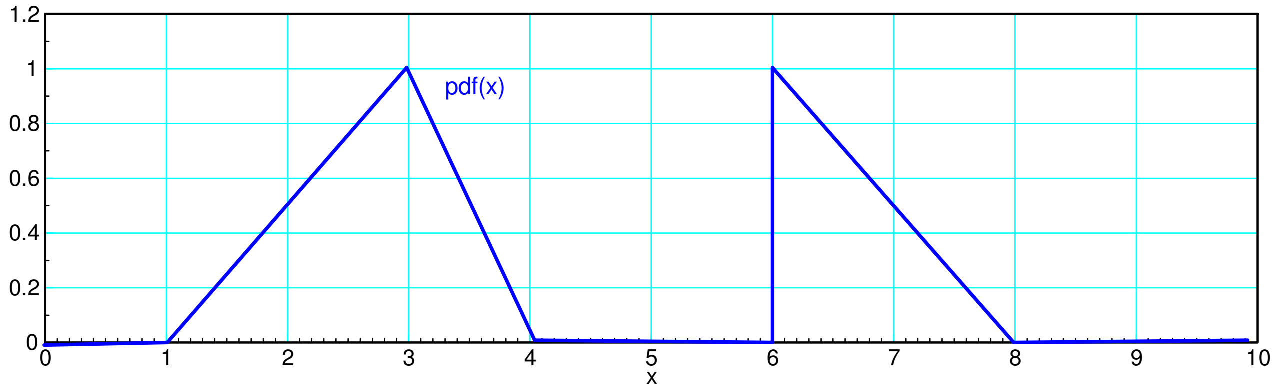
ECE 341 - Test #2

Continuous Probability

1) Continuous PDF

For the following probability density function



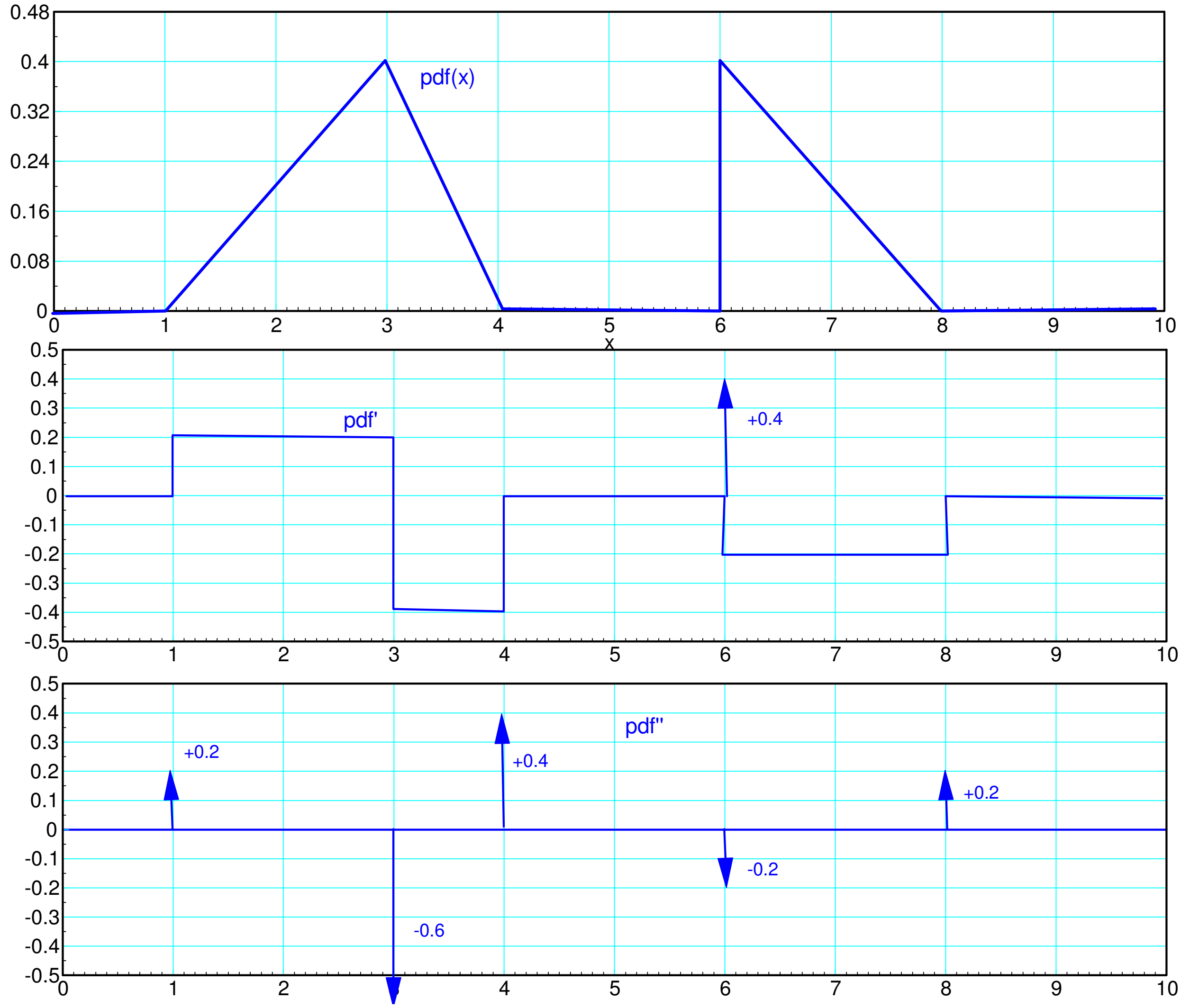
a) Determine the scalar to multiply this curve so that it is a valid pdf (i.e. the total area = 1.0000)

As drawn, the total area = 2.5

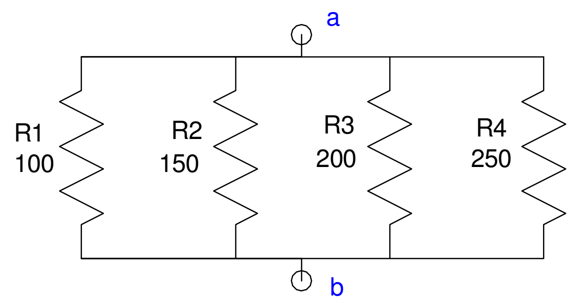
Scalar = 1/2.5 = 0.4

b) Determine the moment generating function (i.e. LaPlace transform)

$$\psi(s) = \left(\frac{1}{s}\right)(0.4e^{-6s}) + \left(\frac{1}{s^2}\right)(0.2e^{-s} - 0.6e^{-3s} + 0.4e^{-4s} - 0.2e^{-6s} + 0.2e^{-8s})$$



2) Uniform PDF



Assume each resistor has 5% tolerance and a uniform distribution. For example:

- R1 = 100 * (1 + (2*rand-1) * 0.05);

Using Matlab and a Monte-Carlo simulation, find

- 10 values for Rab for random resistances R1..R4
- The mean of the resulting Rab, and
- The standard deviation of the resulting Rab

Include

- Your Matlab code
- The ten resistances Rab
- The mean and standard deviation

Note: Resistors in parallel add as the sum of the inverses, inverted

$$R_{ab} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right)^{-1}$$

Matlab Code:

```
Rab = zeros(10,1);
for i=1:10
    R1 = 100 * (1 + (2*rand-1) * 0.05);
    R2 = 150 * (1 + (2*rand-1) * 0.05);
    R3 = 200 * (1 + (2*rand-1) * 0.05);
    R4 = 250 * (1 + (2*rand-1) * 0.05);
    Rab(i) = 1 / (1/R1 + 1/R2 + 1/R3 + 1/R4);
end
[Rab]
mean(Rab)
std(Rab)
```

Results:

```
Rab =

   39.7828
   38.5949
   40.1115
   39.6164
   39.7429
   39.2113
   39.6064
   38.7061
   38.0281
   39.0935


mean =   39.2494

std =     0.6476
```

3) Gamma CDF

Let A, and B be continuous exponential distributions:

- A has a mean of 7 and
- B has a mean of 8

Determine the cdf of Y = A + B using moment generating functions (LaPlace transforms)

Note: The cdf is the pdf times 1/s (integrate)

$$cdf = \left(\frac{1}{s}\right) \cdot A(s) \cdot B(s)$$

Solution

$$cdf = \left(\frac{1}{s}\right)\left(\frac{1/7}{s+1/7}\right)\left(\frac{1/8}{s+1/8}\right)$$

Do a partial fraction expansion

$$cdf = \left(\frac{a}{s}\right) + \left(\frac{b}{s+1/7}\right) + \left(\frac{c}{s+1/8}\right)$$

$$cdf = \left(\frac{1}{s}\right) + \left(\frac{7}{s+1/7}\right) + \left(\frac{-8}{s+1/8}\right)$$

Take the inverse LaPlace transform

$$cdf = (1 + 7e^{-t/7} - 8e^{-t/8})u(t)$$

4) Central Limit Theorem

Let A be a continuous uniform distribution over the range of (1,5)

Let Y be the sum of five samples from population A

- $Y = a_1 + a_2 + a_3 + a_4 + a_5$

a) Determine the mean and variance of A

For a single uniform distribution

$$\mu = \left(\frac{b+a}{2}\right) = 3$$

$$\sigma^2 = \left(\frac{(b-a)^2}{12}\right) = \frac{16}{12}$$

b) Determine the mean and variance of Y

For the sum of five uniform distributions

$$\mu = 5 \cdot 3 = 15$$

$$\sigma^2 = 5 \cdot \frac{16}{12} = \frac{80}{12}$$

$$\sigma = \sqrt{\frac{80}{12}} = 2.5819$$

c) Using a normal approximation, determine

- the z-score for the probability that Y > 15 and
- the probability that Y > 15

$$z = \left(\frac{15-15}{2.5819}\right) = 0$$

$$p(Y > 15) = 0.5$$

5) Testing with Normal PDF

Assume A and B have normal distributions

Population	mean	standard dev
A	100	50
B	150	60

Let W be a random variable which is the difference between A and B

$$W = A - B$$

a) Determine the mean and standard deviation of W

$$\mu_w = \mu_a - \mu_b$$

$$\mu_w = 100 - 150 = -50$$

$$\sigma_w^2 = \sigma_a^2 + \sigma_b^2$$

$$\sigma_w^2 = 50^2 + 60^2$$

$$\sigma_w^2 = 6100$$

b) Determine the probability that W > 0

- i.e. the probability that a random sample from A will be larger than a random sample from B

$$z = \left(\frac{\mu_w}{\sigma_w}\right) = \left(\frac{-50}{\sqrt{6100}}\right) = -0.64018$$

From StatTrek, this corresponds to a probability of 0.26103

There is a 26.201% chance that A will be larger than B