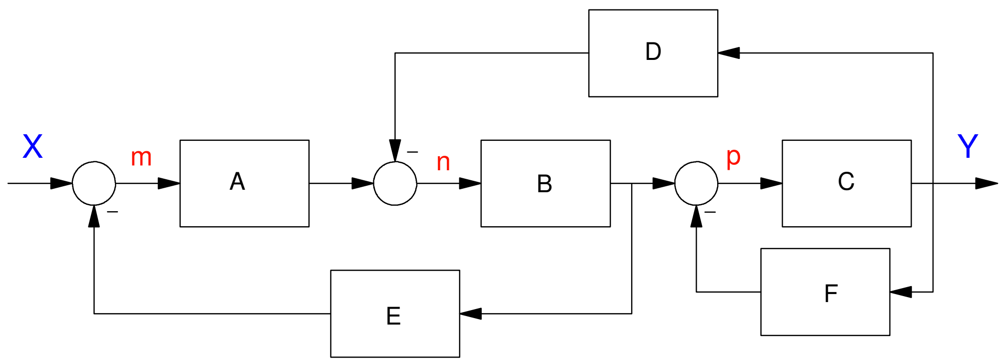
Homework #4: ECE 461/661

Block Diagrams, Canonical Forms, Electrical Circuits. Due Monday, September 20th

Block Diagrams

1) Determine the transfer function from X to Y



Option 1:

$$G = \left(\frac{\sum \text{(gains from X to Y)}}{1 + \sum \text{(loop gains)}} \right) = \left(\frac{ABC}{1+ABE+BCD+CF} \right)$$

Option 2: Write out equations

$$m = X - EBn$$

$$n = Am - DY$$

$$p = Bn - FY$$

$$Y = Cp$$

Simplify (get m, n, p to drop out)

$$m = X - EB(Am - DY)$$

$$(1 + EBA)m = X + EBDY$$

$$m = \left(\frac{1}{1+EBA}\right)X + \left(\frac{EBD}{1+EBA}\right)Y$$

-

$$p = Bn - FY$$

$$p = B(Am - DY) - FY$$

$$p = B\left(A\left(\left(\frac{1}{1+EBA}\right)X + \left(\frac{EBD}{1+EBA}\right)Y\right) - DY\right) - FY$$

$$Y = Cp = C\left(B\left(A\left(\left(\frac{1}{1+EBA}\right)X + \left(\frac{EBD}{1+EBA}\right)Y\right) - DY\right) - FY\right)$$

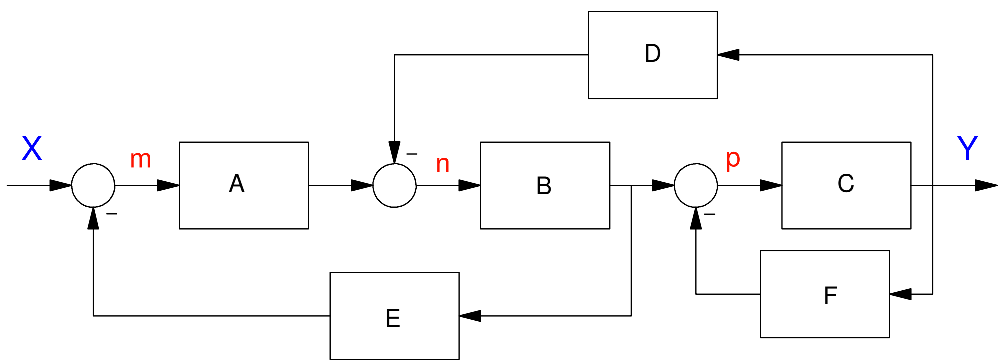
$$Y = \left(\frac{CBA}{1+EBA}\right)X + \left(\frac{CBAEBD}{1+EBA}\right)Y - CBDY - CFY$$

$$(1+EBA)Y = (CBA)X + (CBAEBD)Y - (1+EBA)CBDY - (1+EBA)CFY$$

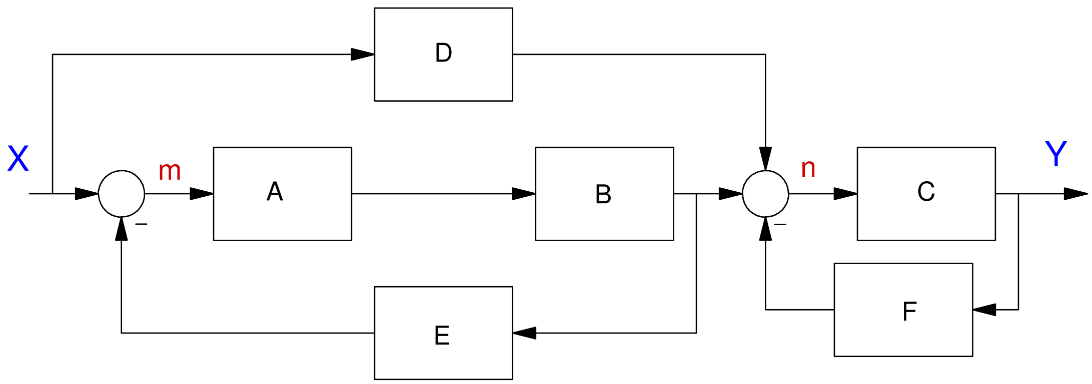
$$(1+EBA+CBD+CF+EBACF)Y = (CBA)X$$

$$Y = \left(\frac{CBA}{1+EBA+CBD+CF+EBACF}\right)X$$

The shortcut missed the EBACF term



2) Determine the transfer funciton from X to Y



Shortcut - note that this is two systems cascaded

$$Y = \left(\frac{AB}{1+ABE}\right)\left(\frac{C}{1+CF}\right) + \left(\frac{DC}{1+CF}\right)$$

Long Way

$$m = X - EBAm$$

$$n = DX + BAm - FY$$

$$Y = Cn$$

simplify

$$m = \left(\frac{1}{1+EBA}\right)X$$

$$n = DX + BAm - FY$$

$$n = DX + BA\left(\left(\frac{1}{1+EBA}\right)X\right) - FY$$

$$Y = Cn$$

$$Y = C\left(DX + BA\left(\left(\frac{1}{1+EBA}\right)X\right) - FY\right)$$

$$(1+EBA)Y = (1+EBA)CDX + CBAX - (1+EBA)CFY$$

$$(1+EBA+CF+EBACF)Y = (CD+EBACD+CBA)X$$

$$Y = \left(\frac{CD+EBACD+CBA}{(1+EBA+CF+EBACF)}\right)X$$

whichis the same as the shortcut

Canonical Forms

3) Give two different state-space models that produce the following transfer function

$$Y = \left(\frac{30}{(s+2)(s+3)(s+4)}\right) U$$

Cascade Form

$$sX = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -3 & 0 \\ 0 & 1 & -4 \end{bmatrix} X + \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X$$

Controller Form

$$Y = \left(\frac{30}{(s+2)(s+3)(s+4)}\right) U = \left(\frac{30}{s^3+14s^2+26s+24}\right) U$$

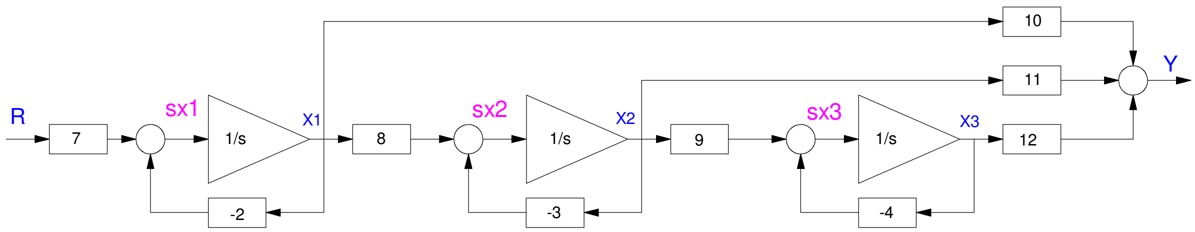
$$sX = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -14 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 30 & 0 & 0 \end{bmatrix} X$$

State-Space

4) Express the following system in state-space form

- Find the transfer function from R to Y



Define the output of each integrator to be a state

The derivative is the input to the integrators

$$sx_1 = 7R - 2x_1$$

$$sx_2 = 8x_1 - 3x_2$$

$$sx_3 = 9x_2 - 4x_3$$

$$y = 10x_1 + 11x_2 + 12x_3$$

Place this in matrix form (state-space form)

$$\begin{bmatrix} sx_1 \\ sx_2 \\ sx_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 8 & -3 & 0 \\ 0 & 9 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix} R$$

$$Y = \begin{bmatrix} 10 & 11 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]R$$

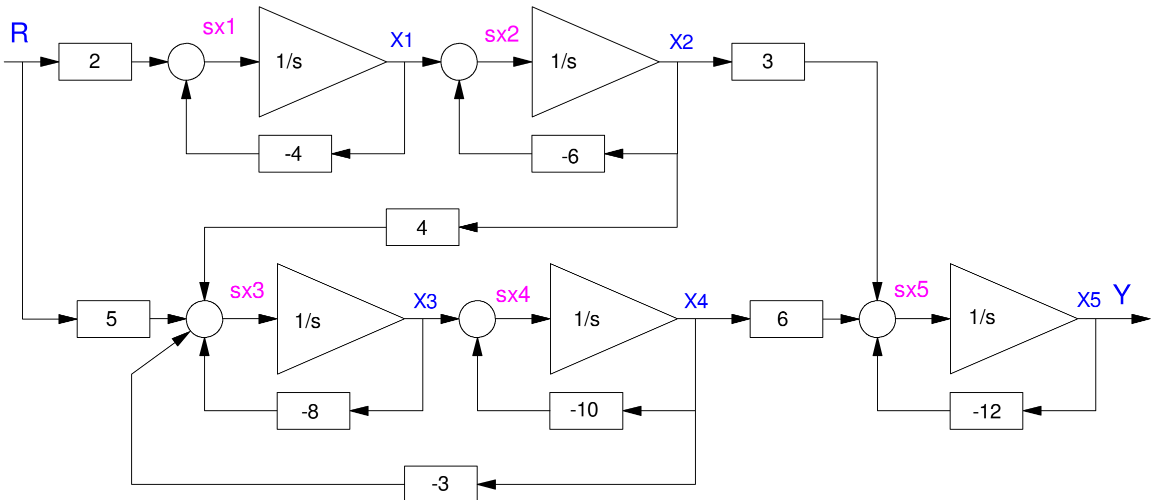
Put this into Matlab to find the transfer funciton

```
>> A = [-2,0,0;8,-3,0;0,9,-4]

    -2     0     0
     8    -3     0
     0     9    -4

>> B = [7;0;0]

     7
     0
     0

>> C = [10,11,12]

    10    11    12

>> D = 0

     0
```

```
>> G = ss(A,B,C,D);
>> tf(G)

70 s^2 + 1106 s + 9352
-----------------------
s^3 + 9 s^2 + 26 s + 24

>> zpk(G)

70 (s^2 + 15.8s + 133.6)
------------------------
   (s+4) (s+3) (s+2)

>>
```

5) Express the following system in state-space form

- Find the transfer function from R to Y



Define the states to be the output of the integrators

The input to the integrators are the derivatives of the states

$$sx_1 = 2R - 4x_1$$

$$sx_2 = x_1 - 6x_2$$

$$sx_3 = 5R + 4x_2 - 8x_3 - 3x_4$$

$$sx_4 = x_3 - 10x_4$$

$$sx_5 = 3x_2 + 6x_4 - 12x_5$$

$$y = x_5$$

Express in matrix form

$$\begin{bmatrix} sx_1 \\ sx_2 \\ sx_3 \\ sx_4 \\ sx_5 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 & 0 & 0 \\ 1 & -6 & 0 & 0 & 0 \\ 0 & 4 & -8 & -3 & 0 \\ 0 & 0 & 1 & -10 & 0 \\ 0 & 3 & 0 & 6 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 5 \\ 0 \\ 0 \end{bmatrix} R$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + [0]R$$

Solve using Matlab

```
>> A = [-4,0,0,0,0;1,-6,0,0,0;0,4,-8,-3,0;0,0,1,-10,0;0,3,0,6,-12]

    -4     0     0     0     0
     1    -6     0     0     0
     0     4    -8    -3     0
     0     0     1   -10     0
     0     3     0     6   -12

>> B = [2;0;5;0;0]

     2
     0
     5
     0
     0

>> C = [0,0,0,0,1]

     0     0     0     0     1

>> D = 0;
>> G = ss(A,B,C,D);
>> tf(G)

               36 s^2 + 408 s + 1266
---------------------------------------------------
s^5 + 40 s^4 + 623 s^3 + 4706 s^2 + 17136 s + 23904

>> zpk(G)


     36 (s^2 + 11.33s + 35.17)
-----------------------------------
(s+12) (s+6) (s+4) (s^2 + 18s + 83)

>>
```