# ECE 463/663 - Homework #5

Full State Feedback. Due Wednesday, February 23rd

1) Write a Matlab m-file which is passed

- The system dynamics (A, B),
- The desired pole locations (P)

and then returns the feedback gains, Kx, so that roots(A - B Kx) = P

```
function [Kx] = ppl(A, B, P)
```

```
function [ Kx ] = ppl( A, B, P0)

N = length(A);

T1 = [];
for i=1:N
   T1 = [T1, (A^(i-1))*B];
end


P = poly(eig(A));
T2 = [];
for i=1:N
    T2 = [T2; zeros(1,i-1), P(1:N-i+1)];
end

T3 = zeros(N,N);
for i=1:N
    T3(i, N+1-i) = 1;
end

T = T1*T2*T3;

Pd = poly(P0);

dP = Pd - P;

Flip = [N+1:-1:2]';
Kz = dP(Flip);
Kx = Kz*inv(T);

end
```

Problems 2-4) Assume the following dynamic system:

$$sX = \begin{bmatrix} -6.2 & 3 & 0 & 0 & 0 \\ 3 & -6.2 & 3 & 0 & 0 \\ 0 & 3 & -6.2 & 3 & 0 \\ 0 & 0 & 3 & -6.2 & 3 \\ 0 & 0 & 0 & 3 & -3.2 \end{bmatrix} X + \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} X$$

2) Find the feedback control law of the form

$$U = K_r R - K_x X$$

so that

- The DC gain is 1.000 and
- The closed-loop poles are at {-1, -10, -10, -10, -10}

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

```
>> A = [-6.2,3,0,0,0 ; 3,-6.2,3,0,0 ; 0,2,-6.2,3,0 ; 0,0,3,-6.2,3 ; 0,0,0,3,-3.2]

   -6.2000    3.0000         0         0         0
    3.0000   -6.2000    3.0000         0         0
         0    2.0000   -6.2000    3.0000         0
         0         0    3.0000   -6.2000    3.0000
         0         0         0    3.0000   -3.2000

>> B = [3;0;0;0;0]

     3
     0
     0
     0
     0

>> C = [0,0,0,0,1];
>> D = 0;
>> xlabel('Time (Seconds)');

>> Kx = ppl(A, B, [-1,-10,-10,-10,-10])

Kx =    4.3333    8.8444   13.7644    8.1899   -0.8387
```

Check that Kx is correct

```
>> eig(A - B*Kx)

 -10.0022
 -10.0000 + 0.0022i
 -10.0000 - 0.0022i
  -9.9978
  -1.0000
```

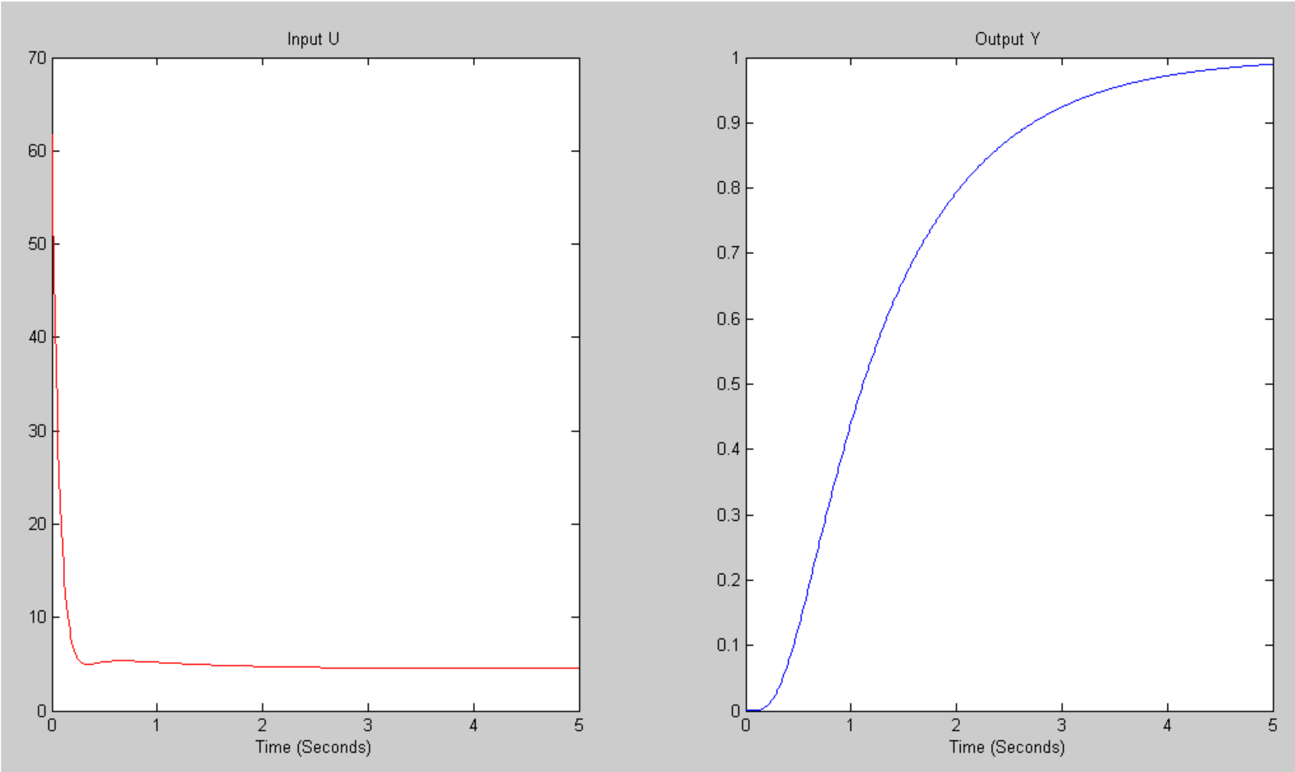
Find Kr to make the DC gain equal to 1.000

```
>> DC = -C*inv(A-B*Kx)*B

DC =    0.0162

>> Kr = 1/DC

Kr =   61.7284
```

Plot the closed-loop step response:

```
>> G2 = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> y2 = step(G2,t);

>> subplot(122);
>> plot(t,y2(:,1),'b');
>> xlabel('Time (Seconds)');
>> title('Output Y');

>> subplot(121);
>> plot(t,y2(:,2),'r');
>> xlabel('Time (Seconds)');
>> title('Input U');
```

Note:  The initial value of U is 51.8284 (Kr)



3) Repeat problem #2 but find Kx and Kr so that

- The DC gain is 1.000 and
- The closed-loop dominant pole is at s = -1 and the other four poles don't move (the are the same as the fast four poles of the open-loop system (eigenvalues of A)

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

```
>> P = eig(A)

  -10.9010
   -8.7840
   -5.2392
   -2.4945
   -0.5813

>> P(5) = -1

  -10.9010
   -8.7840
   -5.2392
   -2.4945
   -1.0000

>> Kx = ppl(A, B, P)

Kx =    0.1396    0.2614    0.5250    0.7218    0.8269
```

Check that Kx is correct

```
>> eig(A - B*Kx)

ans =

  -10.9010
   -8.7840
   -5.2392
   -2.4945
   -1.0000
```

Find Kr to make the DC gain 1.000

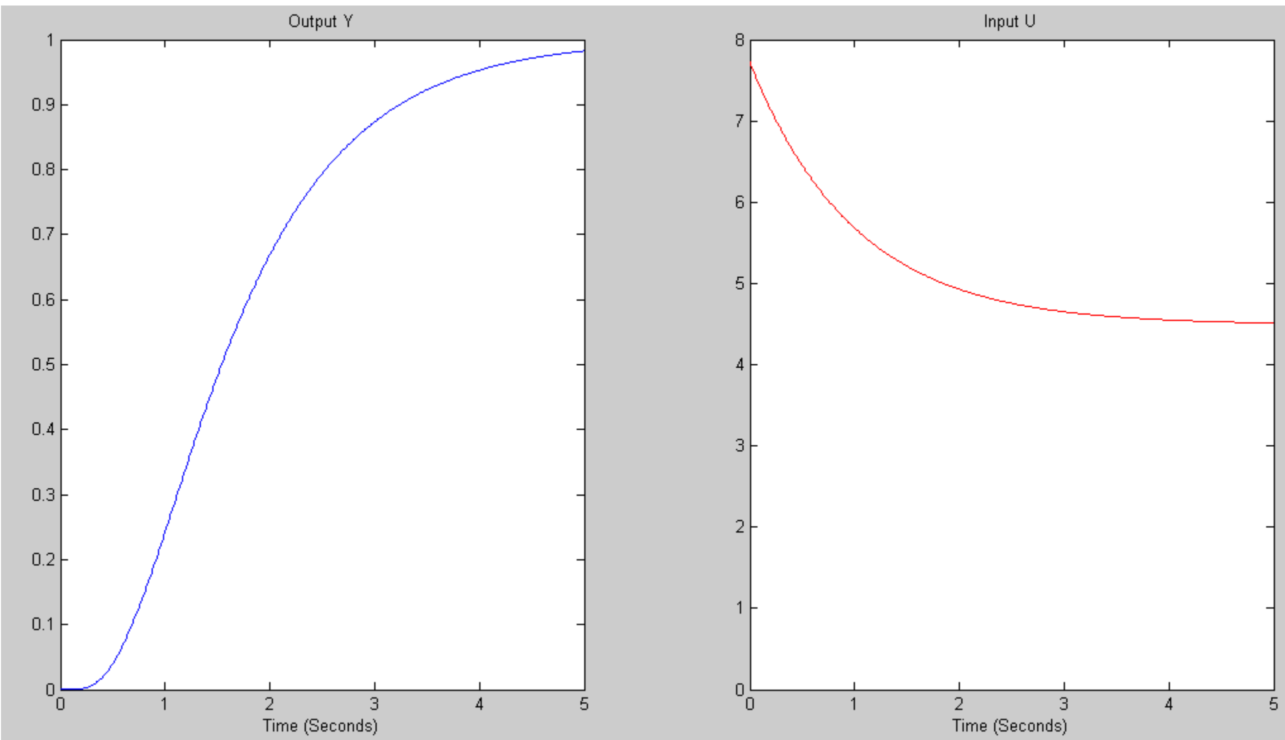
```
>> DC = -C*inv(A-B*Kx)*B

DC =    0.1295

>> Kr = 1/DC

Kr =    7.7249
```

Plot the closed-loop step response:

```
>> G2 = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> y2 = step(G2,t);
>> subplot(121);
>> plot(t,y2(:,1),'b');
>> xlabel('Time (Seconds)');
>> title('Output Y');
>> subplot(122);
>> plot(t,y2(:,2),'r');
>> xlabel('Time (Seconds)');
>> title('Input U');
>>
```

Note:

- The output is almost the same as problem #1 (same dominnt pole)
- The initial value of U is 7.72 (down from 51.828 in problem #2)



4) Repeat problem #2 but find Kx and Kr so that

- The DC gain is 1.000
- The 2% settling time is 2 seconds, and
- There is 5% overshoot for a step input.

Plot

- The resulting closed-loop step reponse, and
- The resulting input, U

Translation: Place the closed-loop dominant pole at s = -2 + j

- The real part is -2 for a 2 second settling time ( real(s) = 4 / Ts )
- The damping ratio is 0.0.6902 for 5% overshoot
- The angle of the dominant pole is 46.3 degrees
- The closed-loop dominant pole is s = -2 + j2.097

```
>> P(5) = -2 + j*2.097;
>> P(4) = conj(P(5));
>> P

 -10.9010
  -8.7840
  -5.2392
  -2.0000 - 2.0970i
  -2.0000 + 2.0970i

>> Kx = ppl(A, B, P)

Kx =    0.3081    1.0927    3.5633    5.9877    7.5162
```

Check that Kx is correct

```
>> eig(A - B*Kx)

 -10.9010
  -8.7840
  -5.2392
  -2.0000 + 2.0970i
  -2.0000 - 2.0970i
```

Find Kr to set the DC gain to 1.000

```
>> DC = -C*inv(A-B*Kx)*B

DC =    0.0385

>> Kr = 1/DC

Kr =   26.0046
```

Plot the step response:

```
>> G2 = ss(A-B*Kx, B*Kr, [C ; -Kx], [D ; Kr]);
>> y2 = step(G2,t);
>> subplot(121);
>> plot(t,y2(:,1),'b');
>> xlabel('Time (Seconds)');
>> title('Output Y');
>> subplot(122);
>> plot(t,y2(:,2),'r');
>> xlabel('Time (Seconds)');
>> title('Input U');
>>
```