ECE 341 - Homework #8

Gamma & Normal Distributions.

Gamma Distributions

Let

- d be a sample from D, an exponential distribution with a mean of 5
- e be a sample from E, an exponential distribution with a mean of 10
- f be a sample from F, an exponential distribution with a mean of 15

1) Use moment generating functions to determine the pdf for d + d + d (i.e. the time for three events to be observed in D)

$$d(t) = 0.2\ e^{-0.2t}\ u(t)$$

$$D(s) = \left(\frac{0.2}{s+0.2}\right)$$

$$y(t) = d(t) * *d(t) * *d(t)$$

$$Y(s) = D(s) \cdot D(s) \cdot D(s)$$

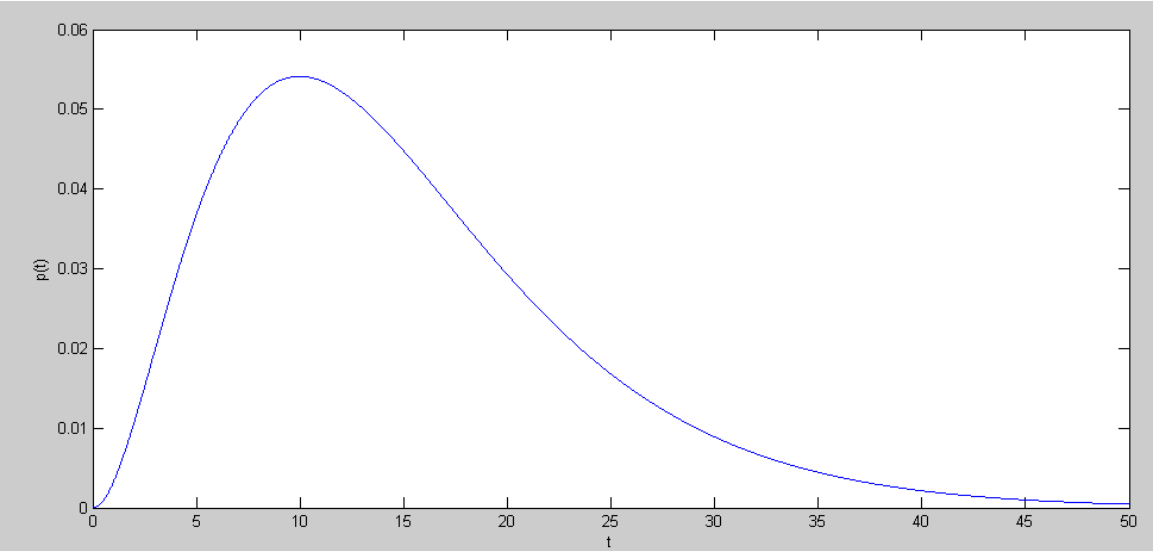
$$Y(s) = \left(\frac{0.2^3}{(s+0.2)^3}\right)$$

From the table of LaPlace transforms

$$\left(\frac{2}{(s+b)^3}\right) \rightarrow t^2\ e^{-bt}\ u(t)$$

$$Y(s) = \left(\frac{0.2^3}{2}\right)\left(\frac{2}{(s+0.2)^3}\right)$$

$$y(t) = \left(\frac{0.2^3}{2}\right)\ t^2\ e^{-0.2t}\ u(t)$$



2) Use moment generating functions to determine the pdf for the sum: d + e + f (i.e. the time for one event from D, E, and F)

$$D(s) = \left(\frac{0.2}{s+0.2}\right)$$

$$E(s) = \left(\frac{0.1}{s+0.1}\right)$$

$$F(s) = \left(\frac{0.0667}{s+0.0667}\right)$$

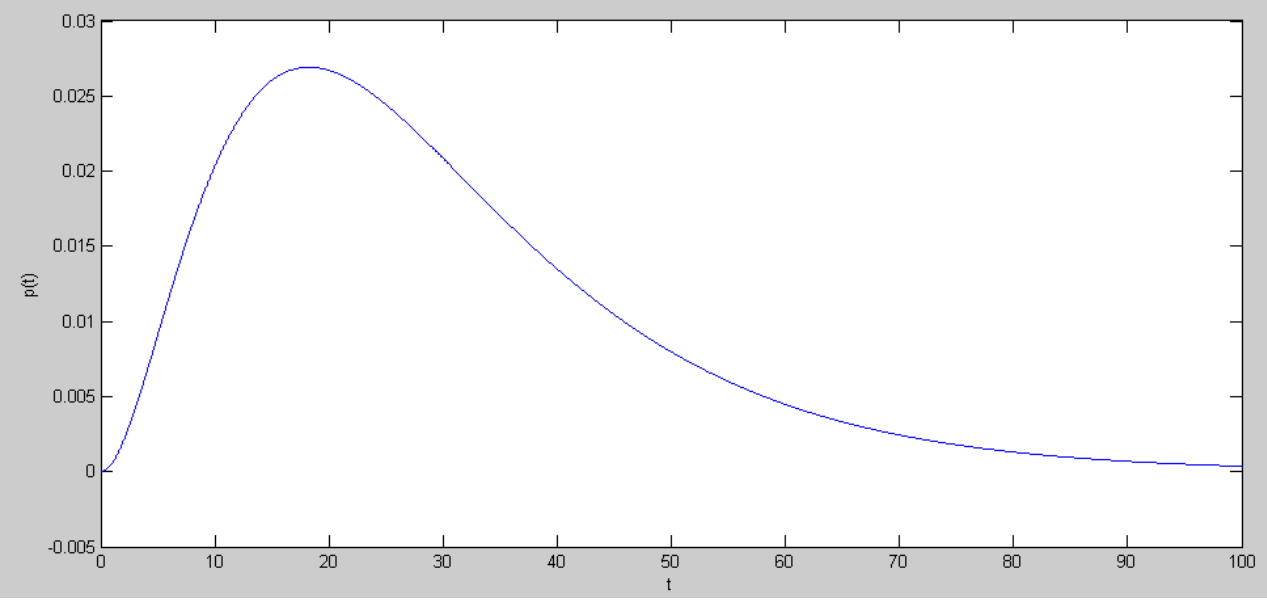
$$Y = \left(\frac{0.2}{s+0.2}\right)\left(\frac{0.1}{s+0.1}\right)\left(\frac{0.0667}{s+0.0667}\right)$$

Expand using partial fractions

$$Y = \left(\frac{0.1}{s+0.2}\right) + \left(\frac{-0.4}{s+0.1}\right) + \left(\frac{0.3}{s+0.0667}\right)$$

Take the inverse LaPlace transform

$$y(t) = (0.1\ e^{-0.2t} - 0.4\ e^{-0.1t} + 0.3\ e^{-0.0667t})u(t)$$



Normal Distribution

The low for the month has been measured at Hector Airport since 1942. The mean and standard deviations are:

Month	May	June	July	Aug	Sept	Oct
Mean	27.4013F	40.2179F	46.2949F	43.2321F	30.5526F	19.3462F
st dev	4.4236F	3.9924F	3.9481F	4.1435F	4.8050F	5.1265F

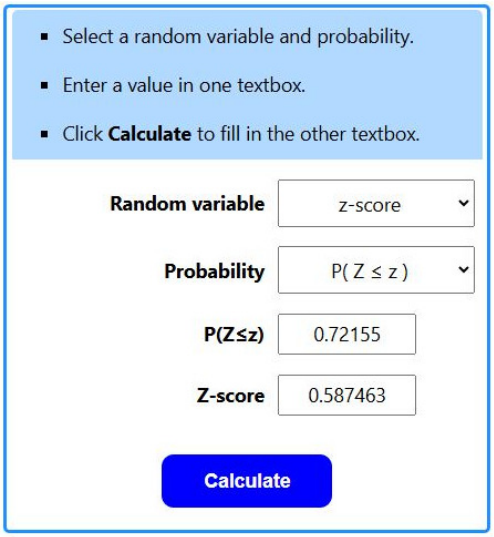
http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Low.txt

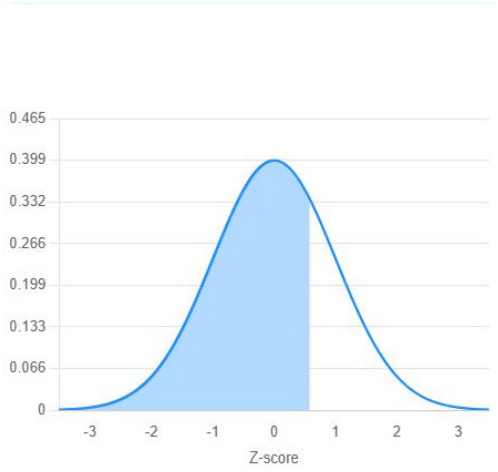
3) What is the probability that we will have a killing frost (temperature drops below 30F) in

May: p = 0.722

$$z = \left(\frac{30-27.4013}{4.4236}\right) = 0.587463$$

From StatTrek, this corresponds to a probability of 0.72155

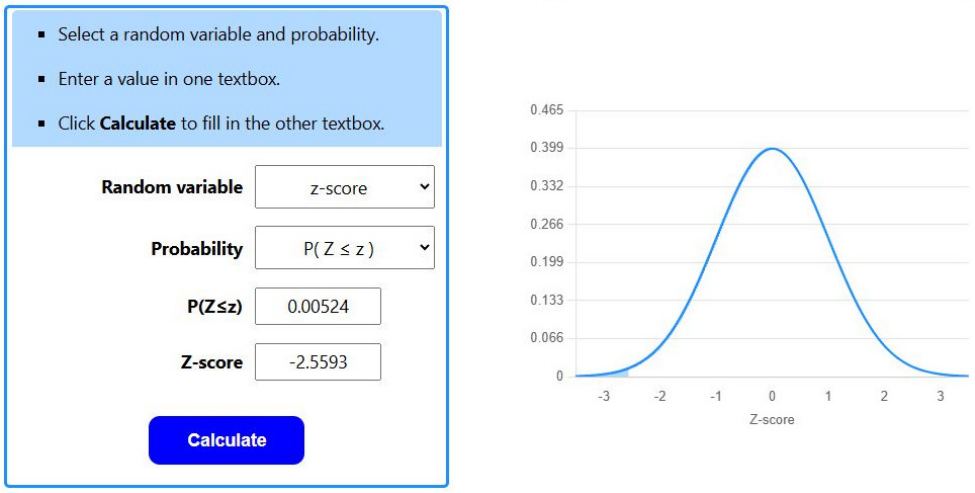




June: p = 0.001

$$z = \left(\frac{30 - 40.2179}{3.9924}\right) = -2.5593$$

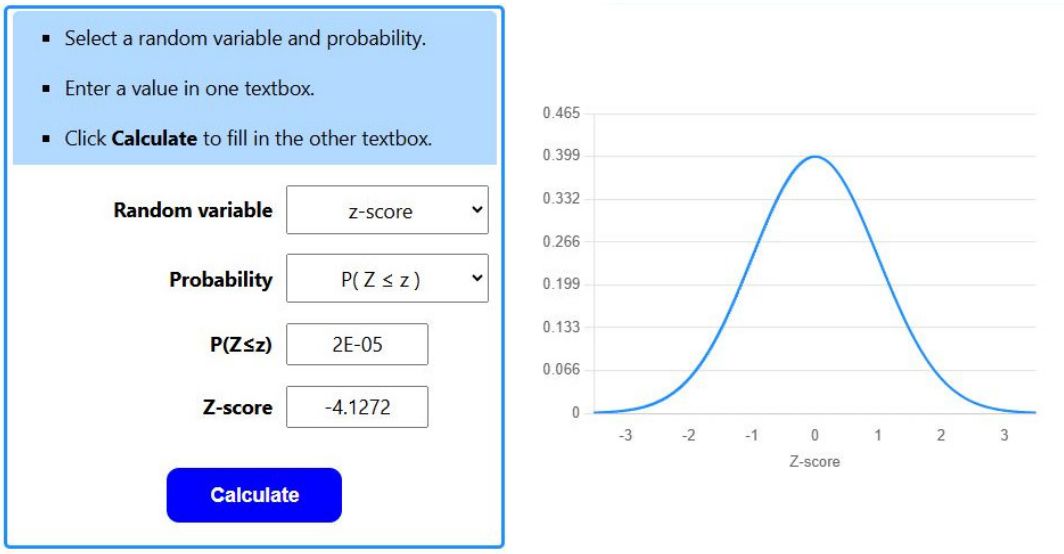
From StatTrek, this corresponds to a probability of 0.00524



July: p < 0.0005

$$z = \left(\frac{30 - 46.2949}{3.9481}\right) = -4.1272$$

From StatTrek, this corresponds to a probability of 0.00002



Rainfall

The rainfall in Fargo each month (in inches) is

Month	May	June	July	Aug	Sept	Oct
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619

4) What is the probability that we will get more than 10 inches of rain in the months of June, July, and August (combined)?

The sum will have a mean and variance equal to the sum

$$\mu = \mu_{june} + \mu_{july} + \mu_{aug}$$

$$\mu = 9.1222$$

$$\sigma^2 = \sigma^2_{\ june} + \sigma^2_{\ july} + \sigma^2_{\ a}$$

$$\sigma^2 = 11.2435$$

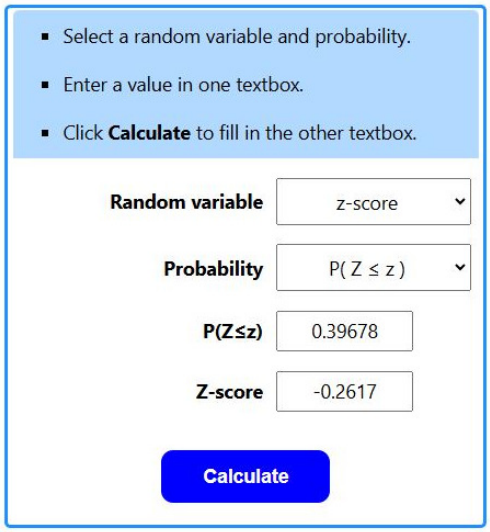
$$\sigma = 3.3531$$

The z-score for 10 inches is

$$z = \left(\frac{9.1222-10}{3.3531}\right) = -0.2617$$

From StatTrek, this corresponds to a probability of 0.39678

There is a 39.678% chance Fargo will get more than 10"+ of rain this summer



5) What is the probability that we will get no rain over these 6 months

Month	May	June	July	Aug	Sept	Oct	Sum
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694	15.6055
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619	4.2824*
Var	2.7344	4.4327	3.8045	3.0064	2.2240	2.1372	18.3391

* standard deviation is the root sum of squares of the rain for the summer

The z-score for 0" if rain is 0.007

$$z = \left(\frac{0-15.6055}{4.2824}\right) = -3.6441$$

There probability of no rain over 6 months is 0.00013