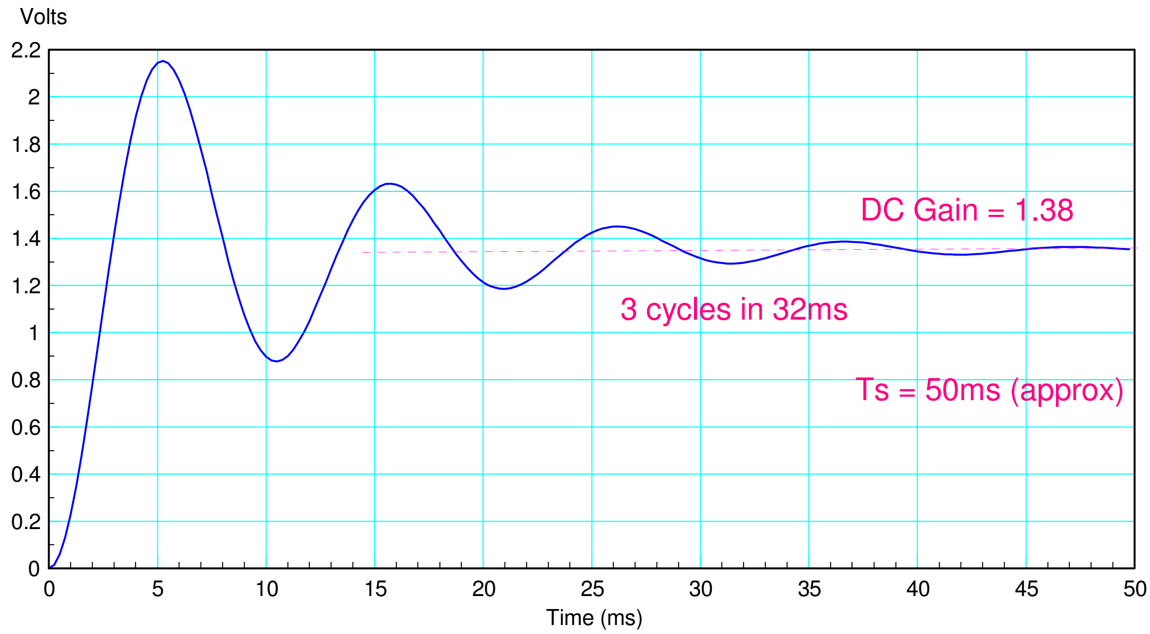
ECE 461 - Final: Name ____________________

Fall - 2023

1) Give the transfer function for a system with the following step response:



DC gain = 1.38

Ts = 50ms

$$\sigma \approx \frac{4}{0.05} = 80$$

3 cycles in 32ms

$$\omega_d \approx \left(\frac{3}{0.032}\right) 2\pi = 589 \frac{rad}{\sec}$$

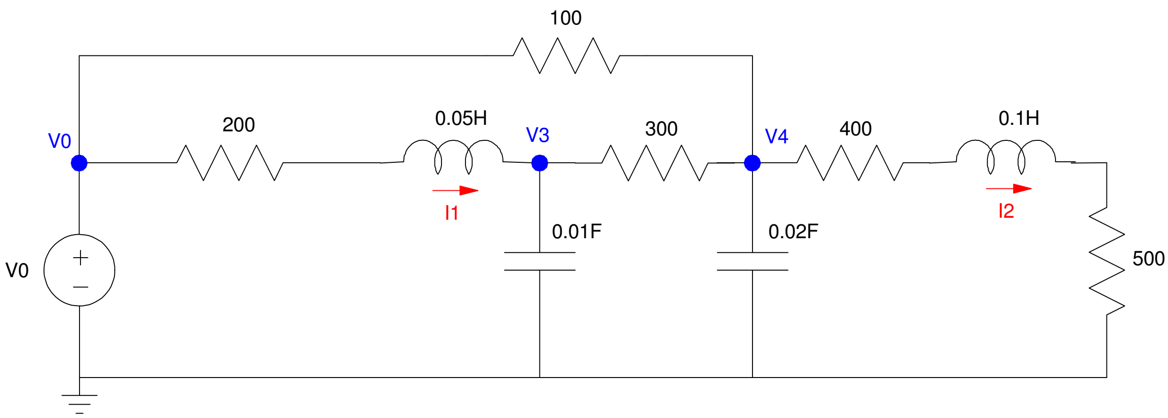
So

$$G(s) \approx \left(\frac{k}{(s+80+j580)(s+80-j580)}\right)$$

Pick 'k' to make the DC gain 1.38

$$\boxed{G(s) \approx \left(\frac{487{,}582}{(s+80+j580)(s+80-j580)}\right)}$$

2) Write the differential equations which describe the following circuit (i.e. write the N differential equations which correspond to the voltage node equations)



Inductors: V = L dI/dt

$$0.05sI_1 = V_0 - 200I_1 - V_3$$

$$0.1sI_2 = V_4 - 400I_2 - 500I_2$$

Capacitors: I = C dV/dt

$$0.01sV_3 = I_1 + \left(\frac{V_4 - V_3}{300}\right)$$

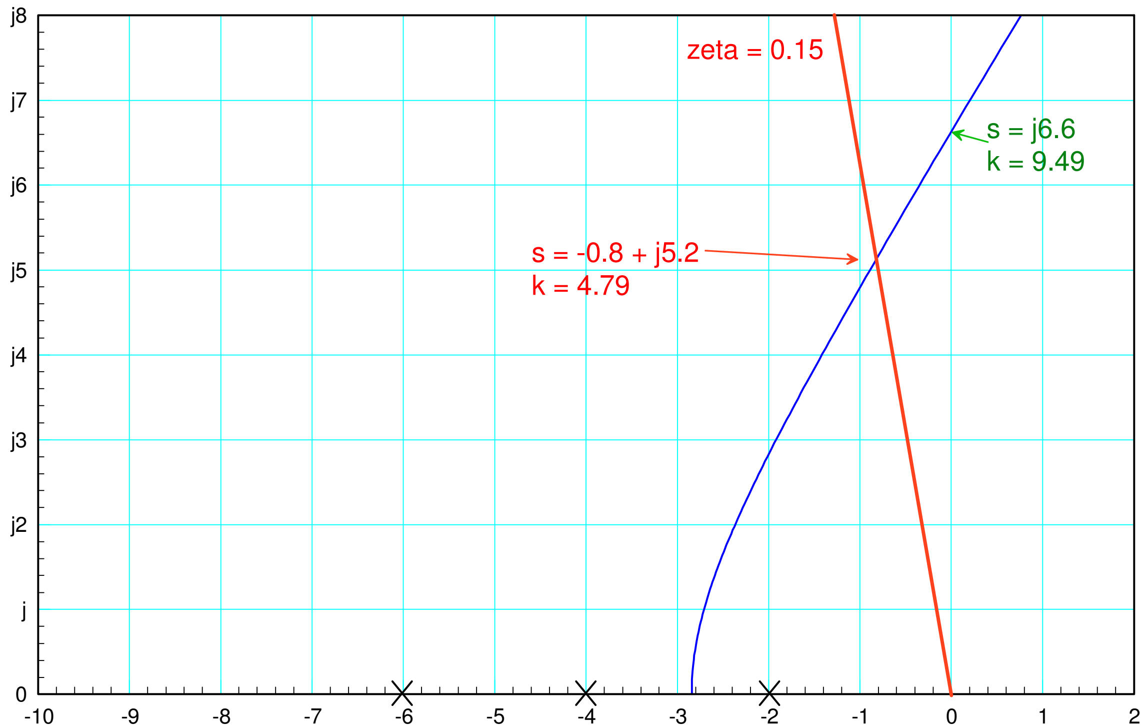
$$0.02sV_4 = \left(\frac{V_3 - V_4}{300}\right) + \left(\frac{V_0 - V_4}{100}\right) - I_2$$

3) Gain Compensation: The root locus for

$$G(s) = \left(\frac{50}{(s+2)(s+4)(s+6)}\right)$$

is shown below. Determine the following:

Maximum gain, k, for a stable closed-loop system	The jw crossing is about s = j6.6 GK(s) = -1 **k = 9.49**
k for a damping ratio of 0.15	**k = 4.79**
Closed-loop dominant pole(s) for a damping ratio of 0.15	**s = -0.8 + j5.2** angle of damping line is 81.73 deg
Closed-Loop DC gain for a damping ratio of 0.15	G*K = (1.0417)(4.79) = 4.99 DC gain = GK / (1+GK) = 0.833 **DC gain = 0.833**



4) Given the following stable system

$$G(s) = \left(\frac{40}{(s+2)(s+4)(s+10)} \right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- No error for a step input, and
- A closed-loop dominant pole at s = -2 + j3

Add a pole at s=0 to make this a type-1 system

Cancel the two slowest poles

Add a pole to put s = -2 + j3 on the root locus

$$K(s) = k \left(\frac{(s+2)(s+4)}{s(s+a)} \right)$$

$$GK = \left(\frac{40k}{s(s+10)(s+a)} \right)$$

Evaluate what we know

$$\left(\frac{40}{s(s+10)} \right)_{s=-2+j3} = 1.298 \angle -144.246^0$$

To make the angles add up to 180 degrees

$$\angle(s+a) = 35.754^0$$

$$a = 2 + \frac{3}{\tan\left(35.754^0\right)} = 6.167$$

Evaluate what we now know

$$\left(\frac{40}{s(s+6.167)(s+10)} \right)_{s=-2+j3} = 0.253 \angle 180^0$$

Pick k to make the gain one

$$k = \frac{1}{0.253} = 3.954$$

so

$$\boxed{K(s) = 3.954 \left(\frac{(s+2)(s+4)}{s(s+6.167)} \right)}$$

(there are other valid solutions)

5) Given the following stable system

$$G(s) = \left(\frac{40}{(s+2)(s+4)(s+7)}\right)$$

Determine a digital compensator, K(z), which results in the closed-loop system having

- No error for a step input,
- A closed-loop dominant pole at s = -2 + j3 (z = 0.78 + j0.24), and
- A sampling rate of T = 0.1

Find the form of K(z)

Add a pole at z = +1 to make this a type-1 system

Cancel the poles at s = {-2, -4}

$$z = e^{sT} = \{0.8187, 0.6703\}$$

$$K(z) = k\left(\frac{(z-0.8187)(z-0.6703)}{(z-1)(z-a)}\right)$$

and the open-loop system is (sample and hold modeled as a 1/2 sample delay)

$$G(s) \cdot delay \cdot K(z)$$

$$\left(\frac{40}{(s+2)(s+4)(s+6)}\right) \cdot e^{-0.05s} \cdot k\left(\frac{(z-0.8187)(z-0.6703)}{(z-1)(z-a)}\right)$$

Evaluate what we know at s = -2 + j3 (z = 0.78 + j0.24)

$$\left(\frac{40}{(s+2)(s+4)(s+6)}\right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.8187)(z-0.6703)}{(z-1)}\right) = 0.1404\angle -154.23^0$$

To make the angles add up to 180 degrees

$$\angle(z-a) = 25.7696^0$$

$$a = 0.7822 - \left(\frac{0.2420}{\tan\left(25.7696^0\right)}\right) = 0.2810$$

Evaluate what we now know

$$\left(\frac{40}{(s+2)(s+4)(s+6)}\right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.8187)(z-0.6703)}{(z-1)(z-0.2810)}\right) = 0.2522\angle 180^0$$

To make the gain one

$$k = \frac{1}{0.2522} = 3.9646$$

and

$$\boxed{K(s) = 3.9646\left(\frac{(z-0.8187)(z-0.6703)}{(z-1)(z-0.2810)}\right)}$$

6) Given the following stable system

$$G(s) = \left(\frac{40}{(s+2)(s+4)(s+7)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- A closed-loop DC gain of 1.000 (i.e. no error for a step input),
- A 0dB gain frequency of 2 rad/sec, and
- A phase margin of 25 degrees

Pick K(s)

Add a pole at s = 0

Cancel the poles at s = {-2, -4}

Add a pole so that GK(j2) = $1\angle -155^0$

$$K(s) = k\left(\frac{(s+2)(s+4)}{s(s+a)}\right)$$

$$GK = \left(\frac{40k}{s(s+7)(s+a)}\right)$$

Evaluate what we knot at s = j2 (the design point)

$$\left(\frac{40}{s(s+7)}\right)_{s=j2} = 2.7472\angle -105.9494^0$$

To make the angle add up to -155 degrees

$$\angle(s+a) = 49.0546^0$$

$$a = \left|\frac{2}{\tan\left(49.0546^0\right)}\right| = 1.7352$$

Evaluate we now know

$$\left(\frac{40}{s(s+1.7352)(s+7)}\right)_{s=j2} = 1.0375\angle -155^0$$

Pick k to make the gain 1.000

$$k = \frac{1}{1.0375} = 0.9638$$

so

$$\boxed{K(s) = 0.9638\left(\frac{(s+2)(s+4)}{s(s+1.7352)}\right)}$$