ECE 341 - Test #1: Name ________________

Combinations, Permitations, and Discrete Probability

1. Enumeration

Assume you roll two 6-sided dice and take the difference. For example, if you roll (1, 5), your score is 4 points (the difference). Using emumeration, determine the odds of score 0 - 6 points. Please show your work.

There are 36 possibilities

		2nd Die					
		1	2	3	4	5	6
1st Die	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

Couting the number of entries

- 0 points: 6/36
- 1 point: 10/36
- 2 points: 8/36
- 3 points: 6.36
- 4 points: 4/36
- 5 points: 2/36

2. Combinations and Permutations

In Bison Poker, you play with a deck of cards containg 72 cards:

- Six suits (clubs, diamonds, hearts, spades, jackrabbits, and bison)
- Each suit contains 12 cards (Ace through queen - no kings).

Each hand in Bison Poker contains five cards.

Using combinatorics, determine the following:

- Total number of 5-card hands (order does't matter)
- Number of ways to be dealt a flush (five cards of one suit)
- Number of ways to be dealt a two-pair (xx yy a)

Number of Hands:

- Choose 5 cards from a 72-card deck

$$N = \binom{72}{5} = 13,991,544$$

Number of ways to be dealt a flush

- 6 suits choose one
- 12 cards of that suit, choose 5 for your hand

$$F = \binom{6}{1}\binom{12}{5} = 4,752$$

Number of ways to be dealt a 2-pair

- 12 values choose 2 for xy
- 6 x's in the deck, choose 2
- 6 y's in the deck, choose 2
- 50 remaining cards that are neither x nor y, choose 1 for a

$$P = \binom{12}{2}\binom{6}{2}\binom{6}{2}\binom{60}{1} = 891,000$$

3. Conditional Probability & Dice Games

Assume you play a game where you roll three 6-sided dice. After rolling the dice, you can then re-roll as many dice as you like one time. Determine the probability or getting three-of-a-kind using conditional probabilities

$$p(3) = p(3|A)p(A) + p(3|B)p(B) + p(3|C)p(C)$$

where

- A: you rolled three of a kind in the first roll — roll = xxx — keep all dice
- B: you rolled two-of-kind in the first roll: — roll = xx y — re-roll y
- C: you rolled no pairs — roll = x y z — re-roll y and z

A:

- 6 values choose 1 for x
- 3 spots for x, choose 3 to place x

$$p(A) = \left(\frac{\binom{6}{1}\binom{3}{3}}{6^3} \right) = 0.027778$$

$$p(3|A) = 1$$

B:

- 6 values for x, choose 1
- 3 spots choose 2 to place xx
- 5 remaining values for y, choose 1

$$p(B) = \left(\frac{\binom{6}{1}\binom{3}{2}\binom{5}{1}}{6^3} \right) = 0.416667$$

$$p(3|B) = 1/6$$

C:

- 6 values choose 3 for xyz
- 3 spots for x, choose 1
- 2 remaining spots for y, choose 1
- 1 remaining spot for z, choose 1

$$p(C) = \left(\frac{\binom{6}{3}\binom{3}{1}\binom{2}{1}\binom{1}{1}}{6^3} \right) = 0.555555$$

$$p(3|C) = \left(\frac{1}{6}\right)^2$$

Add it all together

$$p(3) = p(3|A)p(A) + p(3|B)p(B) + p(3|C)p(C)$$

$$p(3) = (1)(0.027778) + \left(\frac{1}{6}\right)(0.4166666) + \left(\frac{1}{36}\right)(0.555555)$$

$$p(3) = 0.112654$$

4. Binomial Distribution

Assume you are flipping 14 coins where each coin has a probability of a heads being 0.4 (p = 0.4).

Determine the probability of flipping 6 heads two different ways {Monte-Carlo, enumeration, combinatorics, formula, convolution, and/or z-transform). Please show your work.

Monte-Carlo:

```
Win = 0;
for n=1:1e6
    Coin = rand(14,1) < 0.4;
    X = sum(Coin);
    if(X == 6)
        Win = Win + 1;
    end
end
[Win]


Win =      206143

p = 0.206143 (ish)
```

Formula:

The probability is

$$p(6) = \binom{14}{6}(0.4)^6 \cdot (0.6)^8$$

$$p = 0.206598$$

Convolution

```
>> F1 = [0.6,0.4];
>> F2 = conv(F1,F1);
>> F4 = conv(F2,F2);
>> F8 = conv(F4,F4);
>> F12 = conv(F4,F8);
>> F14 = conv(F2,F12);
>> F14(7)

ans =    0.2066
```

z-Transform

A single flip is

$$F = \left(\frac{qz+p}{z}\right)$$

14 flips is

$$F_{14} = \left(\frac{qz+p}{z}\right)^{14}$$

Pull out the z^8 term in the numerator to get the probability of 6 heads

5. Geometric & Pascal Distribution

Let A, B, and C have geometric distributions:

- A = the number of rolls until you get a 1 on a 5-sided die (p = 1/5)
- B = the number of rolls until you get a 1 on a 6-sided die (p = 1/6)
- C = the number of rolls until you get a 1 on a 7-sided die (p = 1/7)

Let X be a Pascal distribution

- X = A + B + C

Determine the probability that X=10 using two different methods

- (Monte-Carlo, enumeration, combinatorics, convolution, z-transform, etc)

Please show your work.

Monte-Carlo:

```
Wins = 0;
for n=1:1e6
    A = 1;
    while(rand > 1/5)
        A = A + 1;
    end
    B = 1;
    while(rand > 1/6)
        B = B + 1;
    end
    C = 1;
    while(rand > 1/7)
        C = C + 1;
    end
    X = A + B + C;
    if(X == 10)
        Wins = Wins + 1;
    end
end
[Wins]

Wins =       47228
```

The probabillity is about 4.7228%

Convolution:

```
k = [0:100]';
A = 1/5 * (4/5).^(k-1) .* (k>0);
B = 1/6 * (5/6).^(k-1) .* (k>0);
C = 1/7 * (6/7).^(k-1) .* (k>0);
AB = conv(A,B);
ABC = conv(AB,C);
[ABC(11)]

ans =    0.0468
```

z-Transforms

$$A(z) = \left(\frac{1/5}{z-4/5}\right)$$

$$B(z) = \left(\frac{1/6}{z-5/6}\right)$$

$$C(z) = \left(\frac{1/7}{z-6/7}\right)$$

$$X = ABC = \left(\frac{1/5}{z-4/5}\right)\left(\frac{1/6}{z-5/6}\right)\left(\frac{1/7}{z-6/7}\right)$$

$$X = \left(\frac{2.5}{z-4/5}\right) + \left(\frac{-6}{z-5/6}\right) + \left(\frac{3.5}{z-6/7}\right)$$

$$zX = \left(\frac{2.5z}{z-4/5}\right) + \left(\frac{-6z}{z-5/6}\right) + \left(\frac{3.5z}{z-6/7}\right)$$

$$zx(k) = \left(2.5\left(\frac{4}{5}\right)^{k} - 6\left(\frac{5}{6}\right)^{k} + 3.5\left(\frac{6}{7}\right)^{k}\right)u(k)$$

$$x(k) = \left(2.5\left(\frac{4}{5}\right)^{k-1} - 6\left(\frac{5}{6}\right)^{k-1} + 3.5\left(\frac{6}{7}\right)^{k-1}\right)u(k-1)$$

$$x(10) = 2.5\left(\frac{4}{5}\right)^{9} - 6\left(\frac{5}{6}\right)^{9} + 3.5\left(\frac{6}{7}\right)^{9}$$

$$x(10) = 0.046776$$