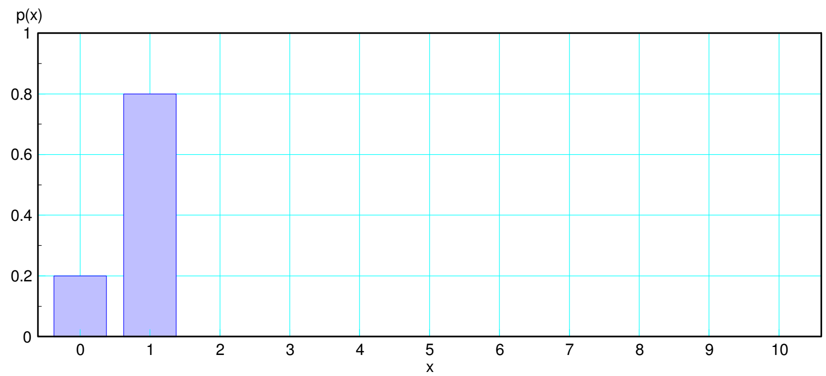
# ECE 341 - Solution to Homework #4

Binomial and Uniform Distributions.

## Binomial Distribution

Assume you toss a coin with a probability of a heads being 0.8

$$X(z) = \left(\frac{0.2z+0.8}{z}\right) = (0.2) + (0.8)\left(\frac{1}{z}\right)$$




1) Determine the probability of tossing 6 heads in 8 tosses

Eight tosses have a binomial distribution with p = 0.8

$$p(m) = \binom{n}{m} p^m q^{n-m}$$

$$p(6) = \binom{8}{6} (0.8)^6 (0.2)^2$$

$$p(6) = 0.2936$$

You can also solve this problem using convolution:

```
X = [0.2,0.8];
X2 = conv(X,X);
X4 = conv(X2,X2);
X8 = conv(X4,X4)
```

| x | p(x) |
|---|---|
| 0 | 0.0000 |
| 1 | 0.0001 |
| 2 | 0.0011 |
| 3 | 0.0092 |
| 4 | 0.0459 |
| 5 | 0.1468 |
| **6** | **0.2936** |
| 7 | 0.3355 |
| 8 | 0.1678 |

2) Determine the probability distribution when tossing this same coin 8 times

- pdf
- mean
- standard deviation

$$pdf = \binom{8}{m}(0.8)^m(0.2)^{8-m}$$

Mean

$$\mu = np = 8 \cdot 0.8 = 6.4$$

Standard deviation

$$\sigma^2 = npq$$

$$\sigma^2 = 8 \cdot 0.8 \cdot 0.2 = 1.28$$

$$\sigma = \sqrt{1.28} = 1.1314$$

You can also solve using convolution in Matlab

```
m1 = sum(x .* X8)
```

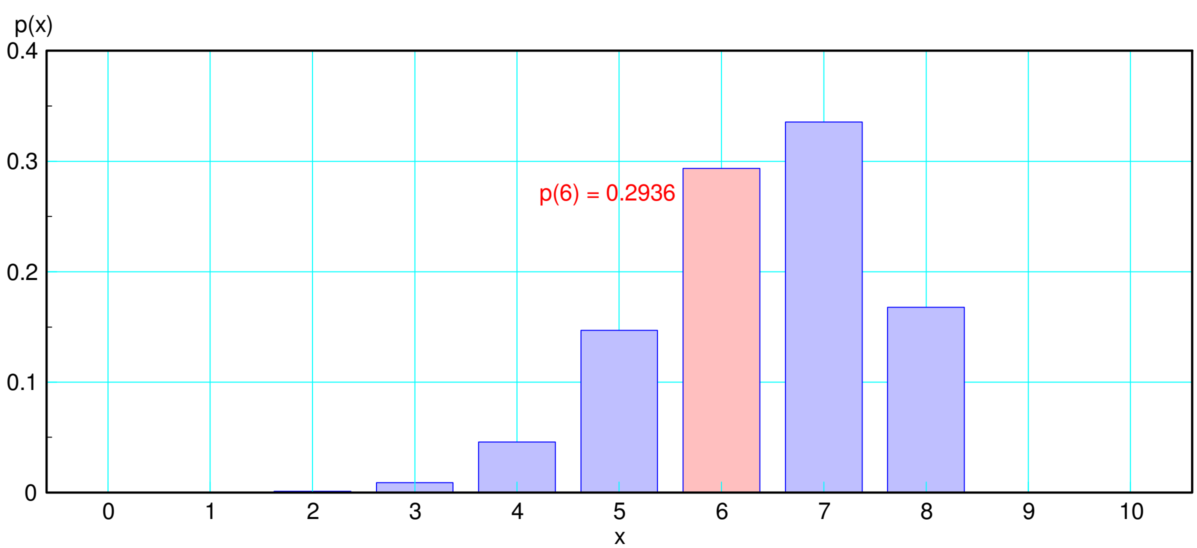
$$\mu = \sum x \cdot p(x)$$

```
m1 =    6.4000

s2 = sum( (x - m1).^2 .* X8 )
```

$$\sigma^2 = \sum (x-\mu)^2 \cdot p(x)$$

```
s2 =    1.2800
```



NOAA has been keeping track of world weather for the past 137 years. 9 of the last 10 years have been the hottest on record.

3a) What is the probability of any given year being one of the 10 hottest on record (i.e. what is p?)

$$p = \left(\frac{10}{137}\right)$$

3b) What is the probability of 9 of the last 10 years being the hottest on record? (i.e. toss a coin and get 9 heads out of 10 tosses)

$$f(9) = \binom{10}{9}\left(\frac{10}{137}\right)^9\left(\frac{127}{137}\right)^1$$

$$f(9) = 5.45 \cdot 10^{-10}$$

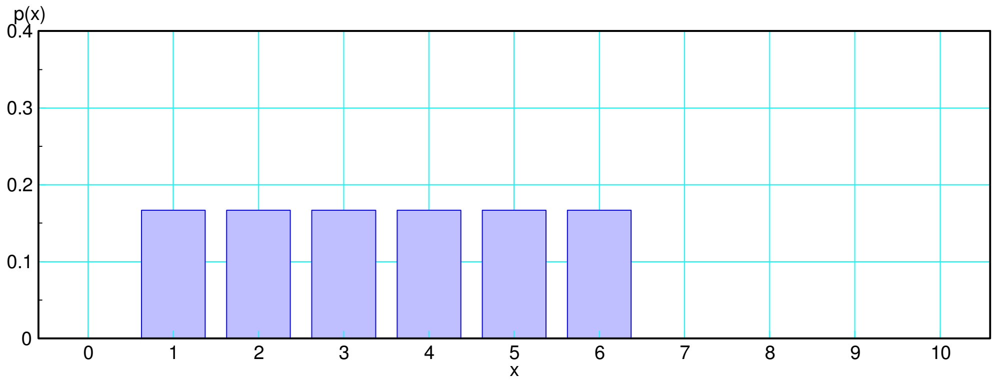
p = 0.000 000 000 545

There is a chance this is just random noise in the data. The odds against this are 1,834,011,061 : 1 against. It's still possible.

## Uniform Distribution

Assume a fair six-sided die:

$$Y(z) = \left(\frac{1}{6}\right)\left(\frac{z^5+z^4+z^3+z^2+z+1}{z^6}\right) = \left(\frac{1}{6}\right)\left(\left(\frac{1}{z}\right)+\left(\frac{1}{z^2}\right)+\left(\frac{1}{z^3}\right)+\left(\frac{1}{z^4}\right)+\left(\frac{1}{z^5}\right)+\left(\frac{1}{z^6}\right)\right)$$




4) Asume you sum four dice (4d6). Determine the

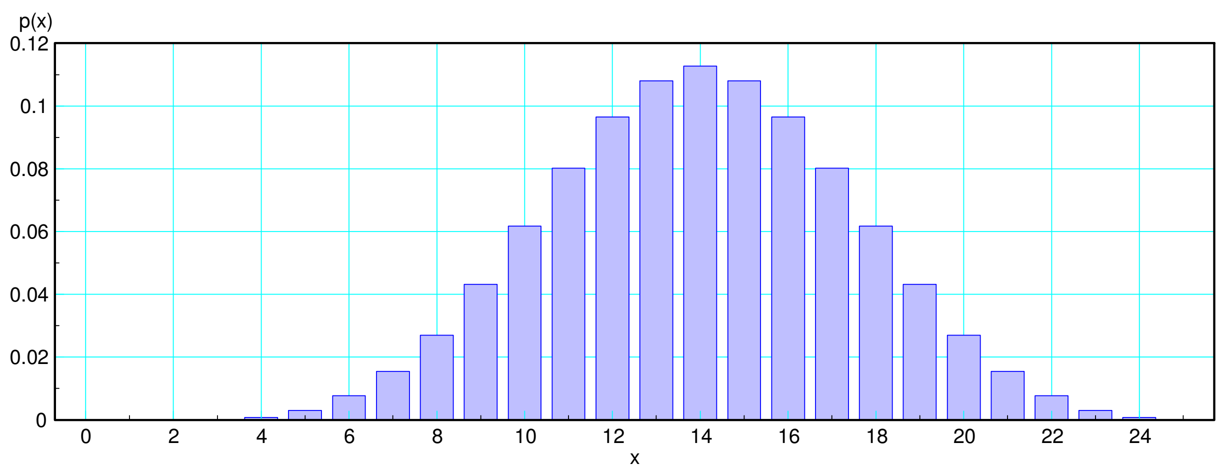
- pdf
- mean, and
- standard deviation

The pdf for rolling four six-sided dice is

$$p_4(z) = \left(\left(\frac{1}{6}\right)\left(\frac{z^5+z^4+z^3+z^2+z+1}{z^6}\right)\right)^4$$

A second (less painful) solution is to use convolution in Matlab

```
d6 = [0,1,1,1,1,1,1]'/6;
d6x2 = conv(d6,d6);
d6x4 = conv(d6x2,d6x2);
```



```
x = [0:24]';

% mean
m1 = sum(d6x4 .* x)
```

$$\mu = \sum x \cdot p(x)$$

```
% variance
s2 = sum((x-m1).^2 .* d6x4)
```

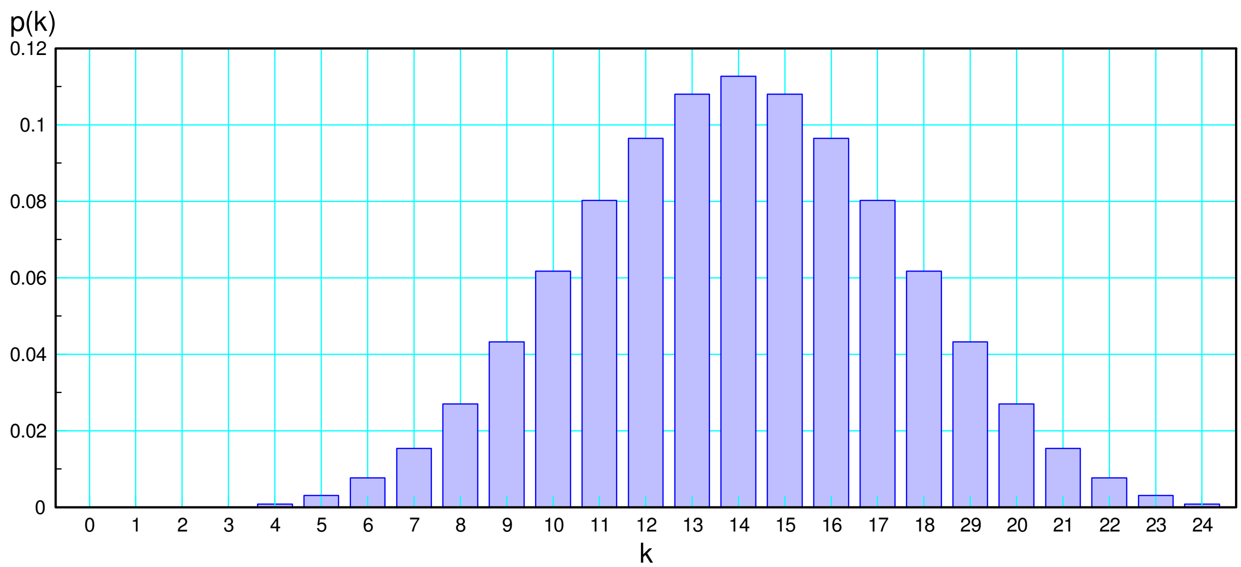
$$\sigma^2 = \sum (x-\mu)^2 \cdot p(x)$$

```
x =    14.0000
s2 =    11.6667
```

This compares to

$\mu = 4 \cdot 3.5 = 14$ *check*

$\sigma^2 = 4 \cdot 2.9167 = 11.6666$ *check*



5) Asume you sum sixteen dice (16d6). Determine the

- pdf
- mean, and
- standard deviation

In Matlab:

```
d6 = [0,1,1,1,1,1,1]' /6;
d6x2 = conv(d6,d6);
d6x4 = conv(d6x2,d6x2);
d6x8 = conv(d6x4,d6x4);
d6x16 = conv(d6x8,d6x8);

x = [0:6*16]';

% mean
m1 = sum(d6x16 .* x)
```

$$\mu = \sum x \cdot p(x)$$

```
% variance
s2 = sum((x-m1).^2 .* d6x16)
```

$$\sigma^2 = \sum (x - \mu)^2 \cdot p(x)$$

```
x =   56.0000
s2 =   46.6667
```

This compares to

$$\mu = 16 \cdot 3.5 = 56$$

$$\sigma^2 = 16 \cdot 2.9167 = 46.6666$$