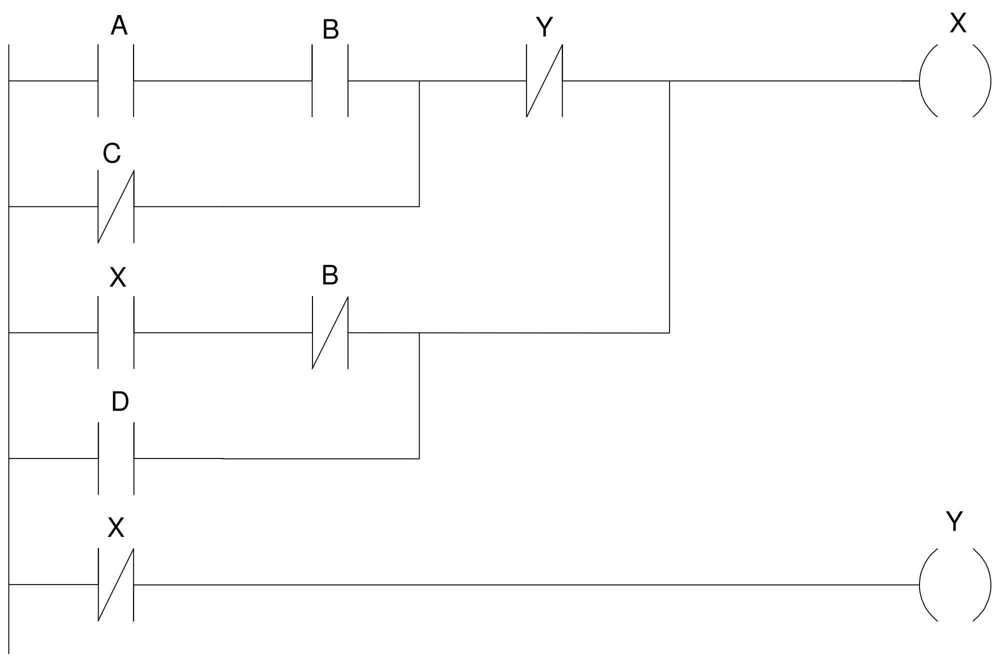
ECE 461/661 - Test #1: Name ____________________

Fall 2021

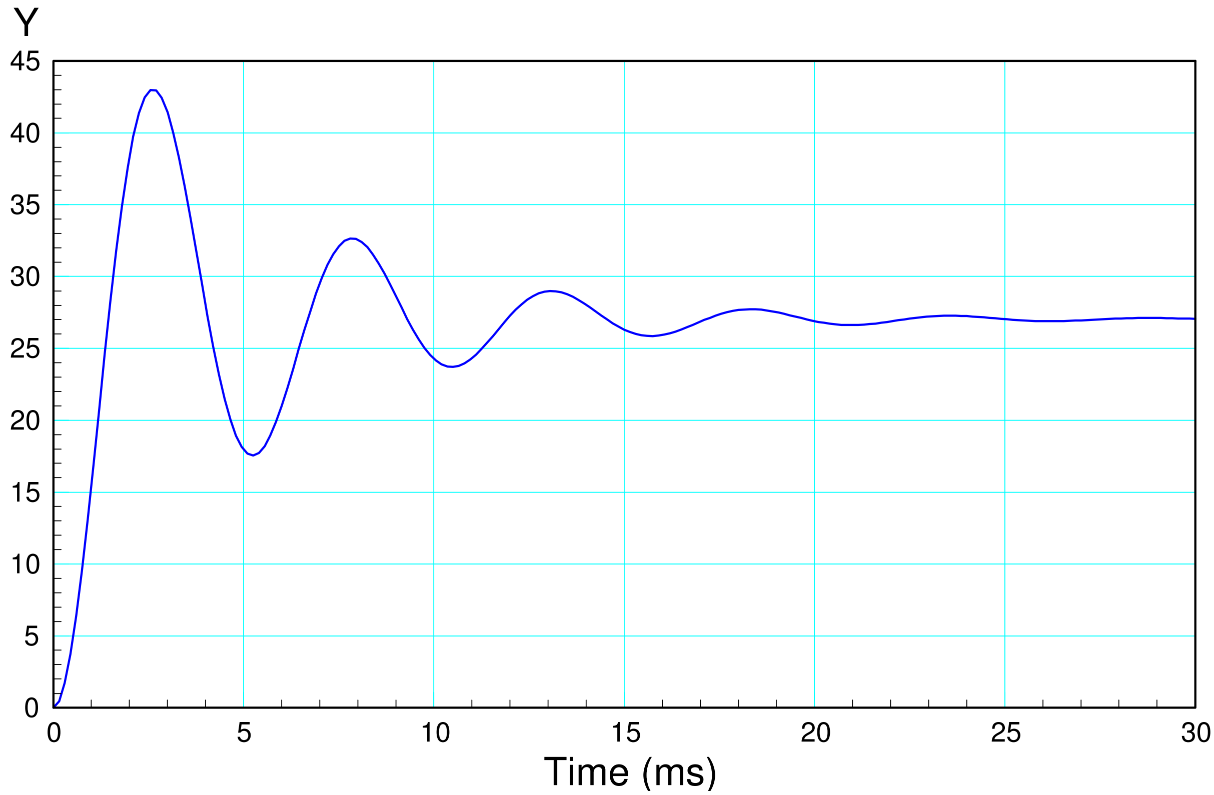
1) Determine the functions for X and Y according to the following ladder diagram. (you don't need to simplify)



$$X = \left(AB + \overline{C}\right)\overline{Y} + X\overline{B} + D$$

$$Y = \overline{X}$$

2) Give the transfer function for a system with the following response to a unit step input:



This has oscillations meaning it's a 2nd order system.

$$G(s) = \left(\frac{a}{(s+b+jc)(s+b-jc)} \right)$$

DC Gain = 27

$$G(s=0) = 27 = \left(\frac{a}{b^2+c^2} \right)$$

Frequency of oscillation (c)

3 cycles in 16ms

$$c = \left(\frac{3 \text{ cycles}}{16\text{ms}} \right) 2\pi = 1178 \frac{rad}{\text{sec}}$$

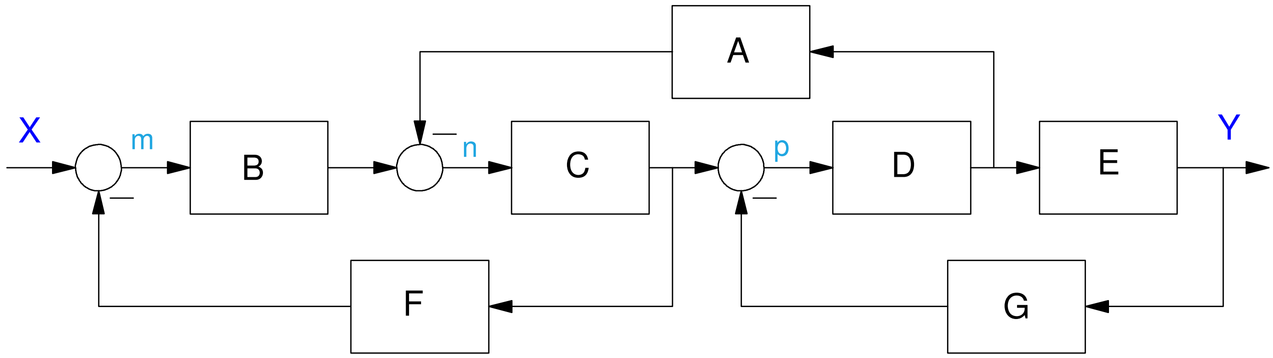
2% Settlimg time = 20ms (ballpark)

$$b = \frac{4}{T_s} = \frac{4}{20ms} = 200$$

so

$$G(s) \approx \left(\frac{27\left(200^2+1178^2\right)}{(s+200+j1178)(s+200-j1178)} \right)$$

3) Find the transfer funciton from X to Y



Shortcut

$$Y = \left(\frac{BCDE}{1+BCF+ACD+DEG}\right)X$$

Long Way

m = X - FCn

n = Bm - ADp

p = Cn - GY

Y = EDp

Solving

n = B (X - FCn) - ADp

(1 + BFC)n = BX - ADp

$$n = \left(\frac{BX-ADp}{1+BFC}\right)$$

$$p = C\left(\frac{BX-ADp}{1+BFC}\right) - GY$$

$$(1+BFC)p = CBX - CADp - (1+BFC)GY$$

$$(1+BFC+CAD)p = CBX - GY - BFCGY$$

$$p = \frac{CBX-GY1-BFCGY}{1+BFC+CAD)}$$

$$Y = ED\left(\frac{CBX-GY1-BFCGY}{1+BFC+CAD)}\right)$$

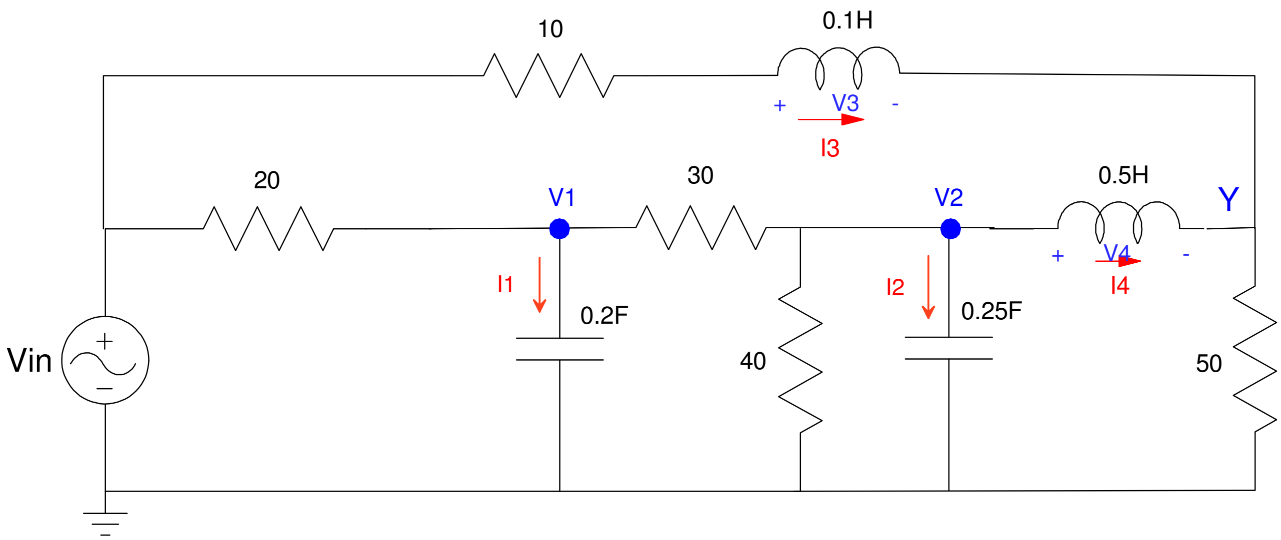
$$(1+BFC+CAD)Y = EDCBX - EDGY - EDBFCGY$$

$$(1+BFC+CAD+EDG+EDBFCG)Y = EDCBX$$

$$Y = \left(\frac{EDCB}{(1+BFC+CAD+EDG+EDBFCG)}\right)X = \left(\frac{EDCB}{(1+BFC)(1+EDG)+ACD}\right)X$$

4) For the following RLC circuit:

- Write the dynamics of this system as four compled differential equations in terms of {Vin, V1, V2, I3, I4}
- Express these dynamics in state-space form



$$I_1 = 0.2sV_1 = \left(\frac{V_{in}-V_1}{20}\right) - \left(\frac{V_1-V_2}{30}\right)$$

$$I_2 = 0.25sV_2 = \left(\frac{V_1-V_2}{30}\right) - \left(\frac{V_2}{40}\right) - I_4$$

$$V_3 = 0.1sI_3 = V_{in} - 10I_3 - 50(I_3 + I_4)$$

$$V_4 = 0.5sI_4 = V_2 - 50(I_3 + I_4)$$

Simplifying

$$sV_1 = 0.25V_{in} - 0.4167V_1 + 0.1667V_2$$

$$sV_2 = 0.133V_1 - 0.233V_2 - 4I_4$$

$$sI_3 = 10V_{in} - 600I_3 - 500I_4$$

$$sI_4 = 2V_2 - 100I_3 - 100I_4$$

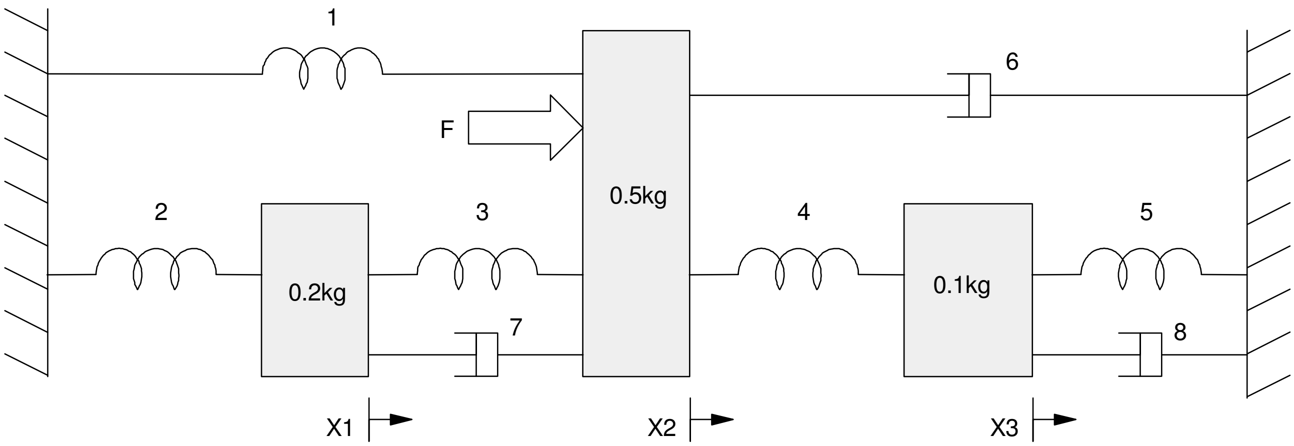
In matrix form

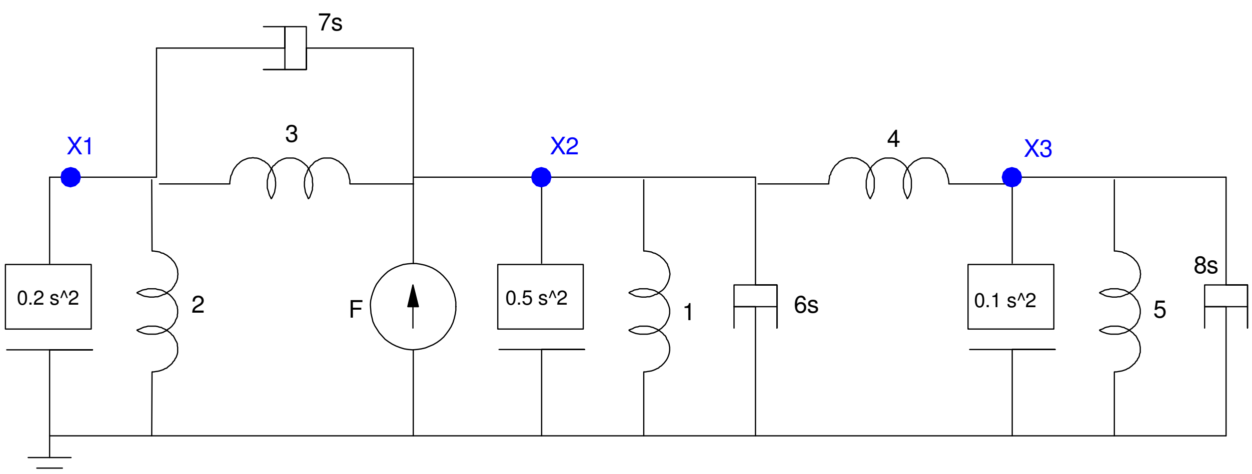
$$\begin{bmatrix} sV_1 \\ sV_2 \\ sI_3 \\ sI_4 \end{bmatrix} = \begin{bmatrix} -0.4167 & 0.1667 & 0 & 0 \\ 0.133 & -0.233 & 0 & -4 \\ 0 & 0 & -600 & -500 \\ 0 & 2 & -100 & -100 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0 \\ 10 \\ 0 \end{bmatrix} V_{in}$$

$$Y = 50(I_3 + I_4) = \begin{bmatrix} 0 & 0 & 50 & 50 \end{bmatrix} \bar{X} + [0]V_{in}$$

5) For the following mass-spring system

- Draw the circuit equivalent for the following mass-spring system
- Write the equations of motion (i.e. write the voltage node equations)





$$(0.2s^2 + 7s + 2 + 3)X_1 - (7s + 3)X_2 = 0$$

$$(0.5s^2 + 6s + 7s + 3 + 4 + 1)X_2 - (7s + 3)X_1 - (4)X_3 = F$$

$$(0.1s^2 + 8s + 5 + 4)X_3 - (4)X_2 = 0$$