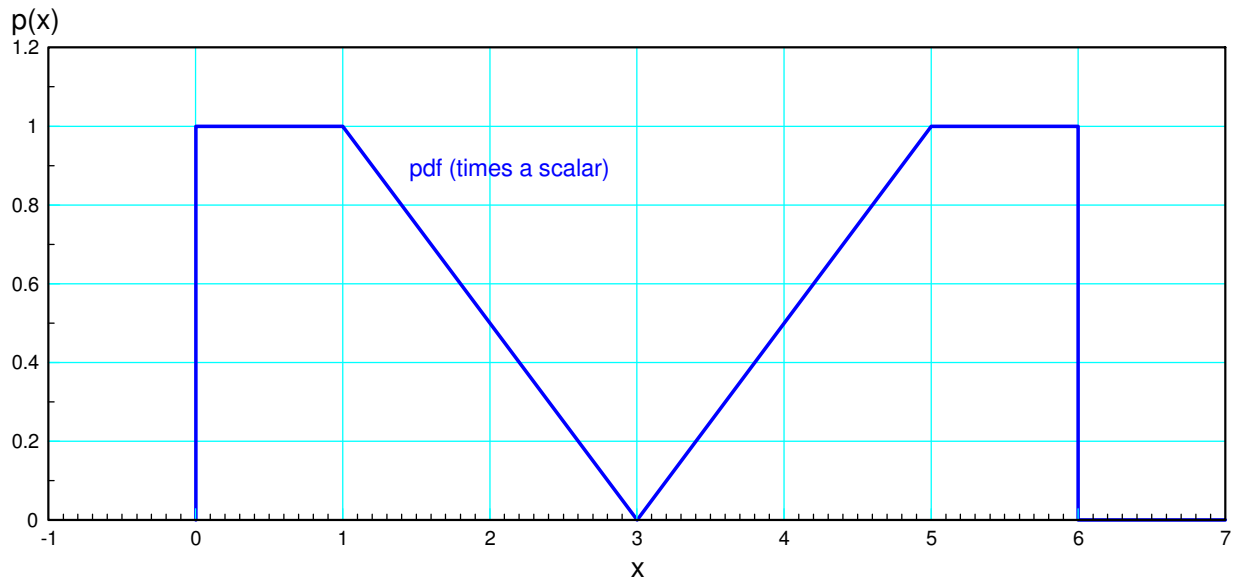


ECE 341 - Homework #6

LaPlace Transforms, Continuous Probability Density Functions.



1) Determine the scalar so that the above function is a valid pdf (i.e. the total area is 1.000)

The area = 4

Scale this by 1/4th

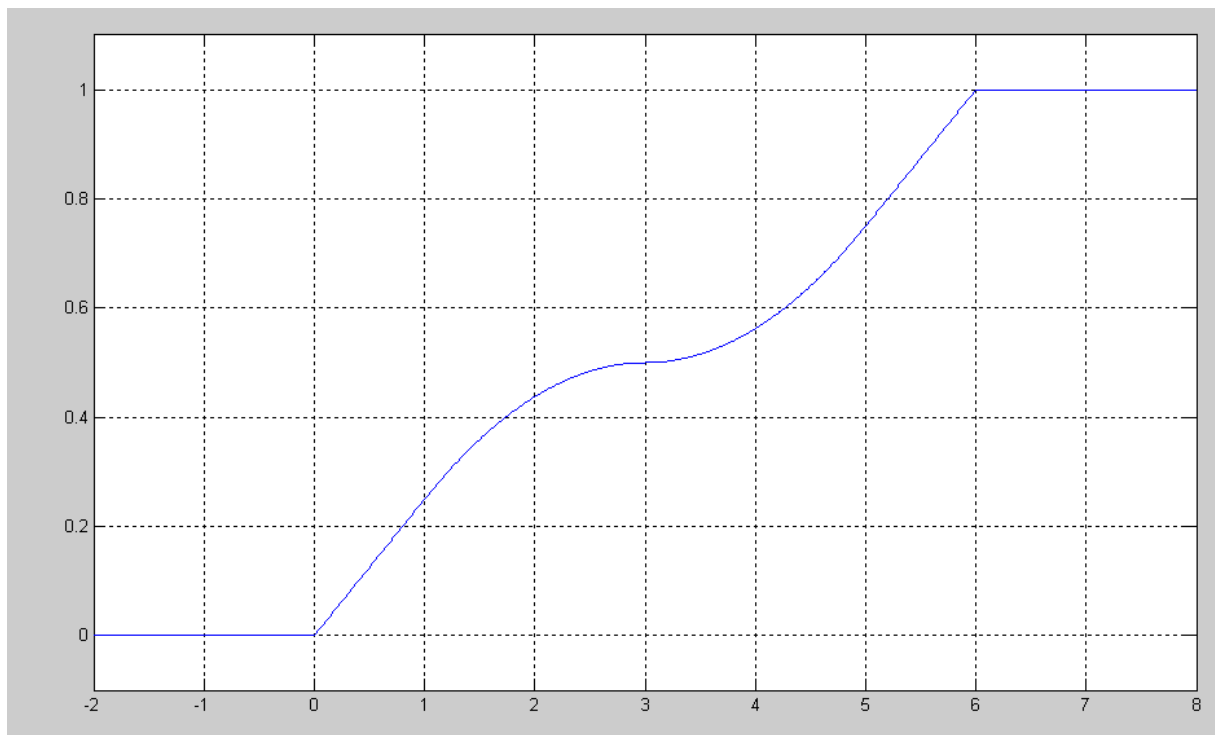
2) Determine the corresponding cdf

$$pdf = \begin{cases} 0 & x < 0 \\ 0.25 & 0 < x < 1 \\ (3-x)/8 & 1 < x < 3 \\ (x-3)/8 & 3 < x < 5 \\ 0.25 & 5 < x < 6 \end{cases}$$

The cdf is the integral of the pdf

$$p = \begin{cases} 0 & x < 0 \\ 0.25x & 0 < x < 1 \\ (-x^2 + 6x - 1)/16 & 1 < x < 3 \\ (x^2 - 6x + 17)/16 & 3 < x < 5 \\ (x-2)/4 & 5 < x < 6 \end{cases}$$

```
x = [-2:0.01:8]';
cdf = 0*x;
cdf = 0.25*x .* (x>0) .* (x<1);
cdf = cdf + (-x.^2 + 6*x - 1)/16 .* (1 <= x) .* (x<3);
cdf = cdf + (x.^2 - 6*x + 17)/16 .* (3 <= x) .* (x<5);
cdf = cdf + (x-2)/4 .* (5<=x) .* (x<6);
cdf = cdf + 1 .* (x>=6);
plot(x, cdf)
```



3) Using Matlab, find 20 random values of x for the above pdf

$$p = \begin{cases} 0 & x < 0 \\ 0.25x & 0 < x < 1 \\ (-x^2 + 6x - 1)/16 & 1 < x < 3 \\ (x^2 - 6x + 17)/16 & 3 < x < 5 \\ (x - 2)/4 & 5 < x < 6 \end{cases}$$

Solve backward

$$x = \begin{cases} 4p & 0 < p < 0.25 \\ 3 - \sqrt{8 - 16p} & 0.25 < p < 0.5 \\ 3 + \sqrt{16p - 8} & 0.5 < p < 0.75 \\ 4p + 2 & 0.75 < p < 1 \end{cases}$$

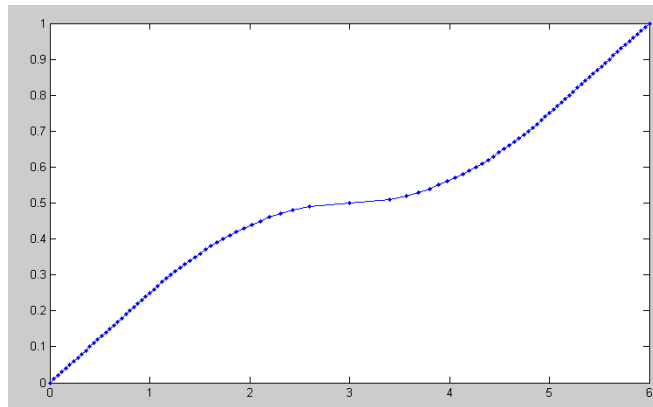
Program this in Matlab

```
function [x] = cdf1(p)

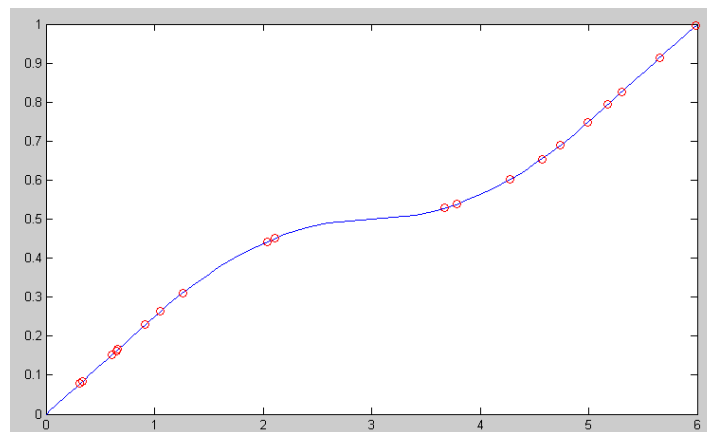
    if (p<0.25)
        x = 4*p;
    elseif (p<0.5)
        x = 3 - sqrt(8-16*p);
    elseif (p<0.75)
        x = 3 + sqrt(16*p-8);
    else
        x = 4*p+2;
    end
end
```

Verify it's correct (sweep p from 0 to 1)

```
p = [0:0.01:1]';
x = 0*p;
for i=1:length(p)
    x(i) = cdf1(p(i));
end
plot(x,p);
```

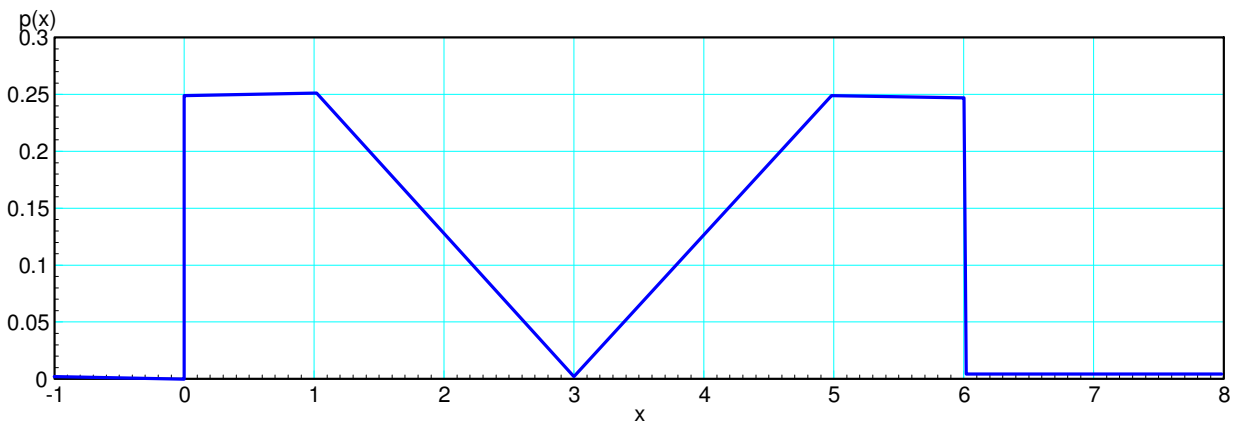


Now that I'm convinced it works, plug in 20 random values for p and find x

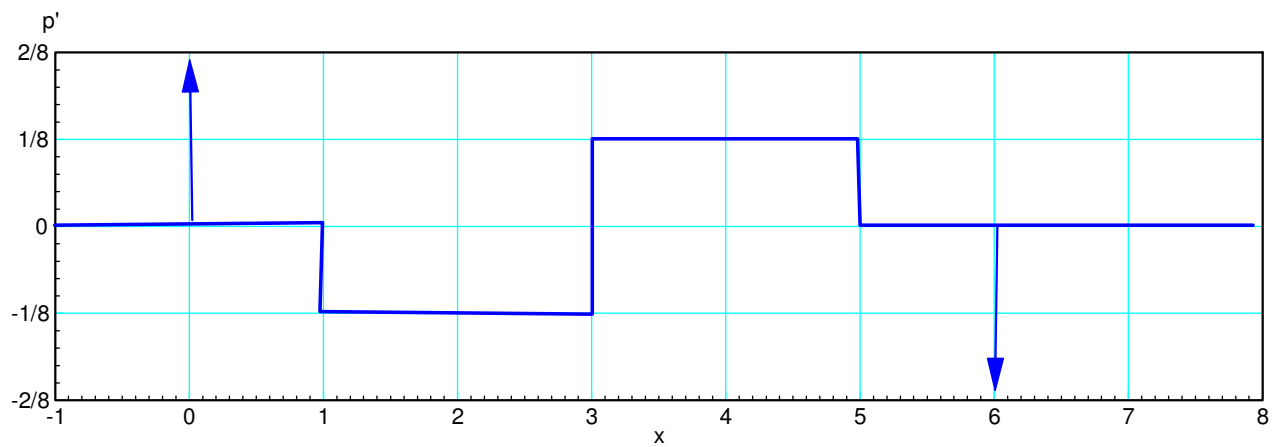


x	$p(x)$
0.6487	0.1622
5.1771	0.7943
1.2620	0.3112
3.6757	0.5285
0.6626	0.1656
4.2774	0.6020
1.0526	0.2630
4.5701	0.6541
4.7400	0.6892
4.9926	0.7482
2.1104	0.4505
0.3353	0.0838
0.9159	0.2290
5.6533	0.9133
0.6095	0.1524
5.3033	0.8258
3.7832	0.5383
5.9845	0.9961
0.3127	0.0782
2.0423	0.4427

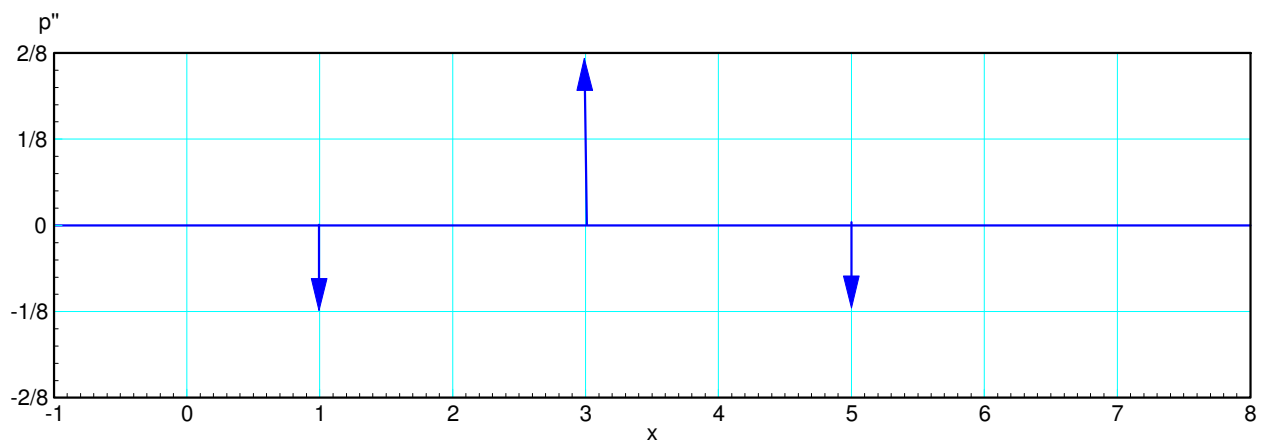
4) Find the moment generating function for $p(x)$



1st derivative



2nd derivative (ignoring delta functions)



$$\psi(s) = \left(\frac{1}{s}\right) \left(\frac{1}{4} - \frac{1}{4}e^{-6s}\right) + \left(\frac{1}{s^2}\right) \left(-\frac{1}{8}e^{-s} + \frac{1}{4}e^{-3s} - \frac{1}{8}e^{-6s}\right)$$