ECE 463/663 - Homework #11

LQR Observers. Due Wednesday, April 20thm 2022

Kalman Filters

Cart and Pendulum (HW #4): The dynamics for a cart and pendulum system with sensor and input noise is as follows

$$
s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -14.7 & 0 & 0 \\ 0 & 24.5 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ -0.5 \end{bmatrix}(F+n_u)
$$

$$
y_1 = x + n_x
$$

$$
y_2 = \theta + n_\theta
$$

where there is Gaussian noise at the input and output

$n_u \sim N(0, 0.02^2)$ *mean zero, standard deviation 0.02*

$n_x \sim N(0, 0.01^2)$ *mean zero, standard deviation 0.01*

$n_\theta \sim N(0, 0.01^2)$ *mean zero, standard deviation 0.01*

1) Use a servo-compensator to force the DC gain to one (i.e. use the servo compensator from homework set #10.

Plot the step response
- Without noise (same as homework set #10)
- With noise

2) Design a full-order observer using pole-placement to place the observer poles at {-3, -4, -5, -6}
- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

3) Design a Kalman filter (i.e. a full-order observer with a specific Q and R)
- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.