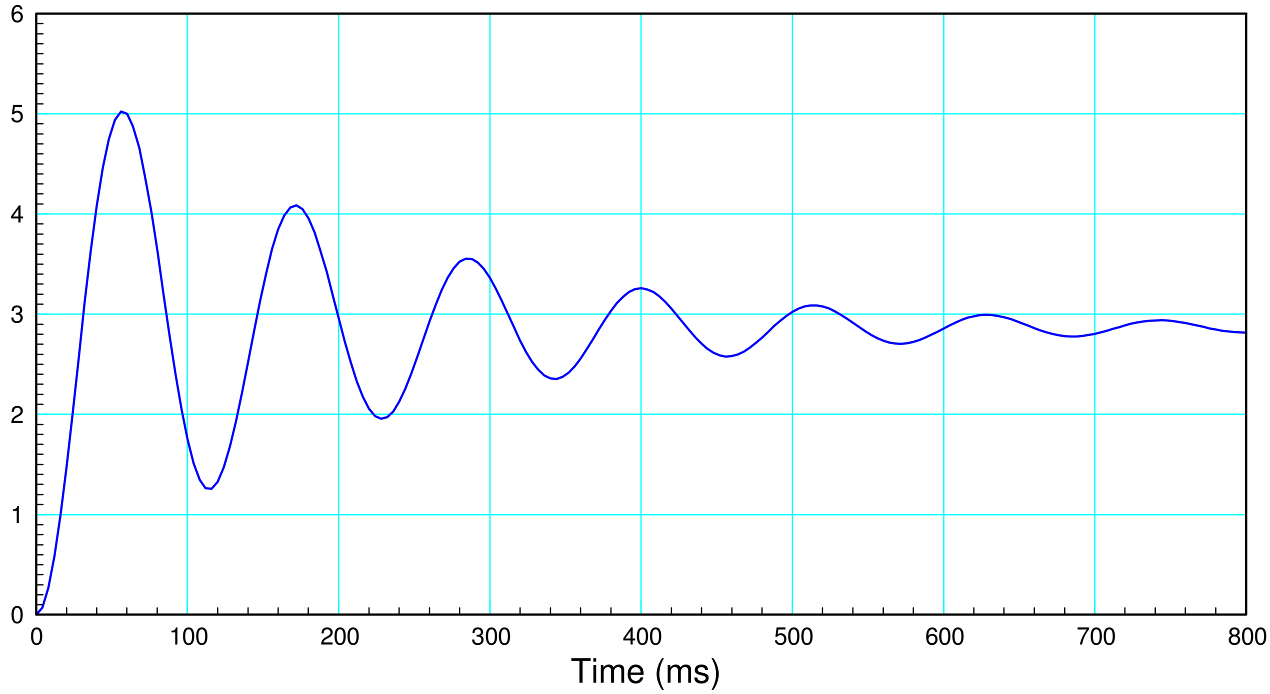
ECE 463/663: Test #1. Name ________________

Spring 2021. Open Book, Open Notes. Calculators & Matlab allowed. Individual Effort

1) Find the transfer funciton for a system with the following step response



DC gain = 2.8

Frequency of oscillation = $\omega_d = \left(\frac{6\text{ cycles}}{680\text{ms}}\right)2\pi = 55.44\ \frac{rad}{\text{sec}}$

2% settling time = 700ms (approx)

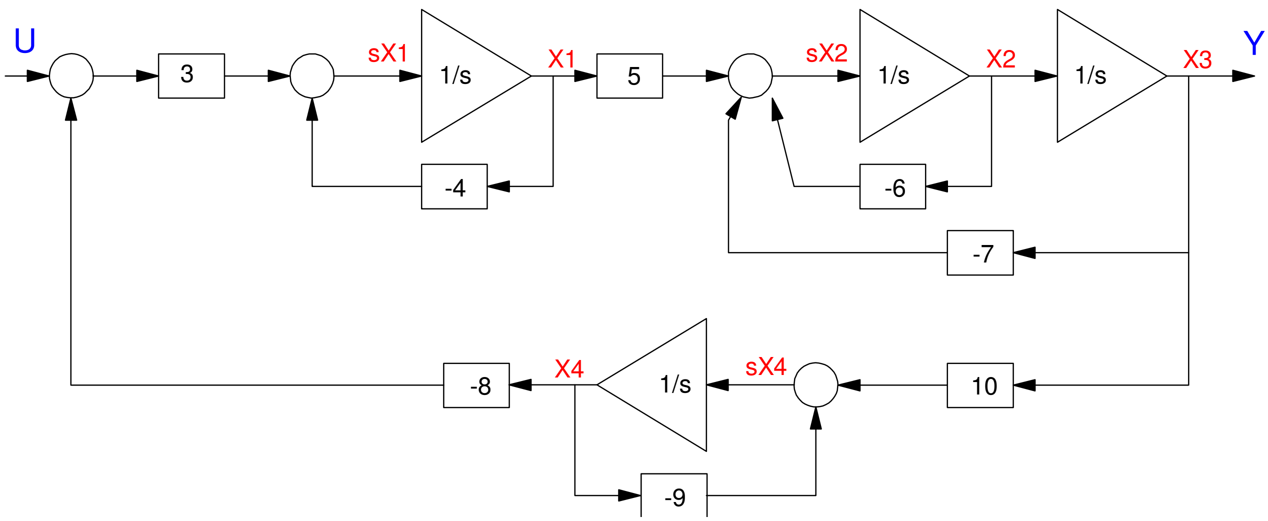
$$\sigma = \frac{4}{0.7} = 5.71$$

making

$$G(s) \approx \left(\frac{8697}{(s+5.71+j55.44)(s+5.71-j55.44)}\right)$$

(the numerator is whatever it takes to make the DC gain equal to 2.8)

2) Give {A and B} for the the state-space model for the following system



$$sX_1 = 3U - 24X_4 - 4X_1$$

$$sX_2 = 5X_1 - 6X_2 - 7X_3$$

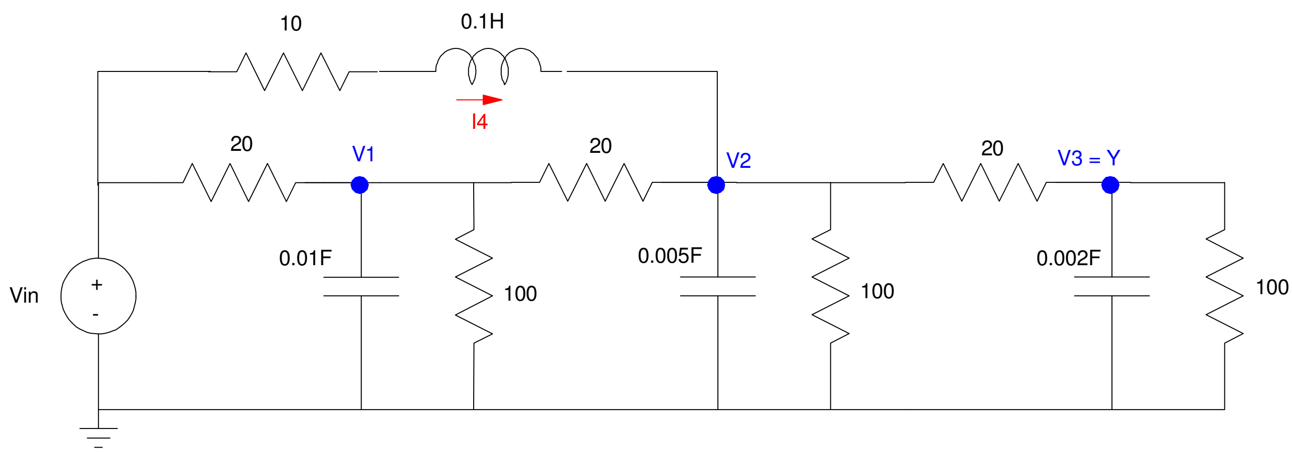
$$sX_3 = X_2$$

$$sX_4 = -9X_4 + 10X_3$$

$$\begin{bmatrix} sX1 \\ sX2 \\ sX3 \\ sX4 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 0 & -24 \\ 5 & -6 & -7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & -9 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

Problem 3) (option #1)

3a) Write four coupled differential equations to describe the following circuit



$$V_4 = 0.1sI_4 = V_{in} - 10I_4 - V_2$$

$$I_1 = 0.01sV_1 = \left(\frac{V_{in}-V_1}{20}\right) + \left(\frac{V_2-V1}{20}\right) - \left(\frac{V_1}{100}\right)$$

$$I_2 = 0.005sV_2 = I_4 + \left(\frac{V_1-V_2}{20}\right) + \left(\frac{V_3-V_2}{20}\right) - \left(\frac{V_2}{100}\right)$$

$$I_3 = 0.002sV_3 = \left(\frac{V_2-V_3}{20}\right) - \left(\frac{V_3}{100}\right)$$

Solving for the highest derivative

$$sV_1 = 5V_{in} - 11V_1 + 5V_2$$

$$sV_2 = 200I_4 + 10V_1 - 22V_2 + 10V_3$$

$$sV_3 = 25V_2 - 30V_3$$

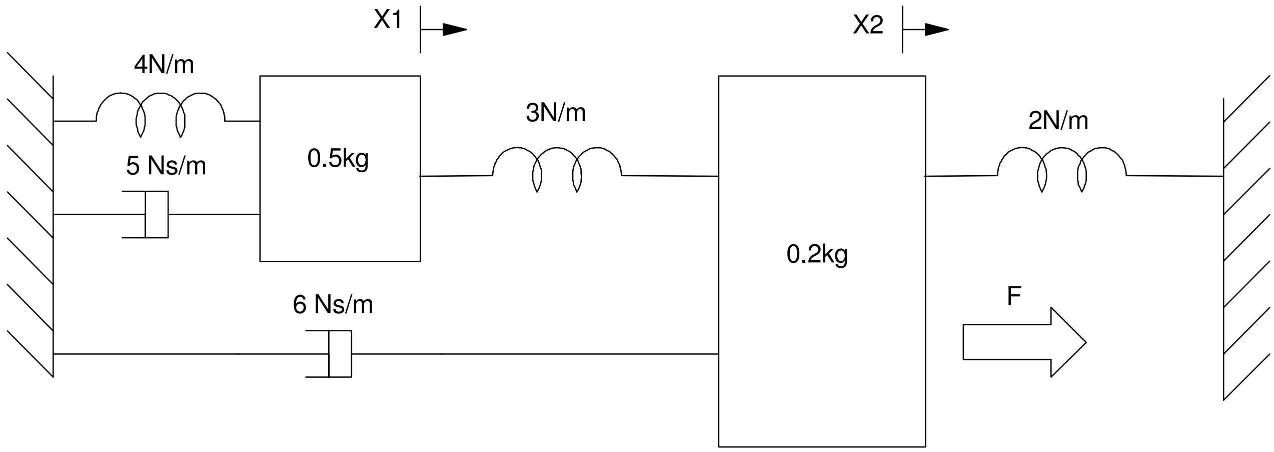
$$sI_4 = 10V_{in} - 100I_4 - 10V_2$$

3b) Express the A and B matricies for the dynamics in state-space form

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \\ sI_4 \end{bmatrix} = \begin{bmatrix} -11 & 5 & 0 & 0 \\ 10 & -22 & 10 & 200 \\ 0 & 25 & -30 & 0 \\ 0 & -10 & 0 & -100 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \\ 10 \end{bmatrix} V_{in}$$

Problem 3) Option #2 (you may work either problem #3 - electrical or mechanical)

3a) Write two coupled differential equations to describe the following mass-spring system



mass #1

$$(0.5s^2 + 4 + 5s + 3)X_1 - (3)X_2 = 0$$

mass #2

$$(0.2s^2 + 3 + 2 + 6s)X_2 - (3)X_1 = F$$

3b) Express the A and B matricies for the dynamics in state-space form

Solve for the highest derivative

$$s^2 X_1 = (-10s - 14)X_1 + 6X_2$$

$$s^2 X_2 = (-30s - 25)X_2 + 15X_1 + 5F$$

in matrix form

$$s\begin{bmatrix} X_1 \\ X_2 \\ sX_1 \\ sX_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -14 & 6 & -10 & 0 \\ 15 & -25 & 15 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ sX_1 \\ sX_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}F$$

4) A ball with a mass of 1kg is rolling on a surface with the shape

$$y = 3 + x \cdot \cos(x)$$

Determine the potential and kinetic energy of the ball in terms of x:

4a) $PE = mgy = f(x)$

$$PE = g(3 + x \cdot \cos(x))$$

4b) $KE = 0.7m\left(\dot{x}^2 + \dot{y}^2\right) = g(x, \dot{x})$

$$\dot{y} = \dot{x}\cos(x) - x\sin(x)\dot{x}$$

$$KE = 0.7\left(\dot{x}^2 + (\dot{x}\cos(x) - x\sin(x)\dot{x})^2\right)$$

Simplifying (not necessary)

$$KE = 0.7\dot{x}^2\left(1 + \cos^2(x) + x^2\sin^2(x) - 2x\sin(x)\cos(x)\right)$$

5) Assume the LaGrangian is:

$$L = 0.7\dot{x}^2\dot{\theta} + 0.5\dot{x}\dot{\theta}\sin(\theta) - gx\cos\theta$$

Determine

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$

$$F = \frac{d}{dt}\left(1.4\dot{x}\dot{\theta} + 0.5\dot{\theta}\sin(\theta)\right) - (-g\cos\theta)$$

$$F = 1.4\ddot{x}\dot{\theta} + 1.4\dot{x}\ddot{\theta} + 0.5\ddot{\theta}\sin(\theta) + 0.5\dot{\theta}^2\cos(\theta) + g\cos\theta$$