ECE 341 - Homework #10

Testing with Normal Distributions.

The low for the month has been measured at Hector Airport since 1942. The mean and standard deviations are:

Month	May	June	July	Aug	Sept	Oct
Mean	27.4013F	40.2179F	46.2949F	43.2321F	30.5526F	19.3462F
st dev	4.4236F	3.9924F	3.9481F	4.1435F	4.8050F	5.1265F

http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Low.txt

The rainfall in Fargo each month (in inches) is

Month	May	June	July	Aug	Sept	Oct
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619

1) What is the 90% confidence interval for the low in June?

For 5% tails, you need a z-score of 1.645

$$\mu - 1.645\sigma < June < \mu - 1.645\sigma \qquad p = 0.9$$

33.65F < June < 46.785

2) What is the probability that it will get colder than 40F in July?

Form the z-score

$$z = \left(\frac{40 - 46.2949}{3.9481} \right) = -1.5944$$

Use a standard normal distribution to convert the to a probability (StatTrek)

p = 0.055

3) What is the probability that the low in June will be less than the low in July?

Month	May	June	July	Aug	Sept	Oct
Mean	27.4013F	40.2179F	46.2949F	43.2321F	30.5526F	19.3462F
st dev	4.4236F	3.9924F	3.9481F	4.1435F	4.8050F	5.1265F

Form a new variable, W

W = July - June

W will have a mean and variance

$$\mu_w = \mu_{july} - \mu_{june} = 6.077$$

$$\sigma^2{}_w = \sigma^2{}_{july} + \sigma^2{}_{june} = 31.52$$

$$\sigma_w = 5.615$$

The z-score is then

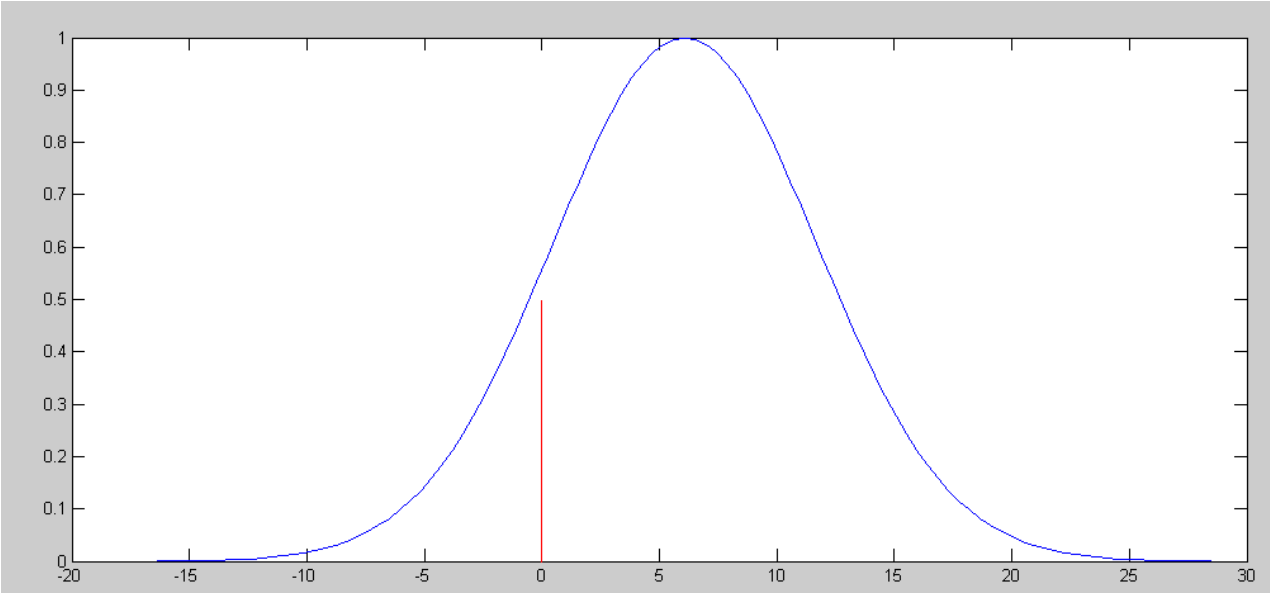
$$z = \frac{\mu_w}{\sigma_w} = 1.082$$

A standard normal table (or StatTrek) converts this z-score to a probability

p = 0.860

86% of the time, the low in June will be less than the low in July (June is colder)

14% of the time, the low in June will be more than the low in July (July is colder - not likely but can happen)



pdf of W: July - June.
86% of the time, July will be warmer than June (area to the right of zero)
14% of the time, June will be warmer than July (area to the left of zero)

4) What is the probability that we will get more rain in June than July?

Month	May	June	July	Aug	Sept	Oct
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619

Create a new variable W

W = June - July

$$\mu_w = \mu_{june} - \mu_{july} = 0.5357$$

$$\sigma^2{}_w = \sigma^2{}_{june} + \sigma^2{}_{july} = 8.237$$

$$\sigma_w = 2.870$$

The z-score for comparing to zero is

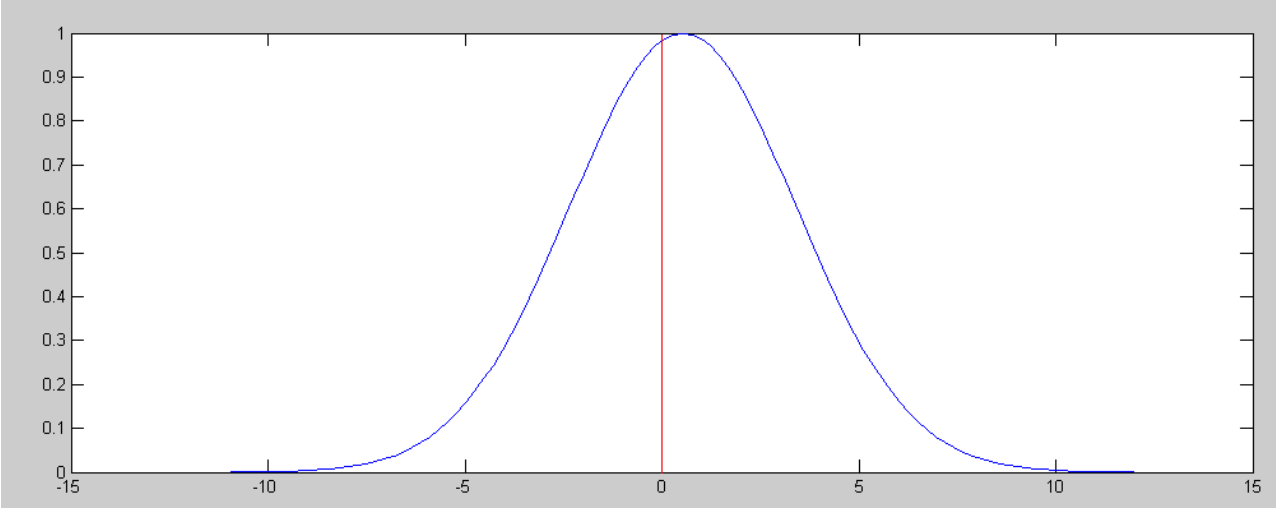
$$z = \left(\frac{\mu_w - 0}{\sigma_w}\right) = 0.1866$$

Using a standard normal table (or StatTrek) to convert this z-score to a probability results in

p = 0574

There is a 57.4% chance that June will be wetter than July

There is a 42.6% chance that July will be wetter than June



pdf for W
There is a 57.4% chance that June will be wetter than July (area to the right of zero)
There is a 42.6% chance that July will be wetter than June (area to the left of zero)

5) What is the probability that we will get more rain in June than any other month?

Month	May	June	July	Aug	Sept	Oct
Mean	2.6549	3.5025	2.9668	2.6529	2.1344	1.694
st dev	1.6536	2.1054	1.9505	1.7339	1.4913	1.4619

That's actually a hard question. If all else fails, resort to a Monte Carlo simulation.

```
N = 1e6;
W = 0;

for i=1:N

   May = randn * 1.6536 + 2.6549;
   June = randn * 2.1054 + 3.5025;
   July = randn * 1.9505 + 2.9668;
   Aug = randn * 1.7339 + 2.6529;
   Sept = randn * 1.4913 + 2.1344;
   Oct = randn * 1.4619 + 1.694;

   if(June > max([May, July, Aug, Sept, Oct]))
       W = W + 1;
   end
end

W / N

ans =  0.3459
```

There is a 34.59% chance that June will be the wettest month of the year.

Sidelight: Doing a comparison of means test for each month gives a number that's much too low:

June vs.	May	July	Aug	Sept	Oct
Mean	0.8476	0.5357	0.8496	1.3681	1.80854
variance	3.3965	3.5505	3.4363	3.3205	3.3074
z-score	0.2496	0.1509	0.2472	0.4120	0.5468
p(June>X)	0.599	0.560	0.598	0.660	0.708

Multiplying all the probabilities together gives

p = 0.0937

which is much too low.

If all else fails, you can always resort to a Monte Carlo simulation....