Homework #7: ECE 461/661

Gain, Lead, PID Compensation. Due Monday, October 19st

The transfer function for a 10-stage RC filter is

$$G_{10}(s) = \left(\frac{9765625}{(s+19.61)(s+18.31)(s+16.28)(s+13.7)(s+10.8)(s+7.825)(s+5.05)(s+2.719)(s+1.04)(s+0.1617)}\right)$$

A 4th-order model (ignore all poles 40x faster than the dominant pole) is

$$G_4(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)$$

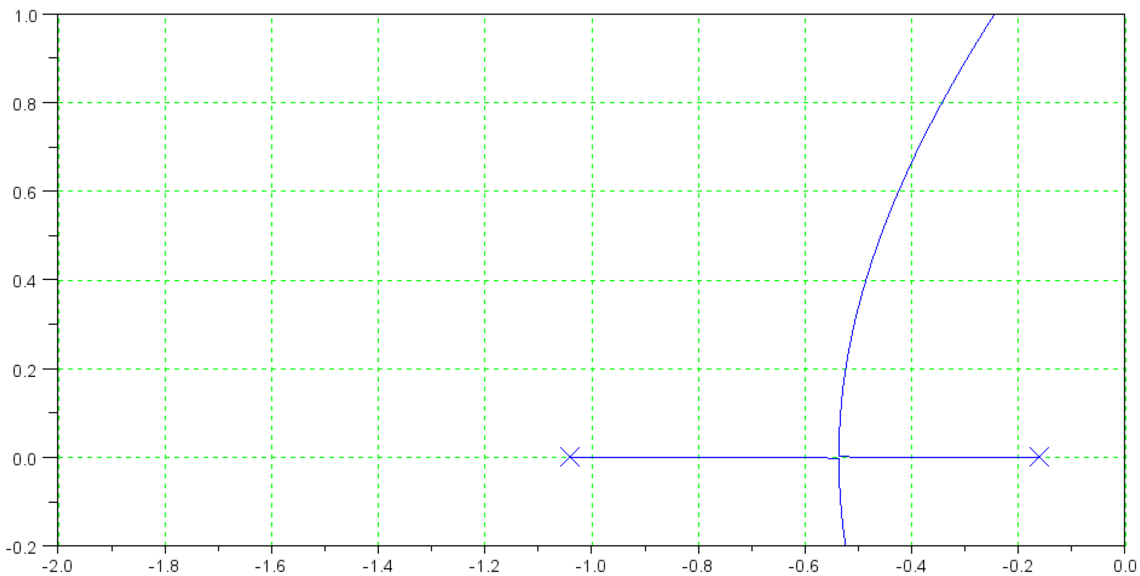
Gain Compensation

1) For the 4th-order model, design a gain compensator (K(s) = k) which results in

- The fastest system possible,
- With no overshoot for a step input (i.e. design for the breakaway point)

Check your design in Matlab or Simulink or VisSim

Step 1: Plot the root locus and determine the breakaway point



Determine the design point (s at the breakaway point)

$$s = -0.5335$$

Pick k so that GK = -1 at this point

$$\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)_{s=-0.533} = 0.7761\angle 180^0$$

$$k = \frac{1}{0.7761} = 1.2884$$

The closed-loop dominant pole(s)

$$s = -0.5335$$

The 2% settling time,

$$T_s = \frac{4}{0.5335} = 7.2437 \text{ sec}$$

The error constant, Kp, and

$$K_p = (GK)_{s=0} = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)_{s=0} (1.2884)$$

$$K_p = (0.6248)(1.2884)$$

$$K_p = 0.8050$$

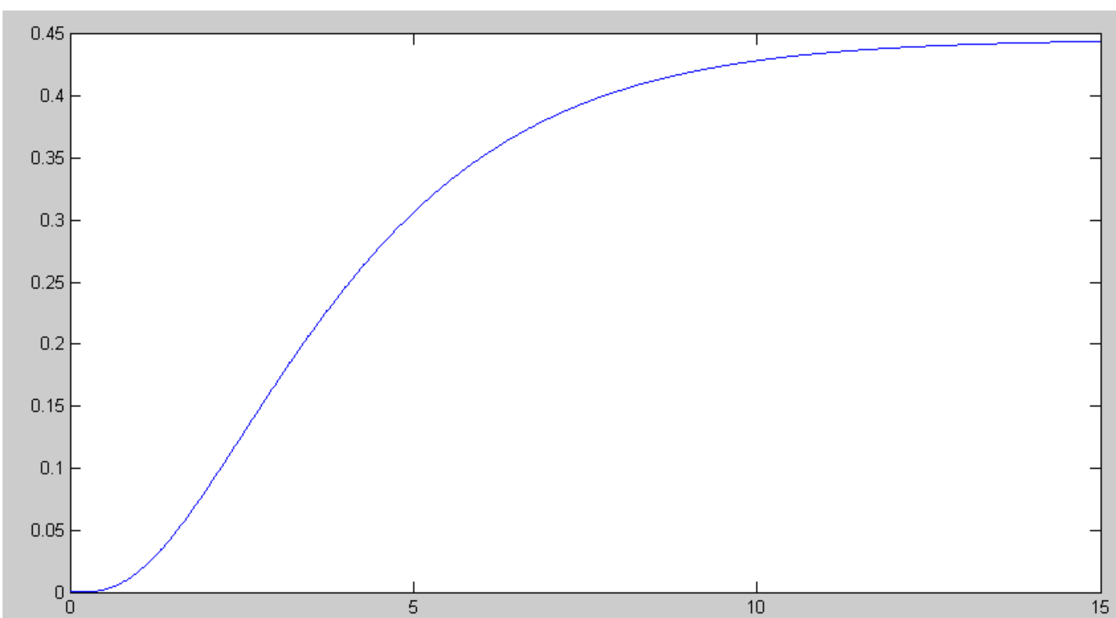
The steady-state error for a step input.

$$E_{step} = \frac{1}{K_p+1}$$

$$E_{step} = 0.5540$$

Checking the result in Matlab

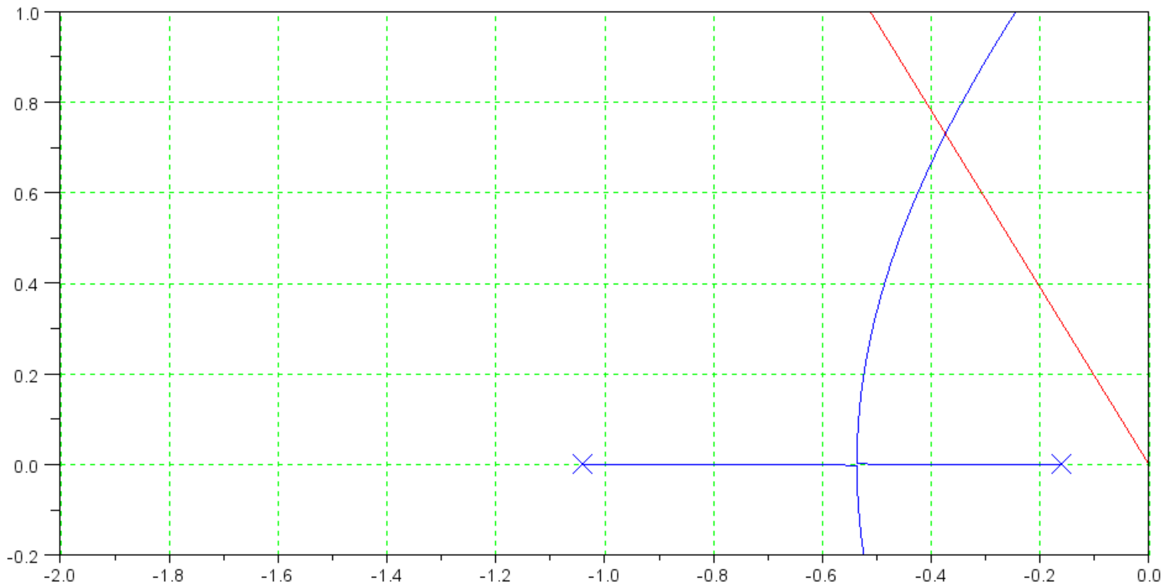
```
>> G = zpk([],[-0.1617,-1.04,-2.719,-5.05],1.4427);
>> evalfr(G, -0.5335)
ans =  -0.7761
>> k = 1 / abs(ans)
k =    1.2884
>> Gcl = minreal(G*k / (1+G*k));
>> zpk(Gcl)

                 1.8588
----------------------------------------
(s+0.5335) (s+0.5399) (s+2.89) (s+5.008)

>> t = [0:0.01:15]';
>> y = step(Gcl, t);
>> plot(t,y);
```



2) Design a gain compensator (K(s) = k) which results in 20% overshoot for a step input.

20% overshoot corresponds to a damping ratio of 0.4559

Draw the root locus and determine the point which intersects the damping line



$$s = -0.3698 + j0.7397$$

At this point

$$\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)_{s=-0.3698+j0.7397} = 0.1612\angle 180^0$$

$$k = \frac{1}{0.1612} = 6.0240$$

The closed-loop dominant pole(s)

$$s = -0.3698 + j0.7397$$

The 2% settling time,

$$T_s = \frac{4}{0.3698} = 10.8167 \text{ sec}$$

The error constant, Kp, and

$$K_p = (GK)_{s=0} = (0.6248)(6.0240)$$

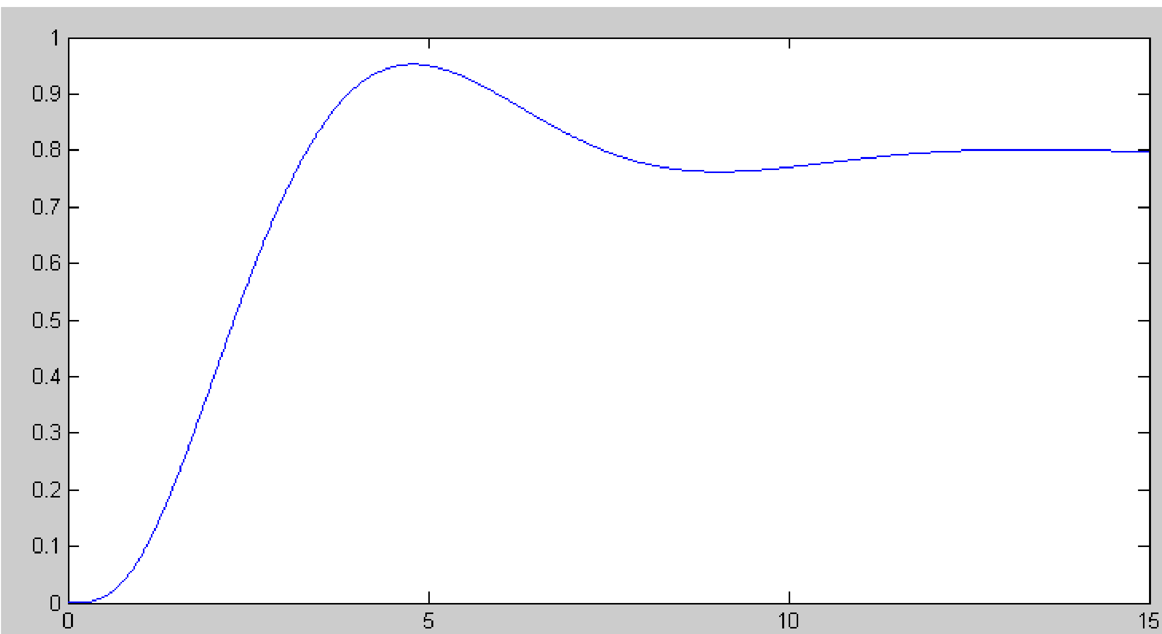
$$K_p = 3.7638$$

The steady-state error for a step input.

$$E_{step} = \frac{1}{K_p+1} = 0.1298$$

Check your design in Matlab or Simulink or VisSim

```
>> G = zpk([],[-0.1617,-1.04,-2.719,-5.05],1.4427);
>> s = -0.3698 + j*0.7397;
>> evalfr(G,s)

ans =  -0.1612 - 0.0000i

>> k = 1/abs(ans)

k =    6.2043

>> Gcl = minreal(G*k / (1+G*k));
>> eig(Gcl)

  -0.3698 + 0.7397i
  -0.3698 - 0.7397i
  -3.4266
  -4.8044

>> t = [0:0.01:15]';
>> y = step(Gcl, t);
>> plot(t,y);
>>
```



Lead Compensation

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)$$

3) Design a lead compensator, $K(s) = k\left(\frac{s+a}{s+10a}\right)$, which results in 20% overshoot for a step input.

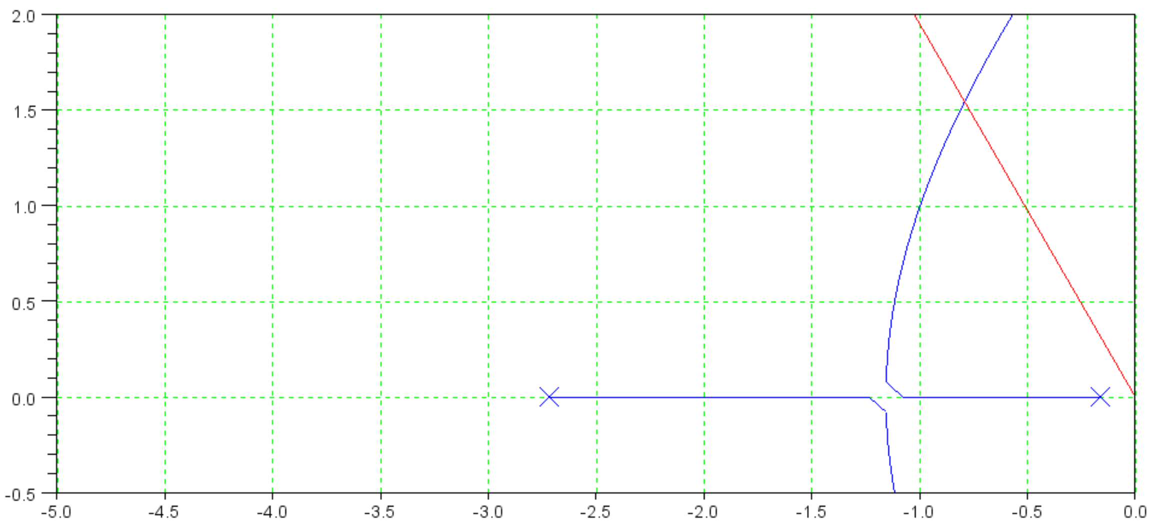
Keep the pole at -0.1617 (keeps the steady state error small)

Cancel the pole at -1.04

$$K(s) = k\left(\frac{s+1.04}{s+10.4}\right)$$

$$GK = \left(\frac{1.4427k}{(s+0.1617)(s+10.4)(s+2.719)(s+5.05)} \right)$$

Sketch the root locus and find the point with a damping ratio of 0.4559



$$s = -0.7829 + j1.5658$$

To find k:

$$\left(\frac{1.4427k}{(s+0.1617)(s+10.4)(s+2.719)(s+5.05)} \right)_{s=-0.7829+j1.5658} = 0.0078\angle 180^0$$

$$k = \frac{1}{0.0078} = 128.76$$

$$K(s) = 128.76\left(\frac{s+1.04}{s+10.4}\right)$$

The closed-loop dominant pole(s)

$$s = -0.7829 + j1.5658$$

The 2% settling time,

$$T_s = \frac{4}{0.7829} = 5.1092 \sec$$

The error constant, Kp, and

$$K_p = G(0)K(0)$$

$$K_p = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)_{s=0} \left(128.76 \left(\frac{s+1.04}{s+10.4} \right) \right)_{s=0}$$

$$K_p = (0.6248)(12.876)$$

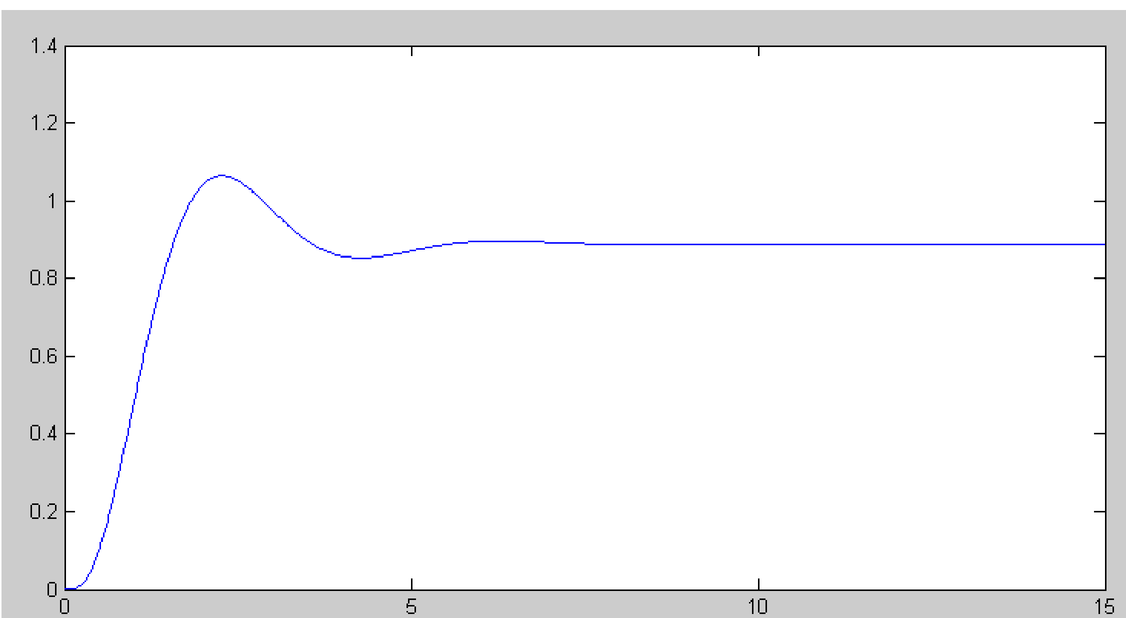
$$K_p = 8.045$$

The steady-state error for a step input.

$$E_{step} = \frac{1}{K_p+1} = 0.1106$$

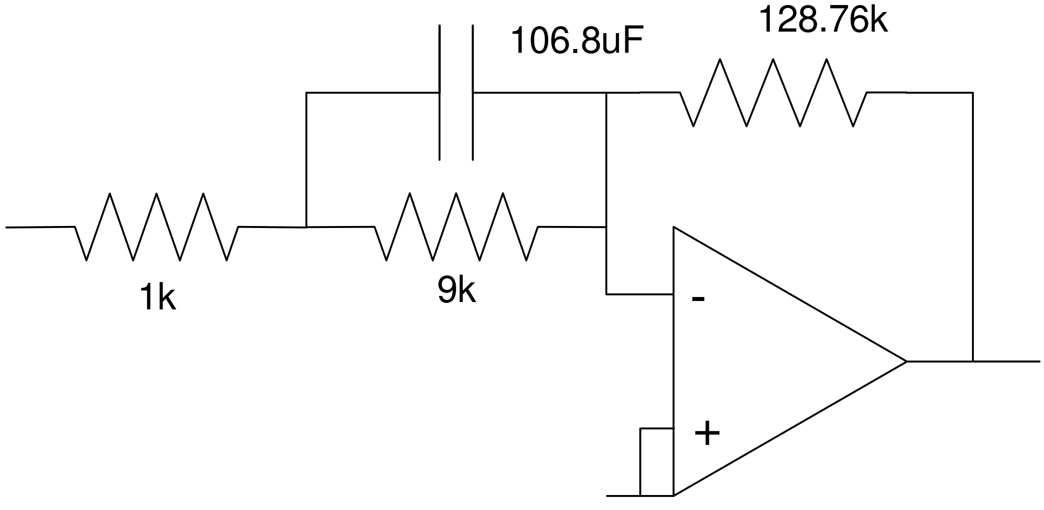
Check your design in Matlab or Simulink or VisSim

```
>> GK = zpk([],[-0.1617,-10.4,-2.719,-5.05],1.4427);
>> s = -0.7829 + j*1.5658;
>> evalfr(GK,s)
ans =  -0.0078 + 0.0000i

>> k = 1 / abs(ans)
k =  128.7637

>> Gcl = minreal(GK*k / (1+GK*k));
>> eig(Gcl)

  -0.7829 + 1.5658i
  -0.7829 - 1.5658i
  -6.9286
  -9.8363

>> t = [0:0.01:15]';
>> y = step(Gcl, t);
>> plot(t,y);
```



Give an op-amp circuit to implement K(s)

$$K(s) = 128.76\left(\frac{s+1.04}{s+10.4}\right)$$



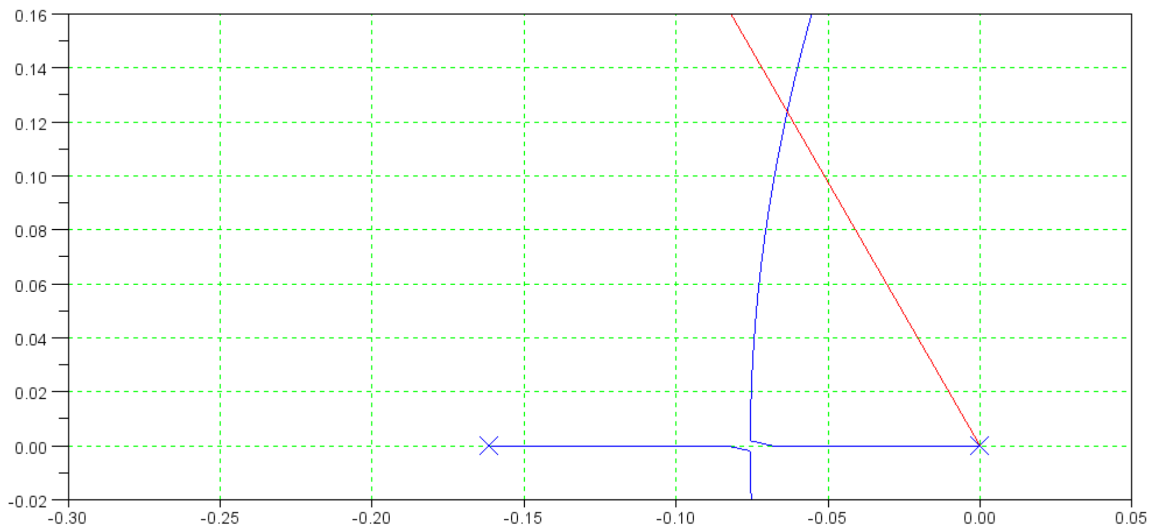
I Compensation

4) Design an I compensator, $K(s) = \frac{I}{s}$, which results in 20% overshoot for a step input.

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)$$

$$GK = \left(\frac{1.4427k}{s(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)$$

Sketch the root locus along with the damping line



$$s = -0.0630 + j0.1260$$

At this point

$$\left(\frac{1.4427k}{s(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)_{s=-0.0630+j0.1260} = 4.8973\angle 180^0$$

$$k = \frac{1}{4.8973} = 0.2042$$

$$K(s) = \left(\frac{0.2042}{s} \right)$$

The closed-loop dominant pole(s)

$$s = -0.0630 + j0.1260$$

The 2% settling time,

$$T_s = \frac{4}{0.0630} = 63.49 \sec$$

The error constant, Kp, and

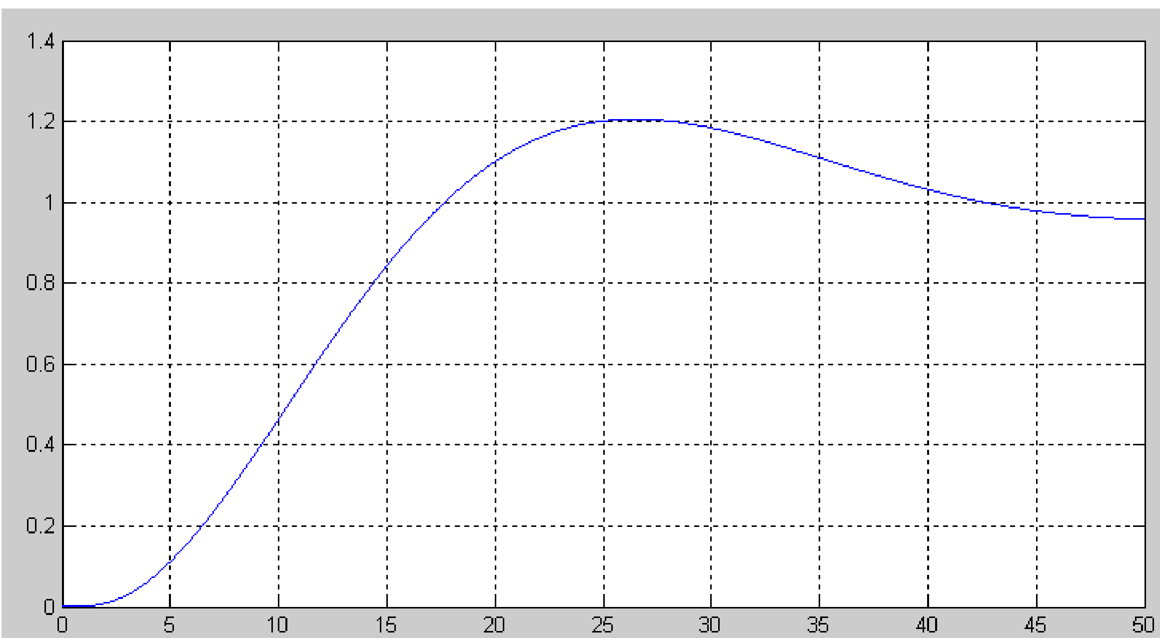
Kp = infinity (type-1 system)

The steady-state error for a step input.

E(step) = 0 (type-1 system)

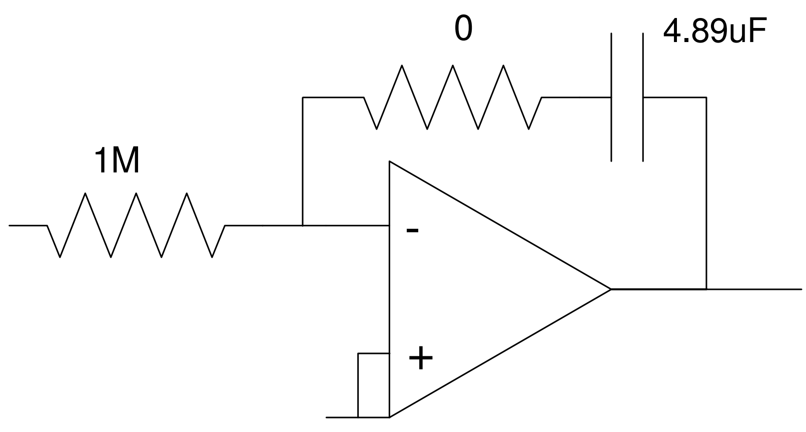
Check your design in Matlab or Simulink or VisSim

```
>> GK = zpk([],[0,-0.1617,-1.04,-2.719,-5.05],1.4427);
>> s = -0.0630 + j*0.1260;
>> evalfr(GK,s)

ans =  -4.8968 + 0.0004i

>> k = 1 / abs(ans)

k =    0.2042

>> Gcl = minreal(GK*k / (1+GK*k));
>> zpk(Gcl)

                       0.29462
------------------------------------------------------
(s+1.085) (s+2.708) (s+5.051) (s^2 + 0.126s + 0.01984)

>> eig(Gcl)

  -0.0630 + 0.1260i
  -0.0630 - 0.1260i
  -1.0854
  -2.7081
  -5.0513

>> t = [0:0.01:50]';
>> y = step(Gcl, t);
>> plot(t,y);
```



Give an op-amp circuit to implement K(s)

$$K(s) = \left(\frac{0.2042}{s}\right)$$



PI Compensation

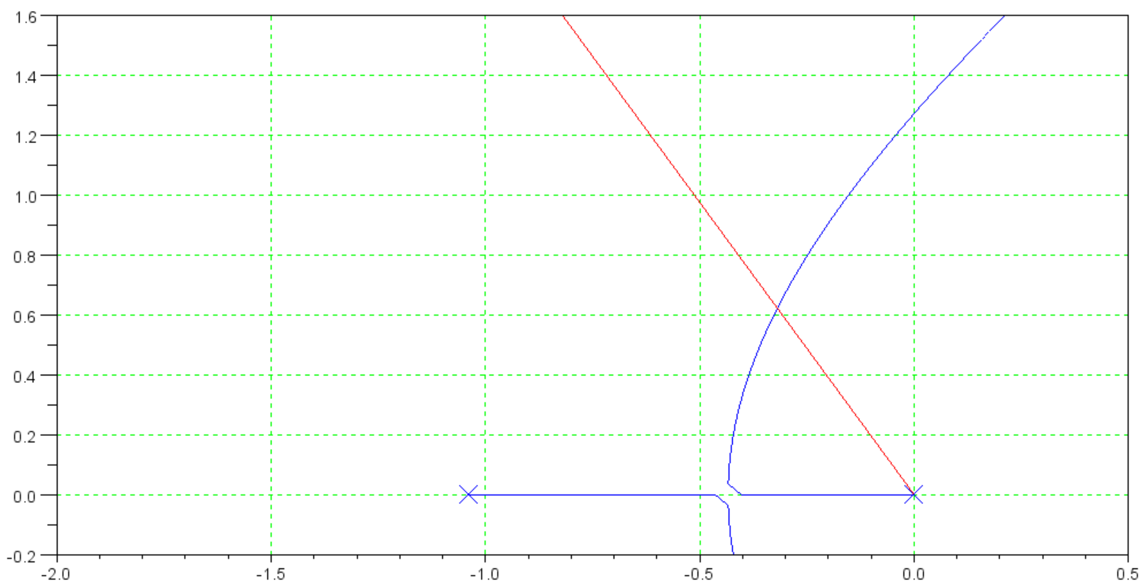
5) Design a PI compensator, $K(s) = k(\frac{s+a}{s})$, which results in 20% overshoot for a step input.

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)$$

$$K(s) = k\left(\frac{s+0.1617}{s}\right)$$

$$GK = \left(\frac{1.4427k}{s(s+1.04)(s+2.719)(s+5.05)}\right)$$

Sketch the root locus along with the 0.4559 damping line



$$s = -0.3159 + j0.6318$$

$$\left(\frac{1.4427k}{s(s+1.04)(s+2.719)(s+5.05)}\right)_{s=-0.3159+j0.6318} = 0.1791\angle 180^0$$

$$k = \frac{1}{0.1791} = 5.5836$$

$$K(s) = 5.5836\left(\frac{s+0.1617}{s}\right)$$

The closed-loop dominant pole(s)

$$s = -0.3159 + j0.6318$$

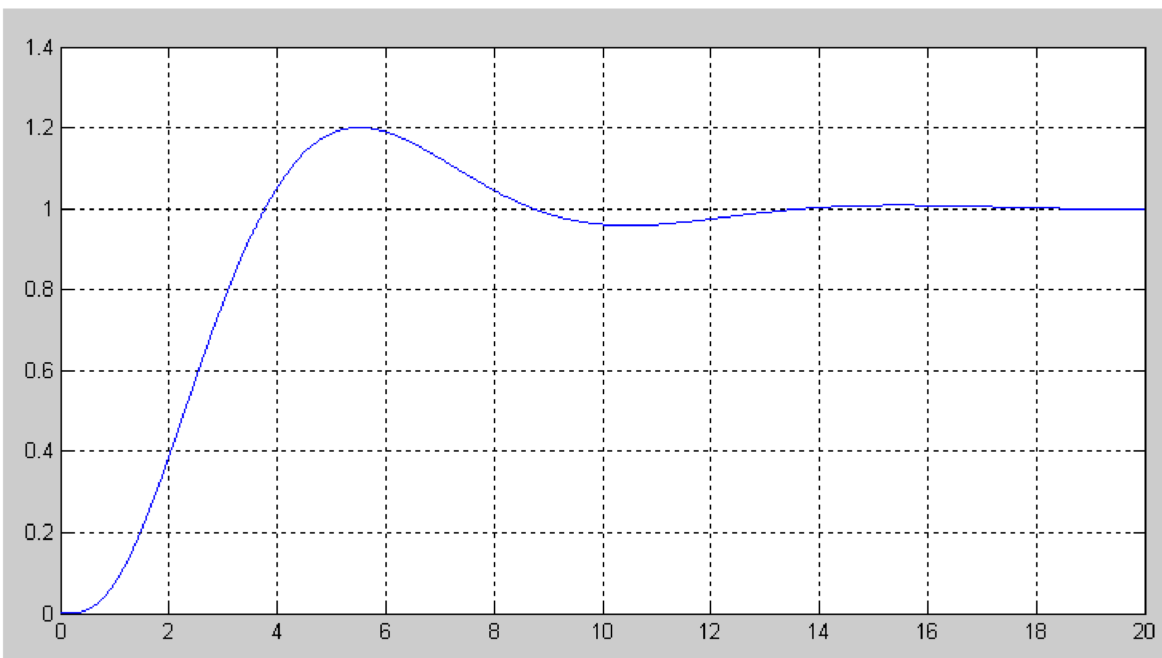
The 2% settling time,

$$T_s = \frac{4}{0.3159} = 12.67 \sec$$

No error for a step input (type-1 system)

Check your design in Matlab or Simulink or VisSim

```
>> GK = zpk([],[0,-1.04,-2.719,-5.05],1.4427);
>> s = -0.3159 + j*0.6318;
>> evalfr(GK,s)

ans =  -0.1791 + 0.0000i

>> k = 1 / abs(ans)

k =    5.5838

>> Gcl = minreal(GK*k / (1+GK*k));
>> eig(Gcl)

  -0.3159 + 0.6318i
  -0.3159 - 0.6318i
  -3.3329
  -4.8443

>> t = [0:0.01:20]';
>> y = step(Gcl, t);
>> plot(t,y);
>> grid on
>>
```



Give an op-amp circuit to implement K(s)

$$K(s) = 5.5836\left(\frac{s+0.1617}{s}\right)$$