ECE 341 - Homework #7

Uniform and Exponential Distributions. Summer 2023

Uniform Distributions

Let

- **a** be a sample from A, a uniform distribution over the range of (0, 3)
- **b** be a sample from B, a uniform distribution over the range of (0, 5)

Let y = a + b

1) Determine the pdf for **a** + **b** using moment generating funcitons (i.e. LaPlace transforms)

$$a(t) = \left(\frac{1}{3}\right)(u(t) - u(3 - t))$$

$$A(s) = \left(\frac{1}{3s}\right)(1 - e^{-3s})$$

$$b(t) = \left(\frac{1}{5}\right)(u(t) = u(5 - t))$$

$$B(s) = \left(\frac{1}{5s}\right)(1 - e^{-5s})$$

$$Y(s) = A(s) \cdot B(s) = \left(\frac{1}{3s}\right)(1 - e^{-3s}) \cdot \left(\frac{1}{5s}\right)(1 - e^{-5s})$$

$$Y = AB = \left(\frac{1}{15s^2}\right)(1 - e^{-3s} - e^{-5s} + e^{-8s})$$

convert back to time

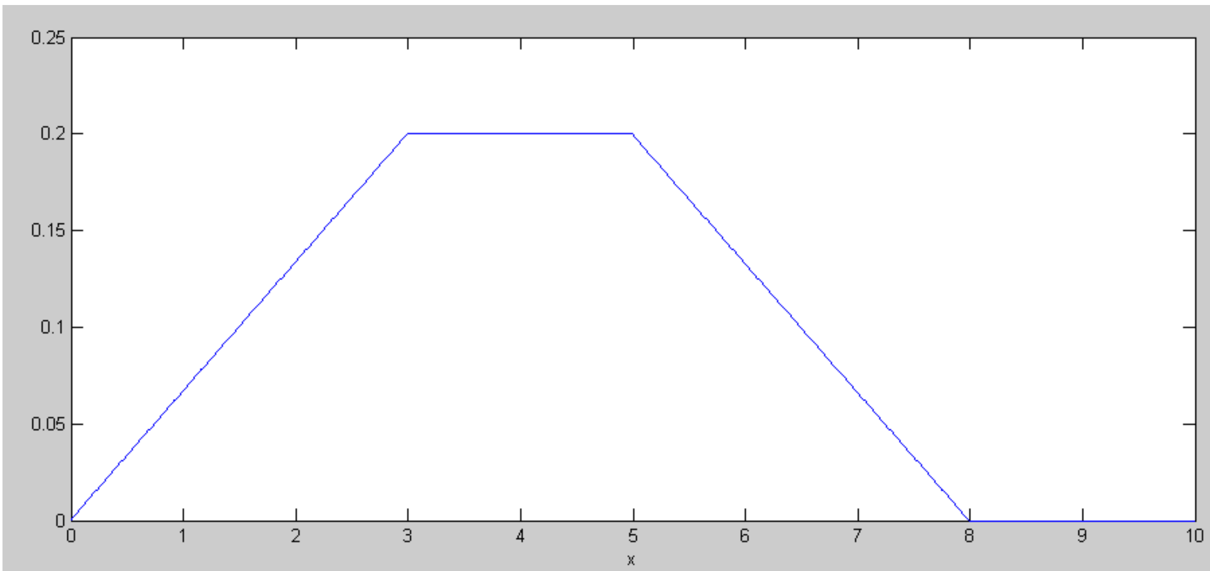
$$y(x) = \left(\frac{1}{15}\right)(xu(x) - (x-3)u(x-3) - (x-5)u(x-5) + (x-8)u(x-8))$$

or

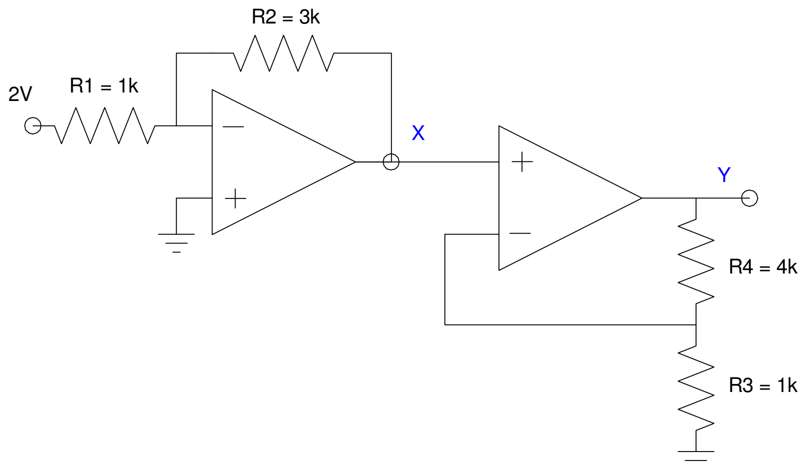
$$y(x) = a + b = \begin{cases} 0 & x < 0 \\ \left(\frac{x}{15}\right) & 0 < x < 3 \\ \left(\frac{3}{15}\right) & 3 < x < 5 \\ \left(\frac{8-x}{15}\right) & 5 < x < 8 \\ 0 & x > 8 \end{cases}$$

2) Determine the pdf for **a** + **b** using convolution (by hand or Matlab)

```
>> dt = 0.01;
>> t = [0:dt:10]';
>> A = (t < 3)/3;
>> B = (t < 5)/5;
>> Y = conv(A,B)*dt;
>> Y = Y(1:length(t));
>> plot(t,Y)
>> ylim([0,0.25])
>> xlabel('x');
>>
```



3) Assume each resistor has a tolerance of 5% (i.e. a uniform distribution over the range of (0.95, 1.05) of the nominal value. Determine the mean and standard deviation for the voltage at Y for the following circuit using a Monte Carlo simulation.



Equations:

$$X = -\left(\frac{R_2}{R_1}\right) 2V$$

$$Y = \left(1 + \frac{R_4}{R_3}\right) X$$

Combining

$$Y = -\left(\frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)(2V)$$

Matlab Code for a Monte-Carlo Simulation

```
y = [];

for i=1:1000
   R1 = 1000 * (1 + (2*rand-1)*0.05);
   R2 = 3000 * (1 + (2*rand-1)*0.05);
   R3 = 1000 * (1 + (2*rand-1)*0.05);
   R4 = 4000 * (1 + (2*rand-1)*0.05);
   Y = -(R2/R1)*(1+R4/R3)*2;
   y = [y ; Y];
end

x = mean(y)
s = std(y)
```

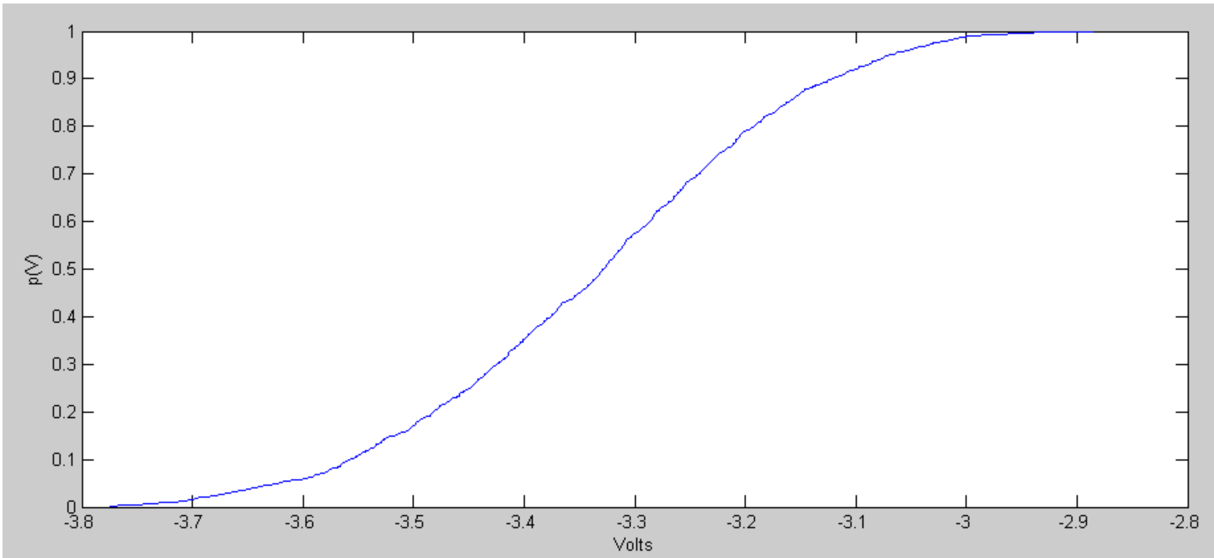
Result

```
x =   -30.0692     mean

s =    1.5930     standard deviation
```

The cdf for y is

```
>> p = [1:1000]' / 1000;
>> plot(sort(y), p);
```



note: save this - we'll use this again when we get to Weibull distribution

Probem 4: Skip (this is a Pascal distribution)

Problem 5: Skip (this is a Pascal distribution)

Queueing Theory

Assume you are running a fast-food restraunt.

- The time between customers arriving at a restaraunt is an exponential distribution with a mean of 50 seconds.
- The time it takes to serve each customer is an exponential distribution with a mean of 40 seconds.

6) Run a single Monte-Carlo simulation for this restaraunt over the span of one hour.

Give the formula for each column in you simulation

What is the longest waiting time for a customer in your simulation?

- Customer #43 waited 443.2 seconds to be served

What is the largest queue over the span of one hour?

- The longest queue length is 9 for customers #43 and #68, #69

B11: raw random times copied from Matlab (exponential distribution)

C11: = B11 + C10

D11: = max(C11, F10)

E11: raw random times copied from Matlab (exponential distribution)

F11: = D11 + E11

G11: = F11 - C11

H11: = 1*(C11<F10) + 1*(C11<F9) + 1*(C11<F8) + 1*(C11<F7) + 1*(C11<F6) + 1*(C11<F5) + 1*(C11<F4) + 1*(C11<F3) + 1*(C11<F2)

	A	B	C	D	E	F	G	H
1		dT1	Tarr	Tserve	dT2	Tdone	Twait	Queue
2		0	0	0	0	0	0	
3		0	0	0	0	0	0	
4		0	0	0	0	0	0	
5		0	0	0	0	0	0	
6		0	0	0	0	0	0	
7		0	0	0	0	0	0	
8	1	58.63	58.63	58.63	20.86	79.49	0	0
9	2	20.22	78.85	79.49	46.75	126.24	0.64	1
10	3	66.12	144.97	144.97	10.64	155.61	0	0
11	4	56.27	201.24	201.24	67.05	268.3	0	0
12	5	18.95	220.2	268.3	13.91	282.2	48.1	1
13	6	9.73	229.93	282.2	124.7	406.91	52.27	2
14	7	62.5	292.44	406.91	15.67	422.58	114.47	1
15	8	119.28	411.71	422.58	72.52	495.09	10.87	1
16	9	15.87	427.59	495.09	10.45	505.55	67.51	1
17	10	44.5	472.09	505.55	16.95	522.5	33.46	2
18	11	23.1	495.18	522.5	4.78	527.27	27.31	2
19	12	71.71	566.89	566.89	42.93	609.82	0	0

20	13	58.59	625.48	625.48	57.5	682.98	0	0
21	14	7.32	632.8	682.98	39.55	722.53	50.18	1
22	15	79.21	712.02	722.53	27.73	750.26	10.51	1
23	16	183.7	895.71	895.71	51.7	947.42	0	0
24	17	28.9	924.61	947.42	52.15	999.57	22.81	1
25	18	86.26	1,010.88	1,010.88	56.82	1,067.69	0	0
26	19	35.47	1,046.35	1,067.69	50.5	1,118.19	21.35	1
27	20	6.73	1,053.07	1,118.19	145.18	1,263.37	65.12	2
28	21	8.92	1,061.99	1,263.37	11.72	1,275.09	201.38	3
29	22	20.9	1,082.89	1,275.09	61.77	1,336.86	192.2	3
30	23	55.25	1,138.14	1,336.86	13.47	1,350.34	198.73	3
31	24	69.85	1,207.99	1,350.34	6.36	1,356.69	142.35	4
32	25	62.42	1,270.41	1,356.69	46.74	1,403.43	86.29	4
33	26	15.34	1,285.75	1,403.43	29.9	1,433.33	117.68	4
34	27	30.55	1,316.29	1,433.33	30.69	1,464.03	117.04	5
35	28	3.77	1,320.07	1,464.03	54.23	1,518.25	143.96	6
36	29	4.74	1,324.8	1,518.25	73.55	1,591.8	193.45	7
37	30	5.86	1,330.66	1,591.8	21.56	1,613.36	261.14	8
38	31	45.41	1,376.07	1,613.36	54.24	1,667.59	237.28	6
39	32	27.34	1,403.42	1,667.59	26.91	1,694.5	264.18	7
40	33	8.41	1,411.83	1,694.5	92.24	1,786.73	282.67	7
41	34	27.33	1,439.16	1,786.73	89.46	1,876.2	347.58	7
42	35	6.39	1,445.55	1,876.2	14.82	1,891.01	430.65	8
43	36	2.26	1,447.81	1,891.01	47.53	1,938.54	443.2	9
44	37	76.08	1,523.88	1,938.54	43.64	1,982.18	414.66	8
45	38	32.89	1,556.77	1,982.18	38.91	2,021.09	425.41	9
46	39	106.15	1,662.92	2,021.09	101.99	2,123.08	358.17	8
47	40	47.72	1,710.64	2,123.08	15.38	2,138.46	412.44	7
48	41	34.97	1,745.6	2,138.46	19.14	2,157.6	392.85	8
49	42	67.58	1,813.19	2,157.6	6.35	2,163.95	344.41	8
50	43	84.48	1,897.67	2,163.95	140.53	2,304.47	266.28	7
51	44	180.07	2,077.74	2,304.47	51.86	2,356.33	226.73	5
52	45	0.02	2,077.77	2,356.33	32.64	2,388.98	278.57	6
53	46	80.23	2,157.99	2,388.98	50.99	2,439.97	230.98	4
54	47	37.93	2,195.92	2,439.97	39.34	2,479.31	244.04	4
55	48	184.01	2,379.93	2,479.31	52.11	2,531.42	99.38	3
56	49	30	2,409.93	2,531.42	39.25	2,570.67	121.48	3
57	50	26.12	2,436.06	2,570.67	63.84	2,634.5	134.61	4
58	51	64.65	2,500.7	2,634.5	36.96	2,671.46	133.8	3
59	52	10.34	2,511.05	2,671.46	253.4	2,924.86	160.42	4
60	53	27.57	2,538.62	2,924.86	12.34	2,937.2	386.24	4
61	54	92.45	2,631.07	2,937.2	5.59	2,942.79	306.13	4
62	55	34.19	2,665.26	2,942.79	5.81	2,948.6	277.53	4
63	56	74.62	2,739.88	2,948.6	3.29	2,951.88	208.72	4
64	57	53.67	2,793.55	2,951.88	25.92	2,977.81	158.33	5
65	58	35.27	2,828.83	2,977.81	29.74	3,007.55	148.98	6
66	59	11.33	2,840.16	3,007.55	22.77	3,030.32	167.39	7

67	60	43.91	2,884.08	3,030.32	72.09	3,102.41	146.25	8
68	61	3.49	2,887.56	3,102.41	49.43	3,151.84	214.85	9
69	62	39.34	2,926.9	3,151.84	73.92	3,225.76	224.94	9
70	63	43.26	2,970.16	3,225.76	135.04	3,360.8	255.6	6
71	64	52.34	3,022.5	3,360.8	180.12	3,540.92	338.3	5
72	65	88.57	3,111.06	3,540.92	10.66	3,551.58	429.86	4
73	66	161.37	3,272.44	3,551.58	7.48	3,559.06	279.14	3
74	67	58.62	3,331.06	3,559.06	59.58	3,618.64	228	4
75	68	34.84	3,365.9	3,618.64	4.93	3,623.56	252.74	4
76	69	105.42	3,471.31	3,623.56	37.26	3,660.83	152.25	5
77	70	34.71	3,506.02	3,660.83	37.79	3,698.62	154.81	6
78	71	0.69	3,506.71	3,698.62	98.71	3,797.33	191.91	7
79	72	5.15	3,511.86	3,797.33	33.17	3,830.5	285.47	8
80	73	79.43	3,591.29	3,830.5	25	3,855.5	239.21	6
81	74	26.49	3,617.78	3,855.5	55.65	3,911.15	237.72	7