ECE 341 - Solution to Homework #2

Card Games

The card game *bridge* uses a 52-card deck. Each person is dealt 13 cards for their hand.

1) How many different hands are possible? (order doesn't matter)

$$N = \begin{pmatrix} 52\\13 \end{pmatrix} = 635,013,559,600$$

different hands

2) What is the probability of having 7 cards of one suit in your hand?

(4 suits, choose 1) * (13 cards in any suit, choose 7) * (39 other cards in the deck, choose 6)

$$M = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 13 \\ 7 \end{pmatrix} \begin{pmatrix} 39 \\ 6 \end{pmatrix}$$

M = 22,394,644,272

The probability of having 7 cards of a suit are then

$$p = \left(\frac{M}{N}\right) = \left(\frac{22,394,644,272}{635,013,559,600}\right) = 0.035266$$

There is a 3.5266% chance of having 7 cards of one suit (28:1 odds against)

3) What is the probability of having all 4 Aces in your hand?

M = (4 aces, choose 4) (48 other cards, choose 9)

$$M = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} 48 \\ 9 \end{pmatrix} = 1,677,106,640$$

The probability of having 4 aces is

$$p = \left(\frac{M}{N}\right) = \left(\frac{1,677,106,640}{635,013,559,600}\right) = 0.002641$$

There is a 0.2641% chance of being dealt all four aces (378:1 odds against)

4) Compute the odds of a flush in 5-card stud.

N = number of hands = (52 cards, choose 5)

$$N = \left(\begin{array}{c} 52\\5 \end{array}\right) = 2,598,960$$

The number of hands that are a flush are

M = (4 suits, choose 1) (13 cards of a suit, choose 5)

$$M = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 13 \\ 5 \end{pmatrix} = 5148$$

The odds of being dealt a flush are thus

$$p = \left(\frac{M}{N}\right) = \left(\frac{5,148}{2,598,960}\right) = 0.001981$$

There is a 0.1981% chance of being dealt a flush. (504.8:1 odds against)

5) Compute the odds of a flush in 5-card draw. Assume you go for a flush if you have four cards of one suit in your opening hand (and draw one card).

This is a conditional probability. To get a flush (A), you could

- B: Be dealt a flush
- C: Be dealt 4 cards of a suit, discard the off card, then draw to a flush, or
- D: Be dealy no pairs, draw 4 cards, and the the 4 cards match the card you kept.

p(A) = p(A|B) p(B) + p(A|C) p(C) + p(A|D) p(D)

Case B: Dealt a flush

p(A|B) p(B) = (1.000)(0.0019079)

198.079 in 100,000 hands

Case C: Dealt four cards of the same suit. The number of hands which have 4 cards of the same suit are

M = (4 suits, choose 1) (13 cards of a suit, choose 4) (39 other cards, choose 1)

$$M = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 13 \\ 4 \end{pmatrix} \begin{pmatrix} 39 \\ 1 \end{pmatrix} = 111,540$$
$$p(C) = \begin{pmatrix} \frac{111,540}{2,598,960} \end{pmatrix} = 0.042971$$

The probability of drawing to a flush is then

$$p(A|C) = \left(\frac{9 \text{ cards of the same suit remaining in the deck, choose 1}}{47 \text{ cards in the deck, choose 1}}\right) = \left(\frac{\begin{pmatrix} 9\\1 \end{pmatrix}}{\begin{pmatrix} 47\\1 \end{pmatrix}}\right) = 0.191489$$

so

$$p(A|C)p(C) = (0.191489)(0.042971) = 0.008218$$

= 821.81 in 100,000 hands

Case D: Dealt no pairs, draw 4 cards, and the 4 cards match the card you kept (assume you keep a card with only one of that suit when discarding)

Number of ways to draw 4 cards

$$N = \left(\begin{array}{c} 47\\4\end{array}\right) = 178,365$$

Number of ways to draw to a flush (12 remaining cards of that suit in the deck)

$$M = \left(\begin{array}{c} 12\\4 \end{array}\right) = 495$$

D contributes

$$p(A|D)p(D) = \left(\frac{495}{178365}\right)(0.5012) = 0.001391$$

= 139.1 in 100,000 hands

Add them all up

$$p(A) = 0.011590$$

= 1158.99 in 100,000 hands

The probability of a flush in 5-card draw is 0.011590 (86.28:1 odds against)

6) Determine the odds of a flush using Matlab and a Monte-Carlo simulation

(kind of tricky - took three tries to get it to work)

5-Card Stud: 100,000 hands

Got 188 flush's

Calculated odds give 0.001981 * 100,000 = 198.1

Experimental is 5.1% lower than calculated

so that's pretty close

5-Card Draw: 100,000 hands (1158 computed)

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# Flushes = {1106, 1089, 1101, 1155}
```

Numbers seem reasonable

Logic for drawing cards:

```
if(NS(1) == 5)
                               % flush
elseif(NS(1) == 4)
   F = zeros(5, 1);
   for i=1:5
      F(i) = F(i) + sum(Suit == Suit(i));
   end
   [a,b] = sort(F, 'descend');
   Value = Value(b);
   Suit = Suit(b);
   Hand = Hand(b);
   Hand(5) = Deck(Top);
   Top = Top + 1;
elseif(N(1) == 4)
                                % 4 of a kind
elseif((N(1)==3)*(N(2)==2)) % full house
elseif((N(1)==3)*(N(2)<2)) % three of a kind
   Hand(4:5) = Deck(Top:Top+1);
   Top = Top + 2;
elseif((N(1)==2)*(N(2)==2)) % two pair
   Hand(5) = Deck(Top);
   Top = Top + 1;
elseif((N(1)==2)*(N(2)<2))
                              % pair
   Hand(3:5) = Deck(Top:Top+2);
   Top = Top + 3;
elseif(N(1) < 2)
                                % High Card
   Hand(2:5) = Deck(Top:Top+3);
   Top = Top + 4;
end
```