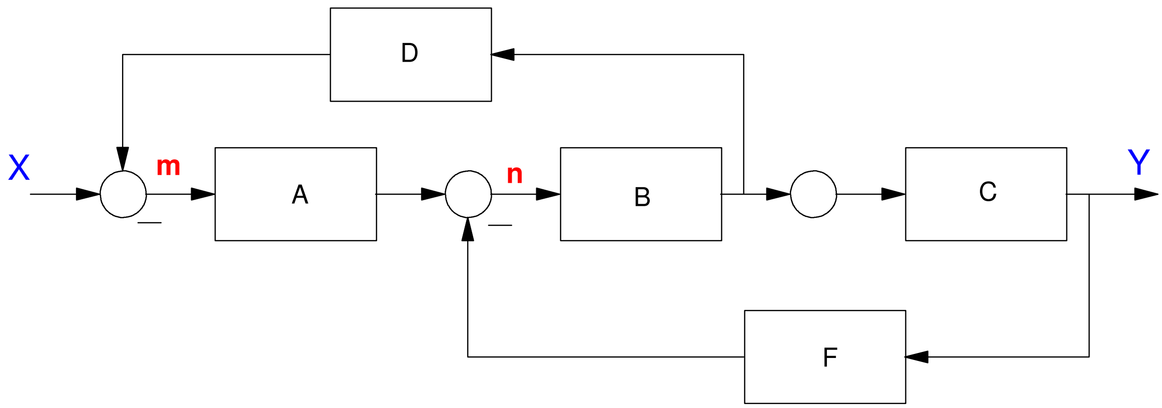
Homework #5: ECE 461/661

Block Diagrams, Canonical Forms, Electrical Circuits. Due Monday, September 18th

Block Diagrams

1) Determine the transfer function from X to Y



Shortcut

$$Y = \left(\frac{ABC}{1+ABD+BCF}\right)X$$

Long Way

$$m = X - DBn$$

$$n = Am - FY$$

$$Y = CBn$$

Substitute and do some algebra

$$n = A(X - DBn) - FY$$

$$(1 + ADB)n = AX - FY$$

$$n = \left(\frac{AX-FY}{1+ADB}\right)$$

$$Y = CBn = CB\left(\frac{AX-FY}{1+ADB}\right)$$

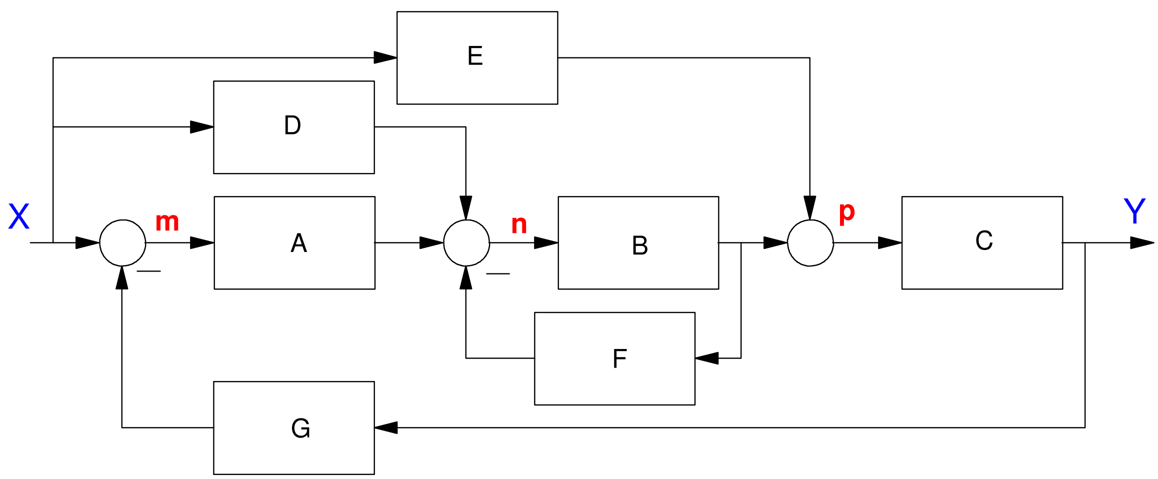
$$(1 + ADB)Y = CB(AX - FY)$$

$$(1 + ADB + CBF)Y = CBAX$$

$$Y = \left(\frac{CBA}{1+ADB+CBF}\right)X$$

same answer

2) Determine the transfer funciton from X to Y



Shortcut

$$Y = \left(\frac{EC+DBC+ABC}{1+BF+ABCG}\right)X$$

Long Way

$$m = X - GY$$

$$n = Am + DX - FBn$$

$$p = Bn + EX$$

$$Y = Cp$$

Substituting and doing some algebra

$$(1+FB)n = A(X-GY) + DX$$

$$n = \frac{AX-AGY+DX}{(1+FB)}$$

$$p = Bn + EX = B\left(\frac{AX-AGY+DX}{(1+FB)}\right) + EX$$

$$Y = Cp = C\left(B\left(\frac{A(X-GY)+DX}{(1+FB)}\right) + EX\right)$$

$$(1+FB)Y = CB(AX-AGY+DX) + (1+FB)CEX$$

$$(1+FB+CBAG)Y = (CBA+CBD+CE+FBCE)X$$

$$Y = \left(\frac{CBA+CBD+CE+FBCE}{1+FB+CBAG}\right)X$$

Almost the same, but there's an extra term in the numberator (FBCE). The shortcut method missed this term.

Canonical Forms

3) Give two different state-space models that produce the following transfer function

$$Y = \left(\frac{2s+20}{(s+1)(s+3)(s+5)+10}\right) U$$

Controller Form: Multiply out

$$Y = \left(\frac{2s+20}{s^3+9s^2+23s+25}\right) U$$

By inspection (controller form)

$$sX = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -25 & -23 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 20 & 2 & 0 \end{bmatrix} X + [0] U$$

Observer form:

$$sX = \begin{bmatrix} 0 & 0 & -25 \\ 1 & 0 & -23 \\ 0 & 1 & -9 \end{bmatrix} X + \begin{bmatrix} 20 \\ 2 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X + [0] U$$

The roots are complex, so cascade and Jordan form are a little trickier. Keep the complex poles together creating a 2x2 block

Cascade:

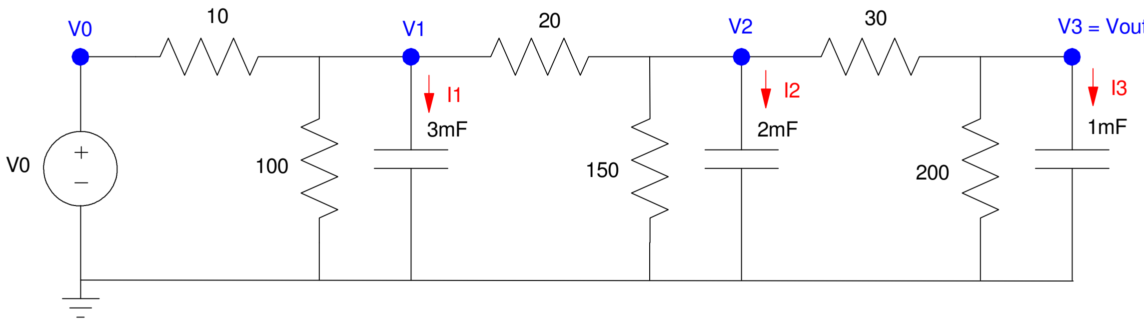
$$\left(\frac{2}{s+5.7608}\right)\left(\frac{s+10}{s^2+3.2392s+4.3397}\right)$$

$$sX = \begin{bmatrix} -5.7608 & \vdots & 0 & 0 \\ \cdots & & \cdots & \cdots \\ 0 & \vdots & 0 & 1 \\ 2 & \vdots & -4.3397 & -3.2392 \end{bmatrix} X + \begin{bmatrix} 1 \\ \cdots \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & \vdots & 10 & 1 \end{bmatrix} X + [0] U$$

Electrical Ciruits

4) Using state-space methods, find the transfer function from V0 to V3



$$I_1 = C_1 s V_1 = \left(\frac{V_0 - V_1}{10}\right) + \left(\frac{V_2 - V_1}{20}\right) + \left(\frac{0 - V_1}{100}\right)$$

$$I_2 = C_2 s V_2 = \left(\frac{V_1 - V_2}{20}\right) + \left(\frac{V_3 - V_2}{30}\right) + \left(\frac{0 - V_2}{150}\right)$$

$$I_3 = C_3 s V_3 = \left(\frac{V_2 - V_3}{30}\right) + \left(\frac{0 - V_3}{200}\right)$$

Plugging in C1, C2, and C3, then simplifying

$$sV_1 = 33.33V_0 - 53.33V_1 + 16.67V_2$$

$$sV_2 = 25V_1 - 45V_2 + 16.67V_3$$

$$sV_3 = 33.33V_2 - 38.33V_3$$

Putting in state-space form

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -53.33 & 16.67 & 0 \\ 25 & -45 & 16.67 \\ 0 & 33.33 & -38.33 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 33.33 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$Y = V_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + [0]V_0$$

Use Matlab to find the transfer function

```
>> A = [-53.33,16.67,0;25,-45,16.67;0,33.33,-38.33]

  -53.3300   16.6700         0
   25.0000  -45.0000   16.6700
         0   33.3300  -38.3300

>> B = [33.33;0;0]

   33.3300
         0
         0

>> C = [0,0,1];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

         27772.2225
----------------------------
(s+76.99) (s+46.8) (s+12.87)

>>
```

5) Using state-space methods, find the transfer function from V0 to V1

Just change the C matrix

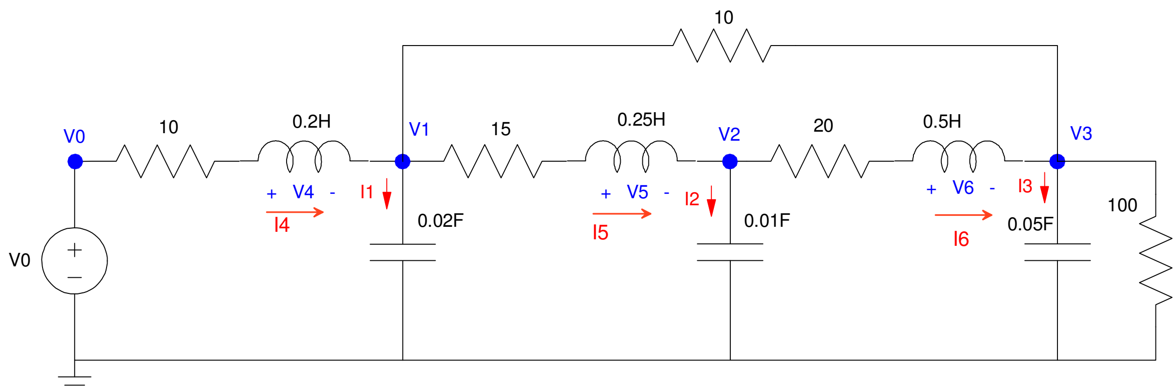
```
>> C = [1,0,0];
>> G = ss(A,B,C,D);
>> zpk(G)

 33.33 (s+65.47) (s+17.86)
----------------------------
(s+76.99) (s+46.8) (s+12.87)
```

Note: When you change what you're measuring

- The poles remain the same
- The zeros change

6) Express the dynamics for the following RLC circuit in state-space form.

- Find the transfr function from V0 to V3



Use the currents in the inductors and voltages across the resistors as the energy states

$$I_1 = 0.02sV_1 = I_4 - I_5 - \left(\frac{V_1 - V_3}{10}\right)$$

$$I_2 = 0.01sV_2 = I_5 - I_6$$

$$I_3 = 0.05sV_3 = I_6 + \left(\frac{V_1 - V_3}{10}\right) - \left(\frac{V_3}{100}\right)$$

$$V_4 = 0.2sI_4 = V_0 - 10I_4 - V_1$$

$$V_5 = 0.25sI_5 = V_1 - 15I_5 - V_2$$

$$V_6 = 0.5sI_6 = V_2 - 20I_6 - V_3$$

Simplify

$$sV_1 = 50I_4 - 50I_5 - 5V_1 + 5V_3$$

$$sV_2 = 100I_5 - 100I_6$$

$$sV_3 = 20I_6 + 2V_1 - 2.2V_3$$

$$sI_4 = 5V_0 - 50I_4 - 5V_1$$

$$sI_5 = 4V_1 - 60I_5 - 4V_2$$

$$sI_6 = 2V_2 - 40I_6 - 2V_3$$

Place in state-space form

$$s\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 5 & 50 & -50 & 0 \\ 0 & 0 & 0 & 0 & 100 & -100 \\ 2 & 0 & -2.2 & 0 & 0 & 20 \\ -5 & 0 & 0 & -50 & 0 & 0 \\ 4 & -4 & 0 & 0 & -60 & 0 \\ 0 & 2 & -2 & 0 & 0 & -40 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 0 \end{bmatrix} V_0$$

Use Matlab to find the transfer function to V3

```
>> A = [-5,0,5,50,-50,0;0,0,0,0,100,-100;2,0,-2.2,0,0,20];
>> A = [A;-5,0,0,-50,0,0;4,-4,0,0,-60,0;0,2,-2,0,0,-40]

   -5.0000         0    5.0000   50.0000  -50.0000         0
         0         0         0         0  100.0000 -100.0000
    2.0000         0   -2.2000         0         0   20.0000
   -5.0000         0         0  -50.0000         0         0
    4.0000   -4.0000         0         0  -60.0000         0
         0    2.0000   -2.0000         0         0  -40.0000

>> B = [0;0;0;5;0;0]

     0
     0
     0
     5
     0
     0

>> C = [0,0,1,0,0,0]

     0     0     1     0     0     0

>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)


                500 (s+60) (s^2 + 40s + 600)
-------------------------------------------------------------
(s+1.053) (s+12.14) (s+39.17) (s+52.64) (s^2 + 52.2s + 804.4)

>>
```

7) Assume V0 = 0. Specify the initial conditions so that V3(0) = 1V and

- The transients decay as slow as possible
- The transients decay as fast as possible

This is an eigenvector & eienvalue problem

```
>> [m,n] = eig(A)

m =

  -0.4337      0.5456     -0.4629 - 0.1752i  -0.4629 + 0.1752i   0.1813      0.8719
  -0.5913      0.7890      0.8425             0.8425            -0.7575     -0.0667
  -0.6784     -0.2616     -0.0464 + 0.0489i  -0.0464 - 0.0489i  -0.0515     -0.1305
   0.0443     -0.0720      0.0937 - 0.0068i   0.0937 + 0.0068i   0.3429     -0.4026
   0.0107     -0.0203     -0.1452 + 0.0269i  -0.1452 - 0.0269i   0.5105      0.1802
   0.0045      0.0754      0.0747 - 0.0667i   0.0747 + 0.0667i   0.1117      0.1541


n =

  -1.0532          0              0                  0                 0            0
        0   -12.1359              0                  0                 0            0
        0          0   -26.0981 +11.1017i            0                 0            0
        0          0              0       -26.0981 -11.1017i           0            0
        0          0              0                  0          -52.6436            0
        0          0              0                  0                 0     -39.1711

>>
```

The slow mode is the 1st eigenvector (eigenvalue = -1.0532)

```
>> X0 = m(:,1)

   -0.4337
   -0.5913
   -0.6784
    0.0443
    0.0107
    0.0045

>> X0 = X0 / X0(3)

V1    0.6393
V2    0.8717
V3    1.0000
I4   -0.0653
I5   -0.0158
I6   -0.0066
```

The fast mode is the 5th eigenvector (eigenvalue = -52.64)

```
>> X0 = m(:,5)

    0.1813
   -0.7575
   -0.0515
    0.3429
    0.5105
    0.1117

>> X0 = X0 / X0(3)

V1   -3.5222
V2   14.7180
V3    1.0000
I4   -6.6618
I5   -9.9181
I6   -2.1700
```