ECE 341 - Homework #12

t-Test with One Population. Due Tuesday, June 10th

Level 10 FlameStrike

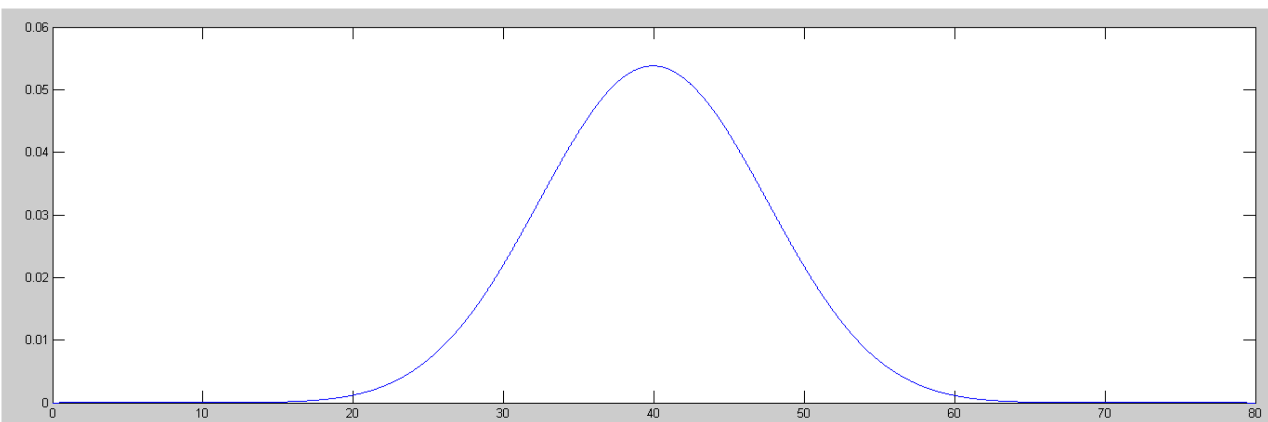
1) Let Y be the sum of ten uniform distributions over the range of (0,8). Calculate the probability that Y > 50.

Use convolution along with Matlab

```
>> dy = 0.01;
>> y = [0:dy:8]';
>> A1 = 0.125 * (y<8);
>> sum(A1) * dy
ans =    1.0000

>> A2 = conv(A1,A1) * dx;
>> A4 = conv(A2,A2) * dx;
>> A8 = conv(A4,A4) * dx;
>> A10 = conv(A2,A8) * dx;
>> sum(A10) * dy
ans =    1.0000

>> size(A10)
ans =   8001          1
>> y = [0:8000]' * dy;
>> plot(y,A10)
>> xlabel('y')
```




```
>> y(5001)
ans =    50

>> sum(A10(5001:8001)) * dy
ans =    0.0857
```

There is a 8.57% chance that the total will be 50 or more.

note:

- This the exact (ish) answer
- It took a *lot* of computations to get this answer

2) Use a normal approximation to determine the probability that X > 50.

A single uniform distrubution over the range of (0,8) has

- $\mu = \left(\frac{b-a}{2}\right) = \left(\frac{8-0}{2}\right) = 4.000$
- $\sigma^2 = \left(\frac{(b-a)^2}{12}\right) = \left(\frac{64}{12}\right) = 5.3333$

The sum of ten uniform distributions has

- $\mu = 10 \cdot 4 = 40$
- $\sigma^2 = 10 \cdot 5.3333 = 53.3333$

The z-score for X=50 is

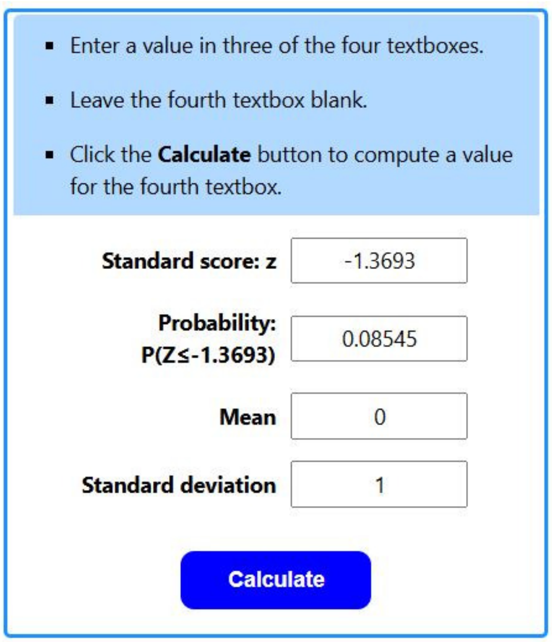
$$z = (\frac{x-\mu}{\sigma}) = \left(\frac{50-40}{\sqrt{53.333}}\right) = 1.3693$$

From Stat-Trek, the area of the tail with a z-score of 1.3693 is 0.08545

Based upon a normal approximation, there is a 8.545% chance that x > 50

Note:

- This took just a few computations to get almost the same result



3) Use a Monte-Carlo simulation with 100,000 trials to find the number of times Y > 50

- Repeat the Monte-Carlo simulation five times
- From these five results, determine the 90% confidence interval for the probability that Y > 50.

Matlab Code:

```
N = 0;

for i=1:1e5
    y = sum(8*rand(10,1));
    if(y >= 50)
        N = N + 1;
    end
end
disp(N/1e5)
```

Running this program five times:

```
    0.0872
    0.0863
    0.0854
    0.0871
    0.0858
```

The t-score for 5% tails and 4 degrees of freedom is 2.132

```
>> Y = [ 0.0872, 0.0863, 0.0854, 0.0871, 0.0858];
>> x = mean(Y);
>> s = std(Y);

>> x + 2.132*s/sqrt(5)
ans =    0.0871

>> x - 2.132*s/sqrt(5)
ans =    0.0856
```

It is 90% likely that the probability of Y>50 is in the range of (8.56%, 8.71%)

- It's actually 8.57% from problem #1

4) Using Matlab, generate ten values for Y

- Cast ten level-10 Flame Strikes

From the mean and standard devaiation of these Y's, use a t-test to determine the probability that Y > 50.

Matlab Code:

```
N = 0;
Y = zeros(10,1);
for i=1:10
    Y(i) = sum(8*rand(10,1));
end

disp(Y)

x = mean(Y)
s = std(Y)
t = (50 - x) / s
```

Result of ten flame-strikes:

```
Y =
   28.3193
   46.7950
   49.8775
   33.3453
   39.4703
   36.1054
   48.1983
   40.6353
   26.4225
   39.6346

x =   38.8804
s =    8.0241
t =    1.3858
```

From StatTrek, a t-score of 1.3858 has a tail with an area of 0.09959

The probability of Y>50 is 9.959%

Method	p(Y > 50)	# Rolls
Convolution	8.57%	0
Normal Approx	8.545%	0
Monte-Carlo	(8.56%, 8.71%)	500,000
t-Test	9.959%	10

Bison Poker

Assume a deck of playing cards has

- Six suits (clubs, diamonds, hearts, spades, jackrabbits, and bison)
- 12 values ranging from ace through queen. (no kings)

5) Calculate the odds of being dealt 4-of-a-kind when dealt a 5-card hand.

The number of possible hands are

$$M = \binom{72}{5} = 13,991,544$$

The number of ways to be dealt 4-of-a-kind are

xxxx y

(12 values for x choose 1)(6 x's choose 4)(11 values for y choose 1)(6 y's choose 1)

$$N = \binom{12}{1}\binom{6}{2}\binom{11}{1}\binom{6}{1}$$

$$N = 11,880$$

The probability of being dealt 4-of-a-kind in Bison Poker is thus

$$p = \left(\frac{N}{M}\right) = \left(\frac{11,880}{13,991,544}\right) = 0.00084908$$

or 1,177.739 : 1 against

6) Run seven Monte-Carlo simulations with 100,000 trials to determine the probability of beind dealt 4-of-a-kind.

Code for a single Monte-Carlo simulation

```
Pair4 = 0;

for i0 = 1:1e5
   X = rand(1,72);

   [a,Deck] = sort(X);

   Hand = Deck(1:5);
   Value = mod(Hand,12) + 1;
   Suit = floor(Hand/12) + 1;

   N = zeros(1,12);
   for n=1:12
       N(n) = sum(Value == n);
   end

   [N,a] = sort(N, 'descend');

   if (N(1) == 4) Pair4 = Pair4 + 1; end

end

disp('4 of a kind')
disp([Pair4])
```

Results:

```
4 of a kind
    90

4 of a kind
    82

4 of a kind
    93

4 of a kind
    80

4 of a kind
   102

4 of a kind
    79

4 of a kind
    85
```

7) From the Monte-Carlo simulation results, calculate using a t-test the odds of being dealt 4-of-a-kind in Bison Poker.

Note: The t-score for 5% tails and 6 d.o.f. is 1.943189

```
>> Data = [90, 82, 93, 80, 102, 79, 85];

>> x = mean(Data)

x =   87.2857

>> s = std(Data)

s =    8.2808

>> x + 1.943189*s/sqrt(5)

ans =   94.4819

>> x - 1.943189*s/sqrt(5)

ans =   80.0895

>>
```

\>>

The 90% confidence interval for the number of hands that have a 4-of-a-kind in 100,000 hands is

(80.0895, 94.4819)

The calculated odds are 84.9048 in 100,000

As the number of trials increases, the 90% confidence interval gets closer and closer to the calculated odds