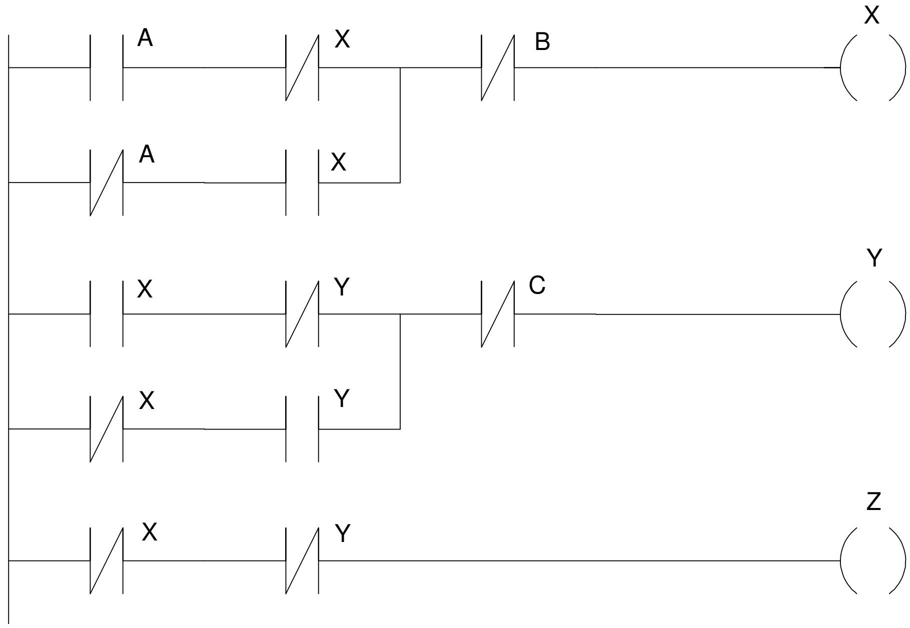
ECE 461/661 - Test #1: Name ____________________

Fall 2025

1) Determine the functions for X, Y, and Z according to the following ladder diagram. (you don't need to simplify)

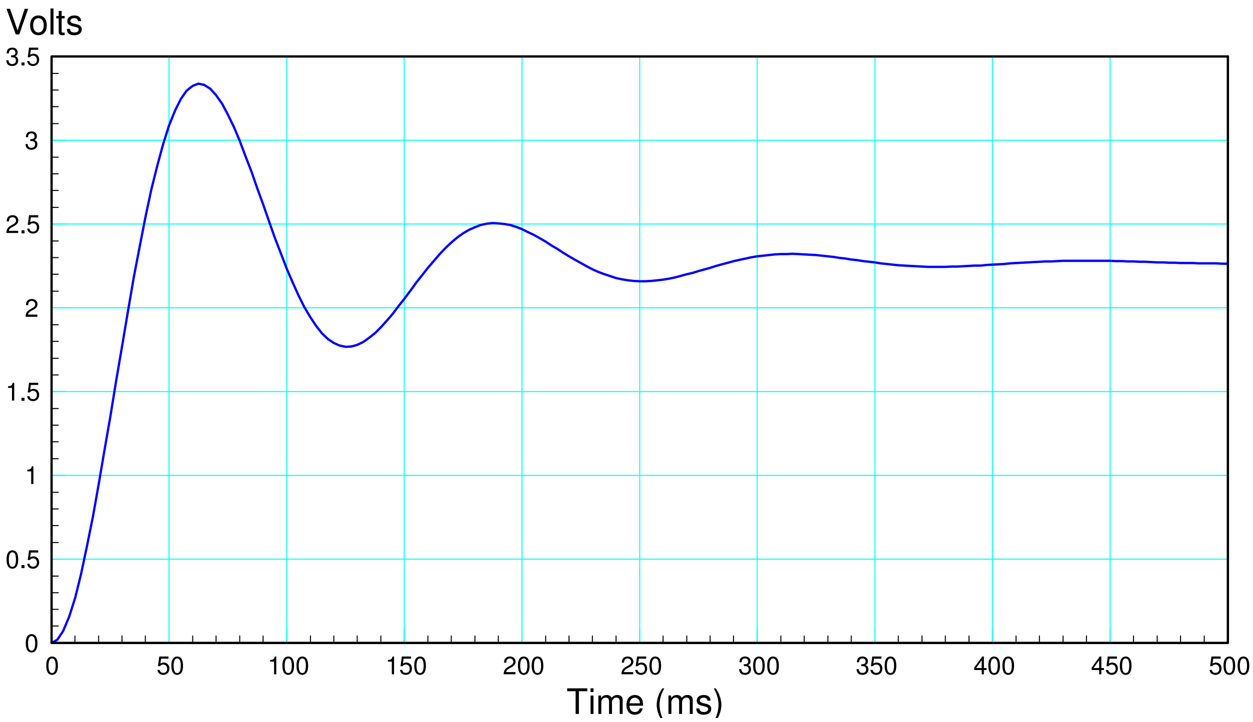


$$X = \left(A\overline{X} + \overline{A}X\right)\overline{B}$$

$$Y = \left(X\overline{Y} + \overline{X}Y\right)\overline{C}$$

$$Z = \overline{X}\,\overline{Y}$$

2) Give the transfer function for a system with the following response to a unit step input:



This is a 2nd-order system (it oscillates, meaning complex poles)

$$G(s) \approx \left(\frac{k}{(s+a+jb)(s+a-jb)}\right)$$

Ts = 350ms (approx)

$$a \approx \frac{4}{0.35} = 11.43$$

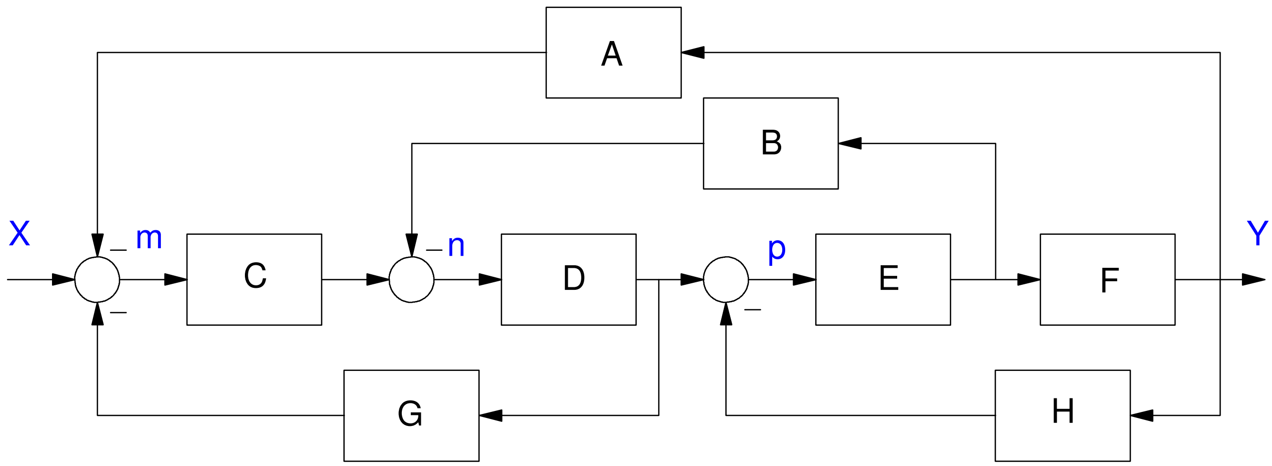
2 cycles in 250ms

$$b \approx \left(\frac{2\text{ cycles}}{0.25\text{s}}\right) 2\pi = 50.27 \frac{rad}{\text{sec}}$$

DC gain = 2.25

$$G(s) \approx \left(\frac{6002.5}{(s+11.43+j50.27)(s+11.43-j50.27)}\right)$$

3) Find the transfer funciton from X to Y



Short-cut

$$Y = \left(\frac{CDEF}{1+CDG+DEB+EFH+CDEFA}\right)X$$

Long method

$$m = X - AY - GDn$$

$$n = Cm - BEp$$

$$p = Dn - HY$$

$$Y = FEp$$

Doing some algebra

$$n = C(X - AY - GDn) - BEp$$

$$(1 + CGD)n = CX - CAY - BEp$$

$$n = \frac{CX-CAY-BEp}{(1+CGD)}$$

$$p = Dn - HY = D\left(\frac{CX-CAY-BEp}{(1+CGD)}\right) - HY$$

$$(1 + CGD)p = D(CX - CAY - BEp) - (1 + CGD)HY$$

$$(1 + CGD + DBE)p = D(CX - CAY) - (1 + CGD)HY$$

$$p = \frac{D(CX-CAY)-(1+CGD)HY}{(1+CGD+DBE)}$$

$$Y = FEp$$

$$Y = FE\left(\frac{D(CX - CAY) - (1 + CGD)HY}{(1 + CGD + DBE)}\right)$$

$$(1 + CGD + DBE)Y = FE(D(CX - CAY) - (1 + CGD)HY)$$

$$(1 + CGD + DBE + FEDCA + FEH + FECDGH)Y = FEDCX$$

$$Y = \left(\frac{FEDC}{1 + CGD + DBE + FEDCA + FEH + FECDGH}\right)X$$

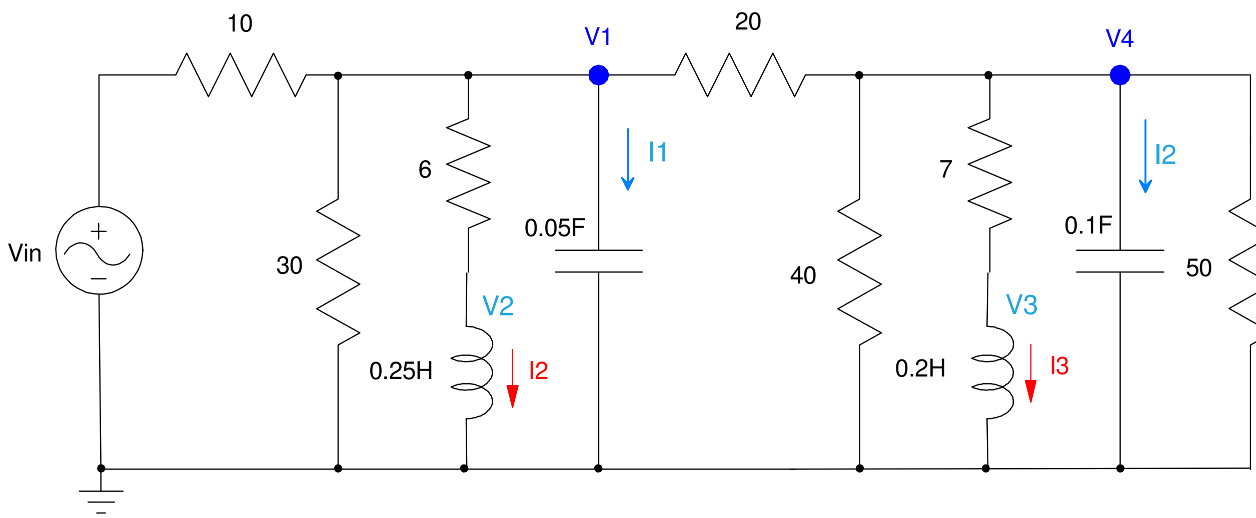
vs the shortcut

$$Y = \left(\frac{CDEF}{1 + CDG + DEB + EFH + CDEFA}\right)X$$

The shortcut missed one term (due to two systems cascaded)

4) For the following RLC circuit:

- Write the dynamics of this system as four compled differential equations in terms of {Vin, V1, I2, I3, V4}
- You don't need to solve or put in state-space form (that's a different problem on the test)



$$I_1 = 0.05sV_1 = \left(\frac{V_{on} - V_1}{10}\right) - \left(\frac{V_1}{30}\right) - I_2 - \left(\frac{V_1 - V_4}{20}\right)$$

$$I_4 = 0.1sV_4 = \left(\frac{V_1 - V_4}{20}\right) - \left(\frac{V_4}{40}\right) - I_3 - \left(\frac{V_4}{50}\right)$$

$$V_2 = 0.25sI_2 = V_1 - 6I_2$$

$$V_3 = 0.2sI_3 = V_4 - 7I_3$$

5) Assume the dynamics of an RLC circuit are:

$$0.5sI_1 = -2I_1 + (V_{in} - V_2) + 3I_3$$

$$0.1sV_2 = 0.4I_1 - \left(\frac{V_{in}-V_2}{6}\right) - 5I_3$$

$$0.2sI_3 = I_1 - 2I_3$$

$$Y = 7V_2 - 8I_1$$

Rewrite as

$$sI_1 = -4I_1 + 2V_{in} - 2V_2 + 6I_3$$

$$sV_2 = 4I_1 - \left(\frac{10}{6}\right)V_{in} + \left(\frac{10}{6}\right)V_2 - 50I_3$$

$$sI_3 = 5I_1 - 10I_3$$

- Give the state-space representation for the dynamics.

$$s\begin{bmatrix} I_1 \\ V_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -4 & -2 & 6 \\ 4 & \left(\frac{10}{6}\right) & -50 \\ 5 & 0 & -10 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \\ I_3 \end{bmatrix} + \begin{bmatrix} 2 \\ \left(\frac{-10}{6}\right) \\ 0 \end{bmatrix} V_{in}$$

$$Y = \begin{bmatrix} -8 & 7 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \\ I_3 \end{bmatrix} + [0]V_{in}$$