ECE 463/663 - Homework #3

Canonical Forms, Similarity Transforms, LaGrangian Dynamics, Block Diagrams

Due Monday, January 31st

Please make the subject "ECE 463 HW#3" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

Canonical Forms

Problem 1-3) For the system

$$Y = \left(\frac{20(s+1.1)}{(s+1)(s+4)(s+5)}\right)X$$

1) Express this system in controller canonical form. (Give the A, B, C, D matrices)

Multiply out

$$Y = \left(\frac{20s+22}{s^3+10s^2+29s+20}\right)X$$

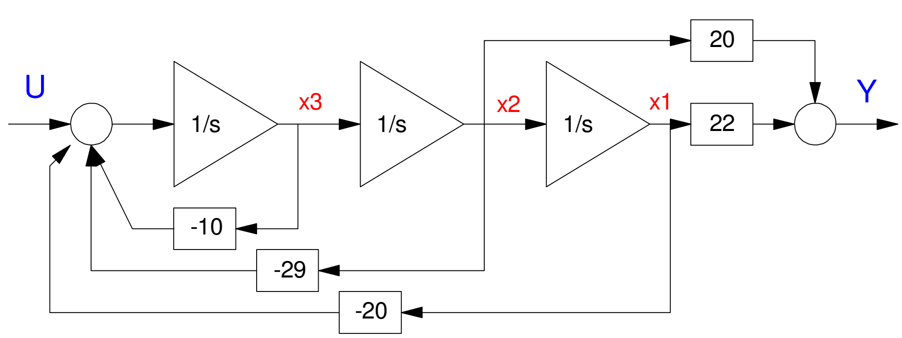
In controller canonical form

$$sX = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -29 & -10 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 22 & 20 & 0 \end{bmatrix} X + [0]U$$

Checking in matlab

```
>> A = [0,1,0;0,0,1;-20,-29,-10];
>> B = [0;0;1];
>> C = [22,20,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)


   20 (s+1.1)
-----------------
(s+5) (s+4) (s+1)
```



2) Express this system in cascade form

$$Y = \left(\frac{20(s+1.1)}{(s+1)(s+4)(s+5)} \right) X$$

Rewrite as

$$Y = \left(\left(\frac{a}{s+1} \right) + \left(\frac{b}{(s+1)(s+4)} \right) + \left(\frac{c}{(s+1)(s+4)(s+5)} \right) \right) X$$

Put over a common denominator

$$Y = \left(\left(\frac{a(s+4)(s+5)}{(s+1)(s+4)(s+5)} \right) + \left(\frac{b(s+5)}{(s+1)(s+4)(s+5)} \right) + \left(\frac{c}{(s+1)(s+4)(s+5)} \right) \right) X$$

Matching terms

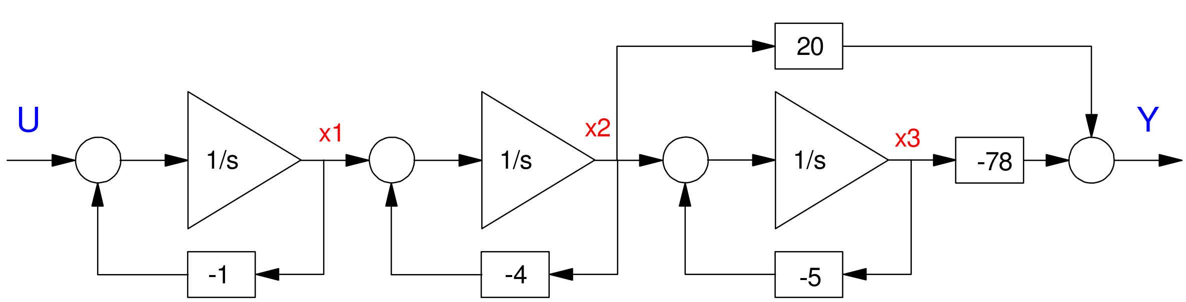
- a = 0
- b = 20
- c = -78

$$sX = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -4 & 0 \\ 0 & 1 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 20 & -78 \end{bmatrix} X + [0] U$$

Checking in Matlab:

```
>> A = [-1,0,0 ; 1,-4,0 ; 0,1,-5];
>> B = [1;0;0];
>> C = [0,20,-78];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)


   20 (s+1.1)
-----------------
(s+5) (s+4) (s+1)
```



3) Express this system in Jordan (diagonal) form

$$Y = \left(\frac{20(s+1.1)}{(s+1)(s+4)(s+5)}\right)X$$

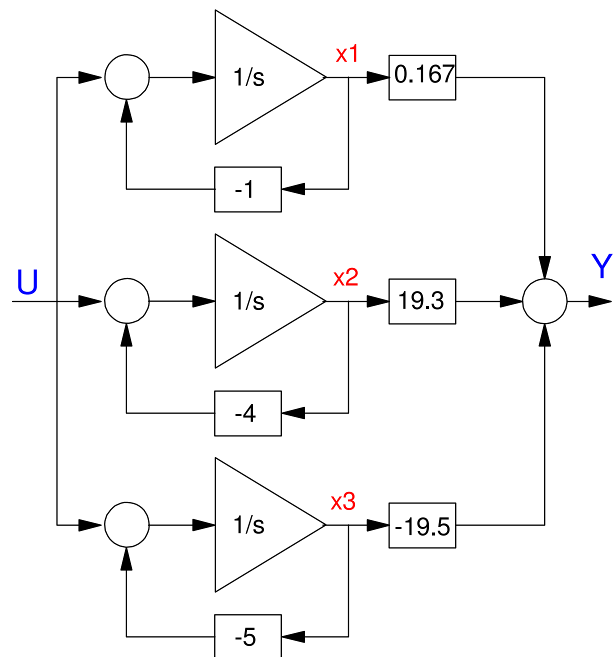
Use partial fraction expansion

$$Y = \left(\left(\frac{0.167}{s+1}\right) + \left(\frac{19.333}{s+4}\right) + \left(\frac{-19.5}{s+5}\right)\right)X$$

In Jordan form...

$$sX = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0.167 & 19.33 & -19.50 \end{bmatrix} X + [0]U$$



Checking in Matlab

```
>> A = diag([-1,-4,-5]);
>> B = [1;1;1];
>> C = [0.167,19.33,-19.50];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

-0.003 (s-6662) (s+1.1)
-----------------------
   (s+5) (s+4) (s+1)

>> C = [0.16666666,19.33333333,-19.50];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

-1e-008 (s-2e009) (s+1.1)
-------------------------
    (s+5) (s+4) (s+1)

>> C = [0.166666666666,19.333333333333,-19.50];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

   20 (s+1.1)
-----------------
(s+5) (s+4) (s+1)
```

4) Assume a system's dynamics are

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -4 & 0 \\ 0 & 1 & -5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} V_0$$

$$Y = V_3$$

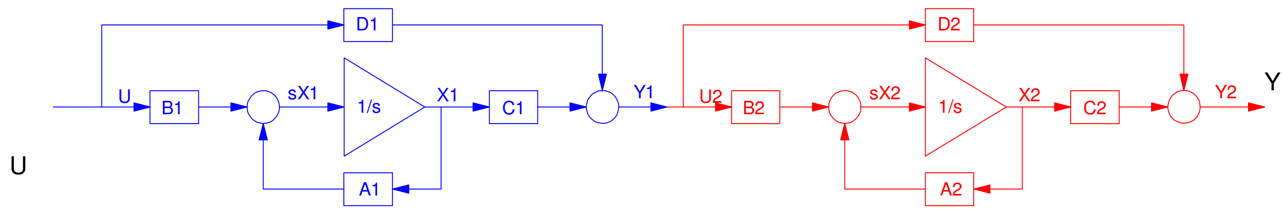
Express these dynamic with the change in variable

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} V_2 \\ V_3 \\ V_1 + V_2 + V_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = T^{-1} X$$

```
>> A = [-1,0,0 ; 1,-4,0 ; 0,1,-5];
>> B = [1;2;3];
>> C = [0,0,1];
>> D = 0;
>> Ti = [0,1,0 ; 0,0,1 ; 1,1,1]

     0     1     0
     0     0     1
     1     1     1

>> T = inv(Ti)

    -1    -1     1
     1     0     0
     0     1     0

>> Az = inv(T)*A*T

    -5    -1     1
     1    -5     0
    -3    -5     0

>> Bz = inv(T)*B

     2
     3
     6

>> Cz = C*T

     0     1     0
```

Block Diagrams

5) Determine the state-space model for two systems in series:



$$sX_1 = A_1 X_1 + B_1 U$$

$$sX_2 = B_2 C_1 X_1 + A_2 X_2 + B_2 D_1 U$$

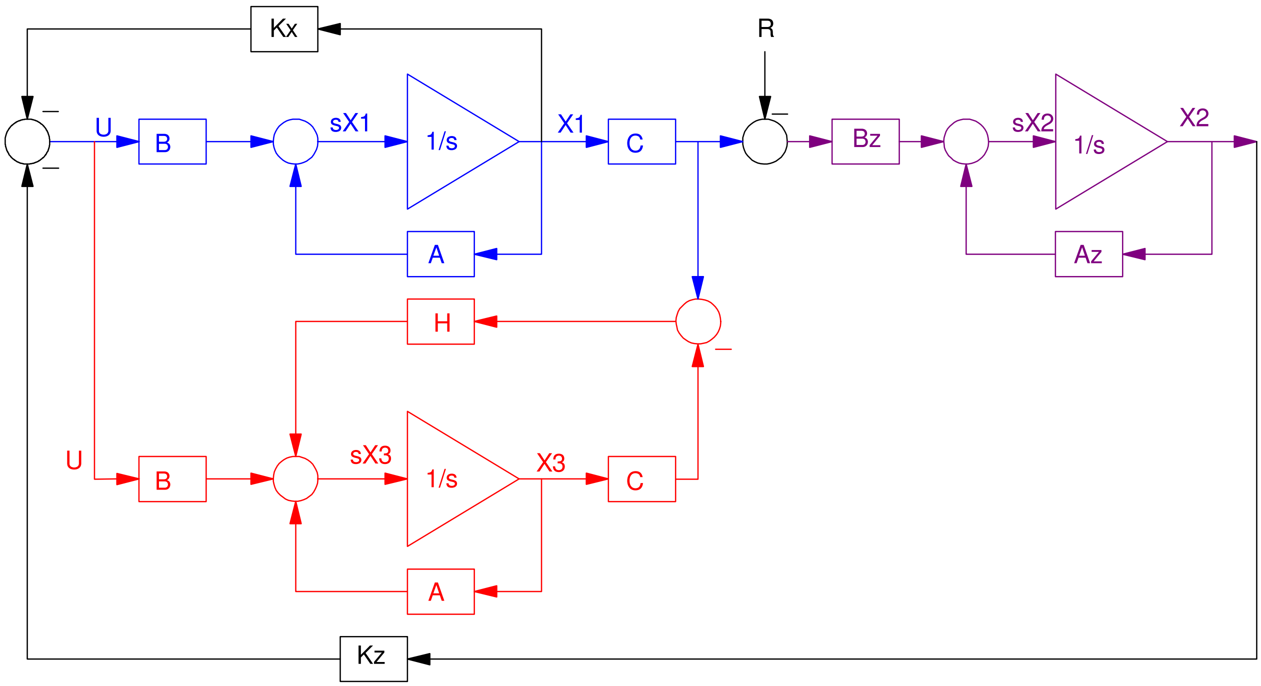
$$Y = C_2 X_2 + D_2 C_1 X_1 + D_2 D_1 U$$

In state-space

$$\begin{bmatrix} sX_1 \\ sX_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [D_2 D_1] U$$

6) Determine the state-space model for the following three interconnected systems:



$$sX_1 = AX_1 - BK_xX_1 - BK_zX_2$$

$$sX_2 = A_zX_2 - B_zR + B_zCX_1$$

$$sX_3 = AX_3 - HCX_3 + HCX_1 - BK_xX_1 - BK_zX_2$$

In state-space

$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} = \begin{bmatrix} (A - BK_x) & -BK_z & 0 \\ B_zC & A_z & 0 \\ (HC - BK_x) & -BK_z & A - HC \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -B_z \\ 0 \end{bmatrix} R$$

LaGrangian Dynamics

A 1kg ball is rolling in a bowl with the shape

$$y = 0.1 \cdot |x|^{2.5}$$

$$\dot{y} = 0.25 \cdot |x|^{1.5} \cdot \dot{x} \cdot sign(x)$$

7) Determine the kinetic and potential energy of this ball as a function of x: Gravity is in the -y direction.

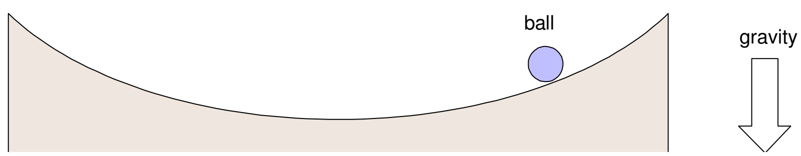
Assuming a solid sphere:

$$PE = mgy = 0.1g \cdot |x|^{2.5}$$

$$KE = 0.7m(\dot{x}^2 + \dot{y}^2)$$

$$KE = 0.7\left(\dot{x}^2 + 0.0625|x|^3\dot{x}^2\right)$$

$$KE = 0.7\left(1 + 0.0625|x|^3\right)\dot{x}^2$$



8) Determine the dynamics for this ball as it rolls in the bowl

Set up the LaGrangian

$$L = KE - PE$$

$$L = 0.7\left(1 + 0.0625|x|^3\right)\dot{x}^2 - 0.1g \cdot |x|^{2.5}$$

Solve the Euler LaGrange equation

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$

$$F = \frac{d}{dt}\left(1.4\left(1 + 0.0625|x|^3\right)\dot{x}\right) - \left(0.1313x^2\dot{x}^2 sign(x) - 0.25g|x|^{1.5} sign(x)\right)$$

$$F = 1.4\left(1 + 0.0625|x|^3\right)\ddot{x} + 0.2625x^2\dot{x}^2 sign(x)$$
$$-0.1313x^2\dot{x}^2 sign(x) + 0.25g|x|^{1.5} sign(x)$$

$$F = 1.4\left(1 + 0.0625|x|^3\right)\ddot{x} + \left(0.1313x^2\dot{x}^2 + 0.25g|x|^{1.5}\right) sign(x)$$