ECE 341 - Homework #9

Weibull Distribution, Central Limit Theorem.

Weibull Distribution

1) Let a be the time you have to wait until the next customer arrives at a store (in minutes). Assume the mean of a is 1.000 minute).

- Determine the pdf for the time it takes for three customers to arrive (the sum of three exponential distributions)
- Determine a Weibill distribution to approximate this pdf.

This is a Gamma distribution with a mgf of

$$\psi(s) = \left(\frac{1}{s+1}\right)^3$$

and a pdf of

$$g(t) = \frac{1}{2}\, t^2\, e^{-t}\, u(t)$$

Fit this to a Weibull distribution with

$$w(t) = \frac{k}{\lambda} \left(\frac{t}{\lambda}\right)^{k-1} e^{-(t/\lambda)}\, u(t)$$

Use Matlab and fminsearch. The cost function is

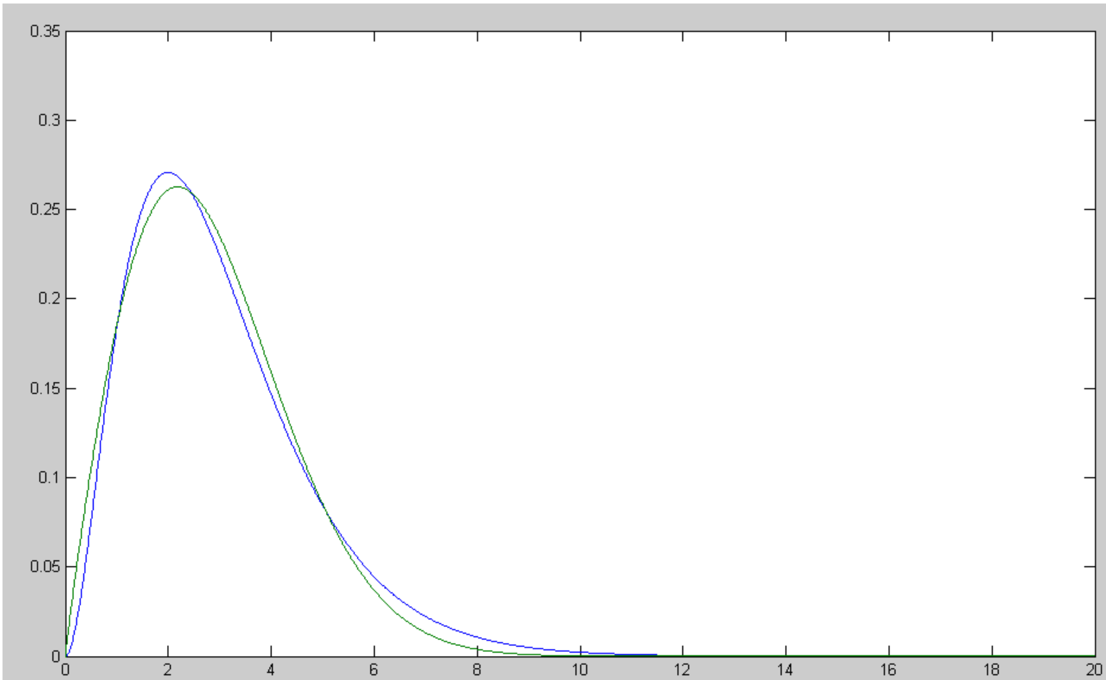
```
function y = costW(z)

   k = abs(z(1));
   L = abs(z(2));

   t = [0:0.01:20]';

   G = 0.5 * t.^2 .* exp(-t);
   W = (k/L) * (t/L).^(k-1) .* exp(- (t/L).^k);

   e = G - W;

   plot(t,G,t,W);
   pause(0.01);

   y = sum(e.^2);

end
```

Calling from Matlab

```
[Z,E] = fminsearch('costW',Z)

Z =    1.9284    3.1902

E =    0.1153
```

meaning

$$w(t) = \frac{1.9284}{3.1902}\left(\frac{t}{3.1902}\right)^{1.9284-1} e^{-(t/1.9284)}\, u(t)$$



Central Limit Theorem

2) Let X be the sum of five 6-sided dice (5d6).

Determine the probability of rolling 22 or higher with 5d6.

There are several ways to do this: Monte-Carlo, enumeration, convoltuion, moment-generating-functions, etc. Let's use enumeration:

```
d6 = [0,1,1,1,1,1,1]' / 6;
d6x2 = conv(d6,d6);
d6x4 = conv(d6x2,d6x2);
d6x5 = conv(d6x4,d6);
sum(d6x5(23:31))

ans =    0.1520
```

Use a Normal approximation and from this, determine the probability that the sum is 21.5 or higher.

The mean and variance are

$$\mu = 5 \cdot 3.5 = 17.5$$

$$\sigma^2 = 5 \cdot 2.91667 = 14.5833$$

$$\sigma = 3.8188$$

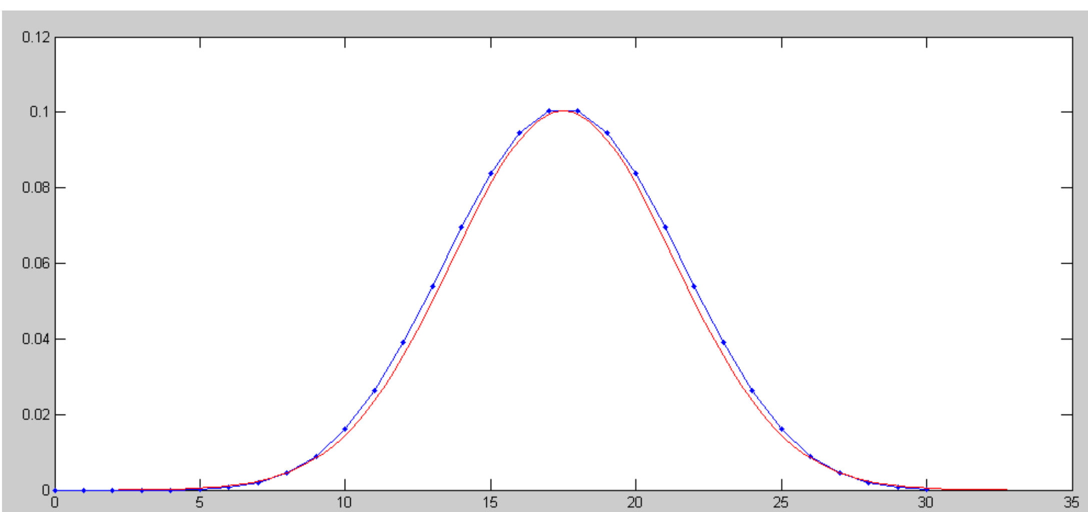
The z-score for 21.5 is

$$z = \left(\frac{21.5-17.5}{3.8188}\right) = 1.047$$

From StatTrek, this corresponds to a probability of 0.852

- The probability of rolling less than 21.5 is 0.852
- The probability of rolling more than 21.5 is 0.148 (close to 0.1520)

The normal approximation isn't exact - but then it's pretty close, even when summing only five distributions.



Binomial pdf (blue) and Normal distribution with the same mean and variance (red)

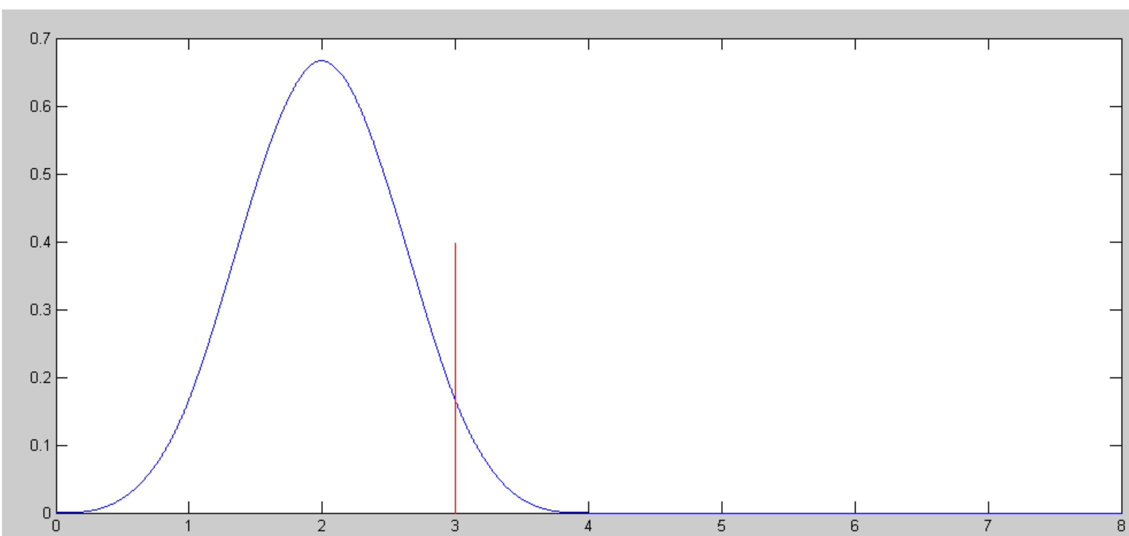
3) Let {a, b, c, d} each be uniformly distributed over the range of (0, 1).

Let X be the sum: a + b + c + d. Determine the probability that the sum is more than 3.00

There are several ways to do this: Monte-Carlo, convolution, LaPlace transforms, etc. Let's use convolution:

```
dt = 0.001;
t = [0:dt:2]';
A = 1 * (t<1);
B = 1 * (t<1);
C = 1 * (t<1);
D = 1 * (t<1);
sum(A)*dt

ans =     1

AB = conv(A,B) * dt;
CD = conv(C,D) * dt;
ABCD = conv(AB, CD) * dt;

t = [0:8000]' * dt;
plot(t,ABCD,'b',[3,3],[0,0.4],'r')

sum(ABCD(3001:8000))*dt
```

ans = 0.0414



Sum of four uniform(0,1) distributions. The area to the right of 3.000 is 0.0414

Use a Normal approximation and from this, determine the probability that the sum is more than 3.00

The mean is

$$\mu = 4 \cdot 0.5 = 2.0$$

The variance of a uniform(0,1) = 1/12. So

$$\sigma^2 = 4 \cdot \tfrac{1}{12} = 0.3333$$

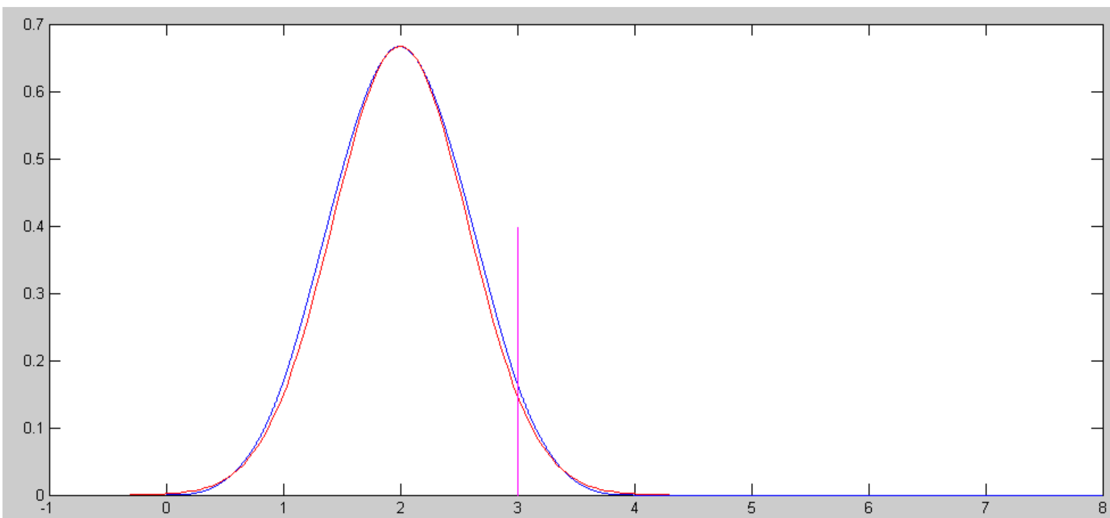
$$\sigma = 0.5773$$

The z-score for 3.00 is

$$z = \left(\frac{3-2}{0.5773}\right) = 1.7321$$

From StatTrek, this corresponds to a probability of 0.042 (vs. 0.0414)

The normal approximation isn't exact, but it's pretty close even with only four summations



pdf for summing four uniform distributions (blue) and its Normal approximation (red)