ECE 376 - Homework #12

Digital Filters - Due Monday, April 22nd

Filters in the z-Plane

1) Assume G(s) is a low-pass filter with real poles:

$$G(s) = \left(\frac{500}{(s+3)(s+7)(s+10)}\right)$$

Design a digital filter, G(z), which has approximately the same gain vs. frequency as G(s). Assume a sampling rate of T = 0.01 second.

Convert to the z-plane

$$s = -3 \qquad z = e^{sT} = e^{-0.03} = 0.9704$$

$$s = -7 \qquad z = e^{sT} = e^{-0.07} = 0.9324$$

$$s = -10 \qquad z = e^{sT} = e^{-0.1} = 0.9048$$

so the form of G(z) is

$$G(z) = \left(\frac{k}{(z-0.9704)(z-0.9324)(z-0.9048)}\right)$$

To find k, match the DC gain

$$DC = \left(\frac{500}{(s+3)(s+7)(s+10)}\right)_{s=0} = 2.3810$$

$$DC = \left(\frac{k}{(z-0.9704)(z-0.9324)(z-0.9048)}\right)_{z=1} = 2.3810$$

$$k = 0.00045355$$

so

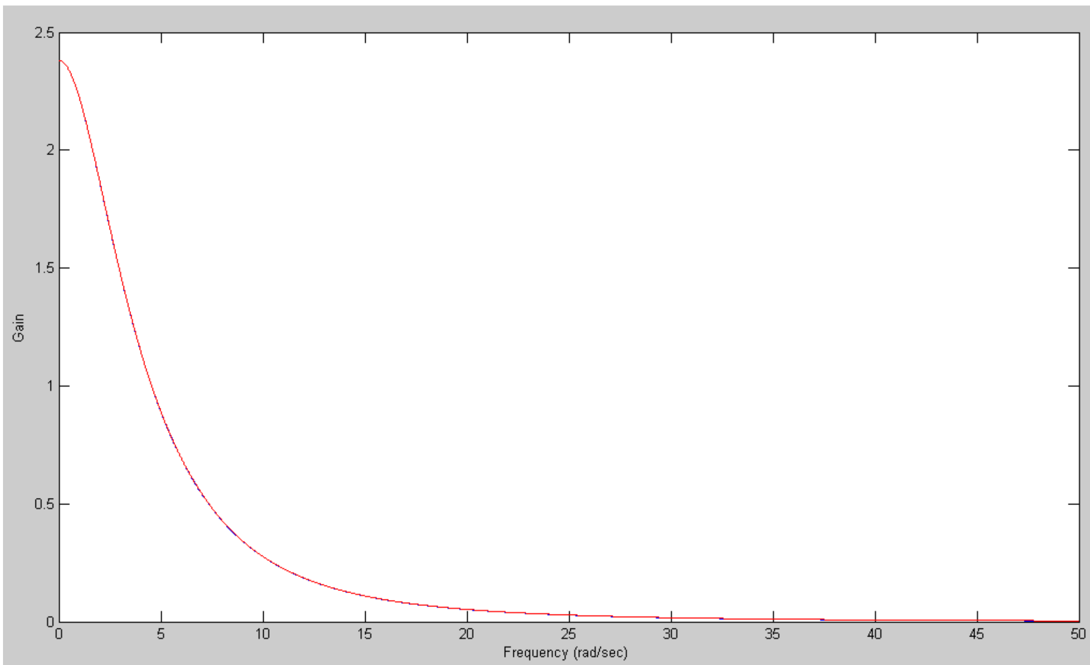
$$\boxed{G(z) = \left(\frac{0.00045355}{(z-0.9704)(z-0.9324)(z-0.9048)}\right)}$$

You could also add three zeros at z=0 to remove the time-delay (optional - delays don't affect the gain vs. frequency)

$$G(z) = \left(\frac{0.00045355z^3}{(z-0.9704)(z-0.9324)(z-0.9048)}\right)$$

Plot the gain vs. frequency for both filters from 0 to 50 rad/sec.

```
>> w = [0:0.01:50]';
>> s = j*w;
>> Gs = 500 ./ ( (s+3).*(s+7).*(s+10) );
>>
>> T = 0.01;
>> z = exp(s*T);
>> Gz = 0.00045355 ./ ( (z-0.9704).*(z-0.9324).*(z-0.9048));
>>
>> plot(w,abs(Gs),'b',w,abs(Gz),'r')
>> xlabel('Frequency (rad/sec) ');
>> ylabel('Gain')
>>
```



2) Assume G(s) is the following band-pass filter:

$$G(s) = \left(\frac{10s}{(s+5+j20)(s+5-j20)}\right)$$

Design a digital filter, G(z), which has approximately the same gain vs. frequency as G(s). Assume a sampling rate of T = 0.01 second.

Same procedure - works for real poles, works for complex poles

$$s = 0 \qquad z = e^{sT} = e^0 = 1$$

$$s = -5 + j20 \qquad z = e^{sT} = 0.9323 + 0.1890i$$

$$s = -5 - j20 \qquad z = e^{sT} = 0.9323 - 0.1890i$$

so the form of G(z) is

$$G(z) = \left(\frac{k(z-1)}{(z-0.9323+j0.1890)(z-0.9323-j0.1890)}\right)$$

Pick k to match the gain at some frequency. DC doesn't work since the gain is zero. Pick some other frequency, like 20 rad/sec

$$\left(\frac{10s}{(s+5+j20)(s+5-j20)}\right)_{s=j20} = 0.9923\angle 7.1250^0$$

$$\left(\frac{k(z-1)}{(z-0.9323+j0.1890)(z-0.9323-j0.1890)}\right)_{z=e^{j0.2}} = 10.4817\angle 1.2997^0$$

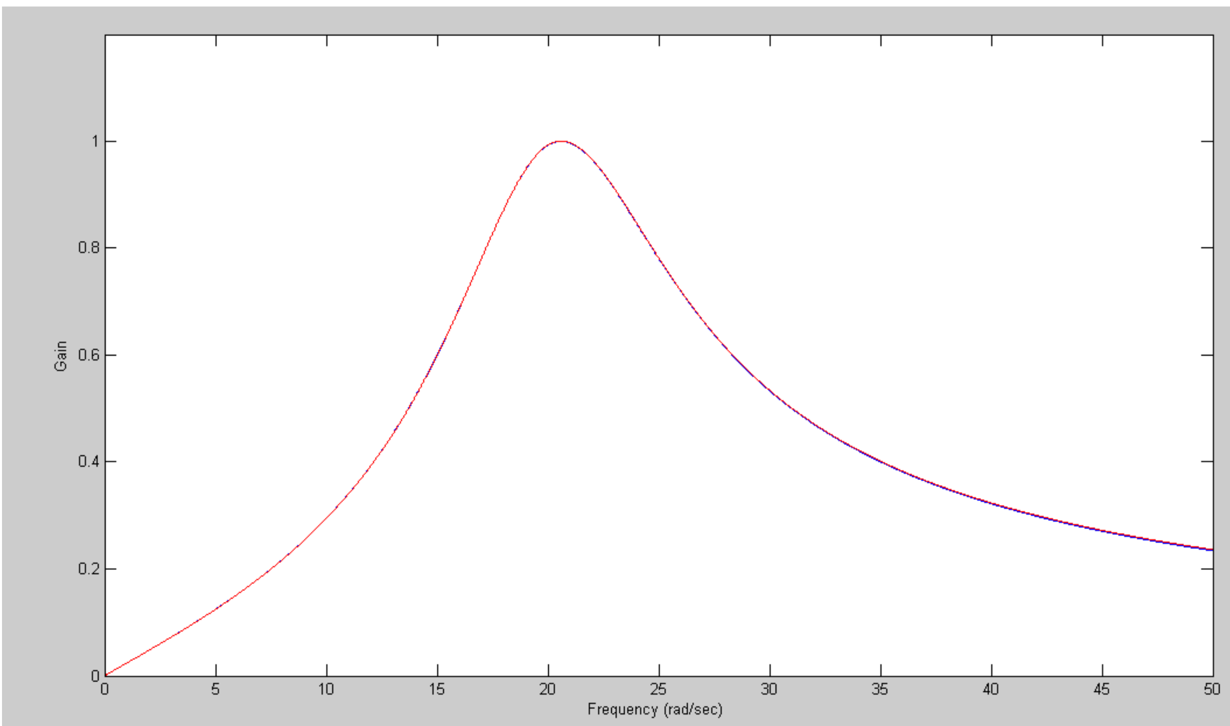
$$k = \left(\frac{0.9923}{10.4817}\right) = 0.0947$$

so

$$G(z) = \left(\frac{k(z-1)}{(z-0.9323+j0.1890)(z-0.9323-j0.1890)}\right)$$

Plot the gain vs. frequency for both filters from 0 to 50 rad/sec.

```
>> w = [0:0.01:50]';
>> s = j*w;
>> Gs = 10*s ./ ( (s+5+j*20).*(s+5-j*20) );

>> T = 0.01;
>> p1 = exp( (-5+j*20)*T );
>> p2 = exp( (-5-j*20)*T );
>> z = exp(s*T);
>> Gz = 0.0947*(z-1) ./ ( (z-p1).*(z-p2) );

>> plot(w,abs(Gs),'b',w,abs(Gz),'r')
>> xlabel('Frequency (rad/sec) ');
>> ylabel('Gain')
>> ylim([0,1.2])
```



3) Write a C program to implement the digital filter, G(z)

$$Y = \left(\frac{0.0947(z-1)}{(z-0.5403+j0.8415)(z-0.5403-j0.8415)} \right) X$$

multiply out

$$Y = \left(\frac{0.0947(z-1)}{z^2-1.8645z+0.9048} \right) X$$

cross multiply

$$(z^2 - 1.8645z + 0.9048)Y = 0.0947(z-1)X$$

meaning

y(k+2) - 1.8645 y(k+1) + 0.9048 y(k) = 0.0947 (x(k+1) - x(k))

time shift

y(k) - 1.8645 y(k-1) + 0.9048 y(k-2) = 0.0947 (x(k-1) - x(k-2))

Solve for y(k)

y(k) = 1.8645 y(k-1) - 0.9048 y(k-2) + 0.0947 (x(k-1) - x(k-2))

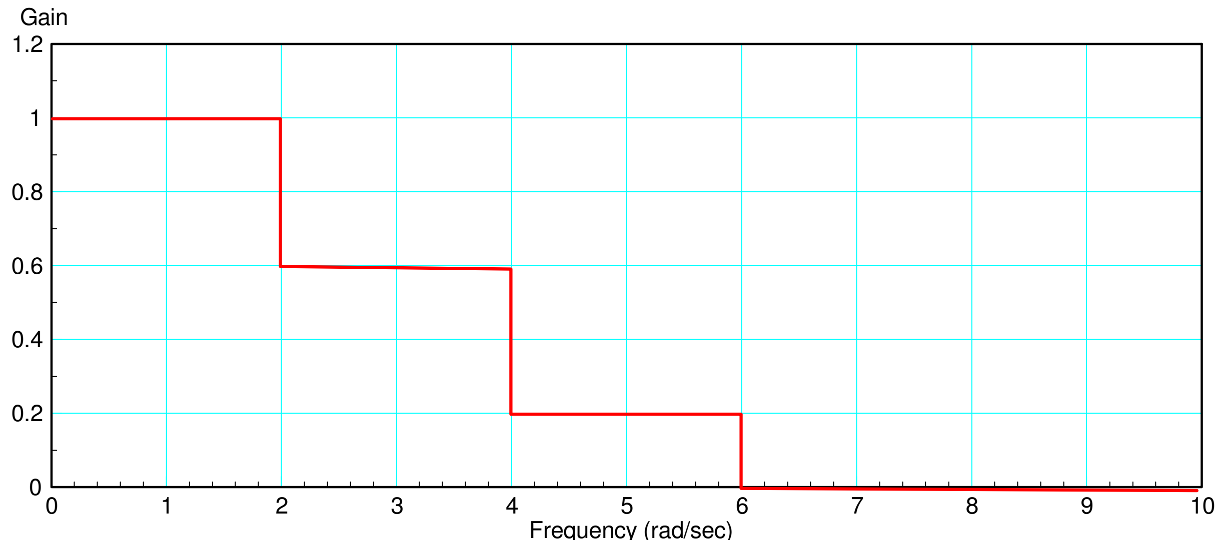
That's your program

```
while(1) {

    x2 = x1;
    x1 = x0;
    x0 = A2D_Read(0);

    y2 = y1;
    y1 = y0;
    y0 = 1.8645*y1 - 0.9048*y2 + 0.0947 * ( x1 - x2 );

   D2A(y0);

   wait_ms(10);
   }
```

FIR Filters

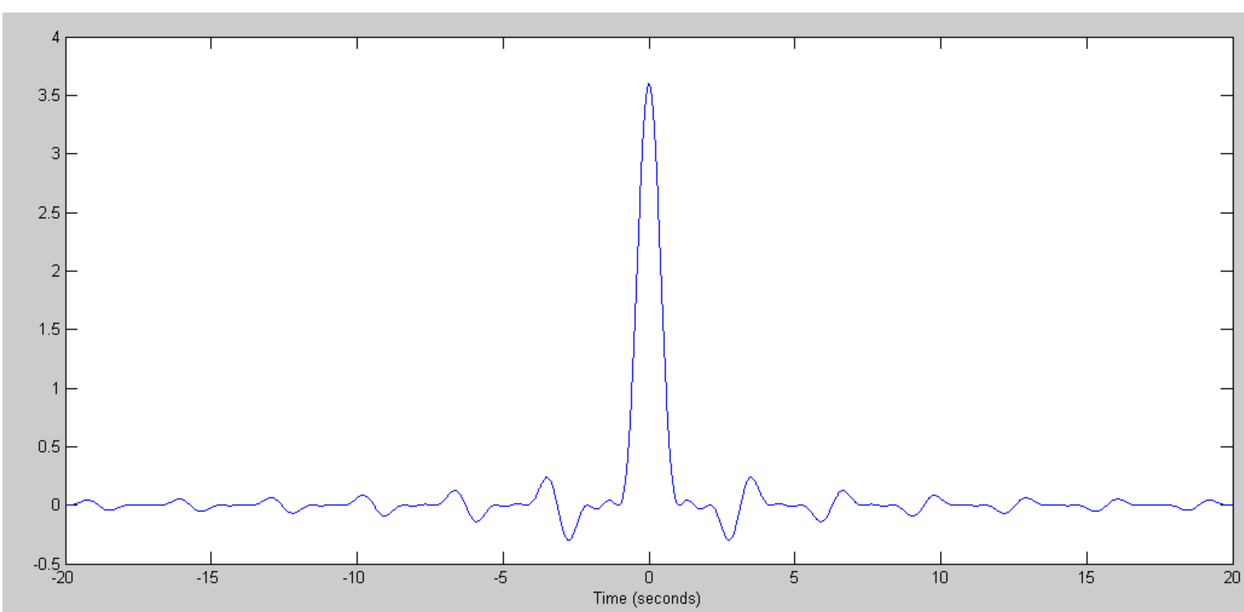
4) Find the impulse response of a filter with the following gain vs. frequency:

- hint: Approximate the waveform by adding up ideal low-pass filters



G(s) = 0.2*LPF(6 rad/sec) + 0.4*LPF(4 rad/sec) + 0.4*LPF(2 rad/sec)

$$H(t) = 0.2\left(\frac{\sin(6t)}{t}\right) + 0.4\left(\frac{\sin(4t)}{t}\right) + 0.4\left(\frac{\sin(2t)}{t}\right)$$



Impulse Response of Filter

5) Design a FIR filter to approximate this impulse reaponse. Include in your design

The sampling rate

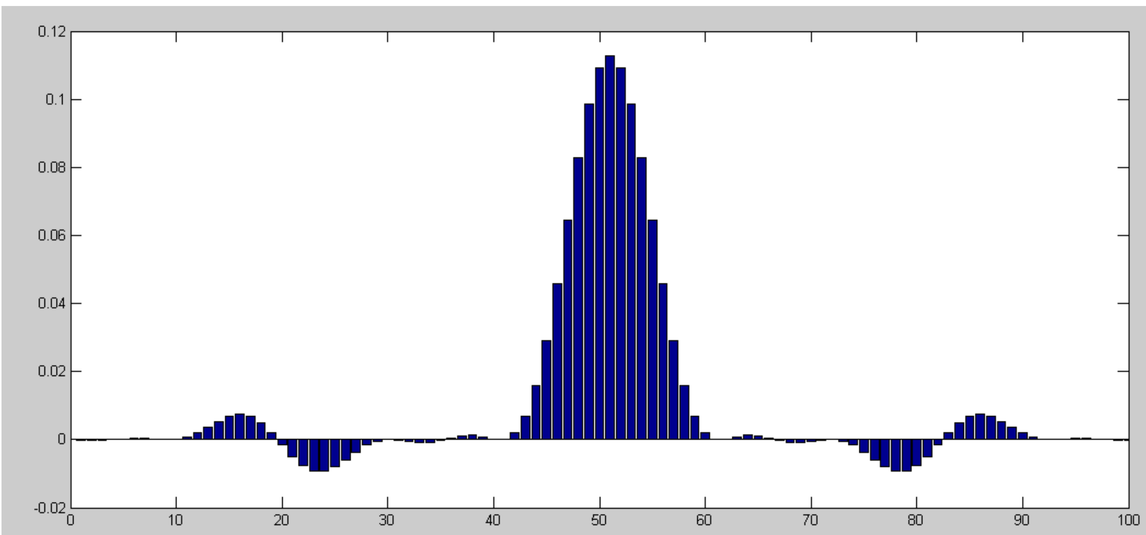
- 100 points for 10 seconds
- T = 0.1 second

The length of the window

- clip the signal from -5s to +5s
- delay the signal by 5 seconds

The impulse response of your FIR fitler.

```
>> t = [-5:0.1:5]';
>> t = [-5:0.1:5]' + 1e-9;
>> H = 0.2*sin(6*t)./t + 0.4*sin(4*t)./t + 0.4*sin(2*t)./t;
>> H = H / sum(H);
>> DC = sum(H)

DC =    1.0000

>> plot(t,H,'.')
>> plot(t+5,H,'.')
>> xlim([0,10])
>> xlabel('Time (seconds)')
```

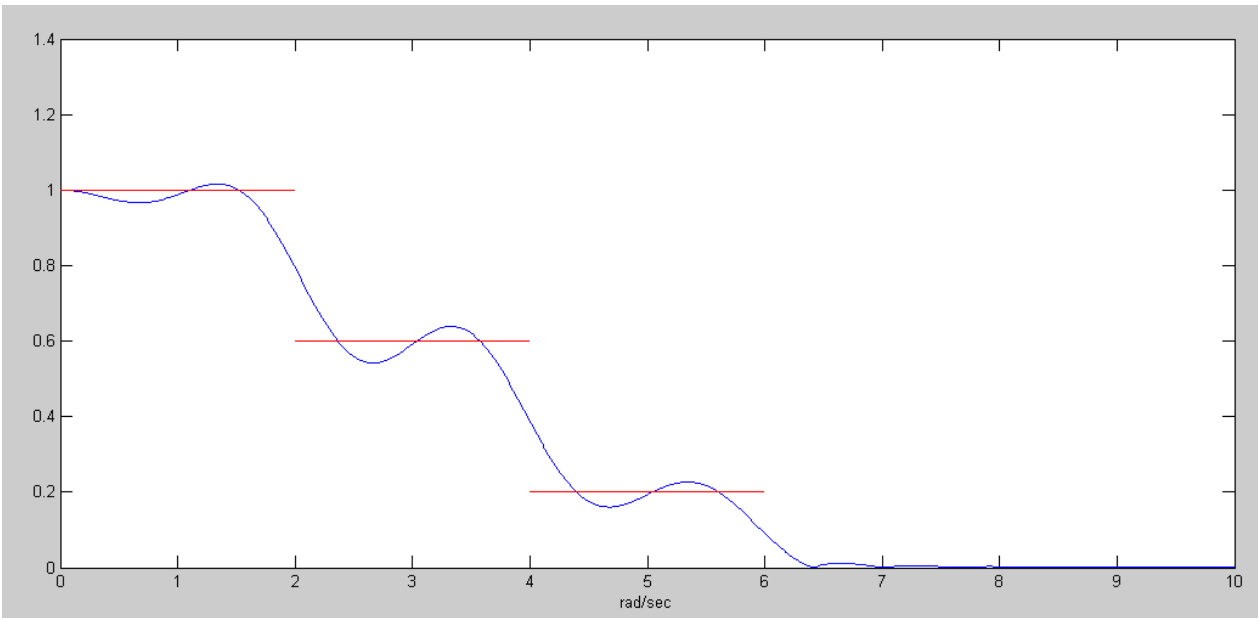


Note: The DC gain is the sum of all terms.

- (the sum should be 1.000 for a DC gain of 1.000)

6) Plot the gain vs. frequency of your filter

```
>> w = [0:0.01:10]';
>> s = j*w;
>> T = 0.1;
>> z = exp(s*T);
>> Gz = 0*s;
>> for i=1:length(t)
    Gz = Gz + H(i) * z.^(-i);
   end
>> plot(w, abs(Gz))
```



It's not perfect, but pretty good. Adding more terms would improve the frequency response