}\right)$$

Assume the DC motor drives an RC car with weight m and wheel with radius r. The acceleration on the car comes from:

Torque = Force x distance

$$F = \frac{T}{r} = m\frac{d^2x}{dt^2}$$

If you integrate acceleration once, you get velocity.

If you integrate acceleration twice, you get position.

Example: A SM23240 motor from the previous lecture drives an RC car with a mass of 10kg with wheels having a diameter of 80mm (radius = 40mm).

Problem 1: Find the velocity and position of the RC car if it starts from a stop and is powered by a constant current source of 1A.

Solution: You could solve analytically. A numerical solution is a little more clear what's going on.

If you use a spreadsheet, such as Xcel, you can compute the stuff you need as follows.

Current = 1A (constant)

Torque = Kt Ia = 0.1133 Nm

Force on the car accelerating it is

$$F = \frac{0.1133Nm}{0.04m \text{ radius}} = 2.8325N$$

The acceleration is

$$F = m\frac{d^2x}{dt^2} = m\ddot{x}$$

$$\ddot{x} = \frac{2.8325N}{10kg} = 0.28325\frac{m}{s^2}$$

Integrate to get velocity

$$\dot{x} = 0.28325t \text{ (m/s)}$$

Integrate to get position

$$x = 0.1416t^2 \text{ (meters)}$$

In a spreadsheet you can do this using the following

= means 'formula'

First, input the initial values. When computing a term, enter = first to tell Xcel this is a forumla

	A	B	C	D	E	F	G	H
1	Time	Current	Torque	Force	Acceleration	Velocity	Position	
2	0	1	=0.1133*B2	=c2/0.04	=d2/10	0	0	
3								
4								

For the following rows, use numerical integration for velocity and position

velocity = integral of acceleration

$$= \text{previous velocity} + \left(\frac{dv}{dt}\right)dt$$

	A	B	C	D	E	F	G	H
1	Time	Current	Torque	Force	Acceleration	Velocity	Position	
2	0	1	0.1133	2.8325	0.2832	0	0	
3	=A2+0.1	1	=0.1133*B3	=C3/0.04	=D3/10	=F2 + E3*0.1	=G2 + F3*0.1	
4								

If you copy row 3 and past to rows 4..20, the formulas are copies with the equations shifted down accordingly

	A	B	C	D	E	F	G	H
1	Time	Current	Torque	Force	Acceleration	Velocity	Position	
2	0	1	0.1133	2.8325	0.2832	0	0	
3	=A2+0.1	1	=0.1133*B3	=C3/0.04	=D3/10	=F2 + E2*0.1	=G2 + F2*0.1	
4	=A3+0.1	1	=0.1133*B4	=C4/0.04	=D4/10	=F3 + E3*0.1	=G3 + F3*0.1	
5	=A4+0.1	1	=0.1133*B5	=C5/0.04	=D5/10	=F4 + E4*0.1	=G4 + F4*0.1	

or with numbers

	A	B	C	D	E	F	G	H
1	Time	Current	Torque	Force	Acceleration	Velocity	Position	
2	0	1	0.11	2.83	0.28	0	0	
3	0.1	1	0.11	2.83	0.28	0.03	0	
4	0.2	1	0.11	2.83	0.28	0.06	0	
5	0.3	1	0.11	2.83	0.28	0.08	0.01	
6	0.4	1	0.11	2.83	0.28	0.11	0.02	
7	0.5	1	0.11	2.83	0.28	0.14	0.03	
8	0.6	1	0.11	2.83	0.28	0.17	0.04	
9	0.7	1	0.11	2.83	0.28	0.2	0.06	
10	0.8	1	0.11	2.83	0.28	0.23	0.08	
11	0.9	1	0.11	2.83	0.28	0.25	0.1	
12	1	1	0.11	2.83	0.28	0.28	0.13	

You can then plot the position and speed vs. time

Problem 2: Change the problem so that the voltage is contant, 30V

Solution: Add a column for the back EMF, Ea

	A	B	C	D	E	F	G	H	I
1	Time	Vin	w (rad/sec)	Ea	Ia	Force	Acceleration	Velocity	Position
2	0	30	0	=C2*0.1133	=(B2-D2)/ 2.739	=E2*0.1133/ 0.04	=F2/10	0	0
3									
4									
5									

Copy to the next row

	A	B	C	D	E	F	G	H	I
1	Time	Vin	w (rad/sec)	Ea	Ia	Force	Acceleration	Velocity	Position
2	0	30	0	=C2*0.1133	=(B2-D2)/2.739	=E2*0.1133/0.04	=F2/10	0	0
3	=A2 + 0.1	30	=H3	=C3*0.1133	=(B3-D3)/2.739	=E3*0.1133/0.04	=F3/10	=H2+0.1*G2	=I2 + 0.1*H2
4									
5									

Copy to the rest of the table

	A	B	C	D	E	F	G	H	I
1	Time	Vin	w (rad/sec)	Ea	Ia	Force	Acceleration	Velocity	Position
2	0	30	0	=C2*0.1133	=(B2-D2)/2.739	=E2*0.1133/0.04	=F2/10	0	0
3	=A2 + 0.1	30	=H3	=C3*0.1133	=(B3-D3)/2.739	=E3*0.1133/0.04	=F3/10	=H2+0.1*G2	=I2 + 0.1*H2
4	=A3 + 0.1	30	=H4	=C4*0.1133	=(B4-D4)/2.739	=E4*0.1133/0.04	=F4/10	=H3+0.1*G3	=I3 + 0.1*H3
5	=A4 + 0.1	30	=H5	=C5*0.1133	=(B5-D5)/2.739	=E5*0.1133/0.04	=F5/10	=H4+0.1*G4	=I4 + 0.1*H4

With numbers:

	A	B	C	D	E	F	G	H	I
1	Time	Vin	w (rad/sec)	Ea	Ia	Force	Acceleration	Velocity	Position
2	0	30	0	0	10.95	31.02	3.1	0	0
3	0.1	30	0.31	0.04	10.94	30.99	3.1	0.31	0
4	0.2	30	0.62	0.07	10.93	30.95	3.1	0.62	0.03
5	0.3	30	0.93	0.11	10.91	30.92	3.09	0.93	0.09
6	0.4	30	1.24	0.14	10.9	30.88	3.09	1.24	0.19
7	0.5	30	1.55	0.18	10.89	30.84	3.08	1.55	0.31
8	0.6	30	1.86	0.21	10.88	30.81	3.08	1.86	0.46
9	0.7	30	2.16	0.25	10.86	30.77	3.08	2.16	0.65
10	0.8	30	2.47	0.28	10.85	30.73	3.07	2.47	0.87
11	0.9	30	2.78	0.31	10.84	30.7	3.07	2.78	1.11

Problem: Vary the input votlage and the wheel diameter.

Solution: In Xcel, using a dollar sign in the formula locks the location of the variable

= b2

keeps the reference to cell (B2) even when you copy the cell.

So, define two cells to be the wheel diameter and input voltage

	A	B	C	D	E	F	G	H	I
1	**Vin**	**30**							
2	**radius**	**0.04**							
3	Time	Vin	w (rad/sec)	Ea	Ia	Force	Acceleration	Velocity	Position
4	0	**=b1**	0	=C4*0.1133	=(B4-D4)/ 2.739	=E4*0.1133/**b2**	=F4/10	0	0
5	=A4 + 0.1	30	=H5	=C5*0.1133	=(B5-D5)/ 2.739	=E5*0.1133/**b2**	=F5/10	=H4 + 0.1*G4	=I4 + 0.1*H4

This allows you to adjust a parameter by changing one cell and seeing what happens.