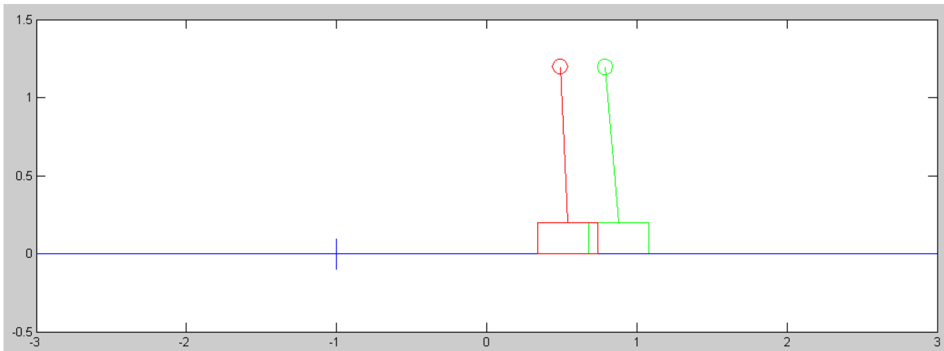
ECE 463: Homework #8

Linear Observers. Due Monday March 20th
Please submit as a hard copy, emai lto jacob.glower@ndsu.edu, or submit on BlackBoard



Cart and Pendulum from homework #4 with a state estimator (green)

Use the dynamics for the cart and pendulum from homework set #4

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.4 & 0 & 0 \\ 0 & 26.133 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.667 \end{bmatrix}F$$

1) Design a full-state feedback control law of the form

$$U = F = K_r R - K_x X$$

so that the closed-loop system has

- A 2% settling time of 8 seconds, and
- 5% overshoot for a step input.

Plot the step response of the linarized system in Matlab.

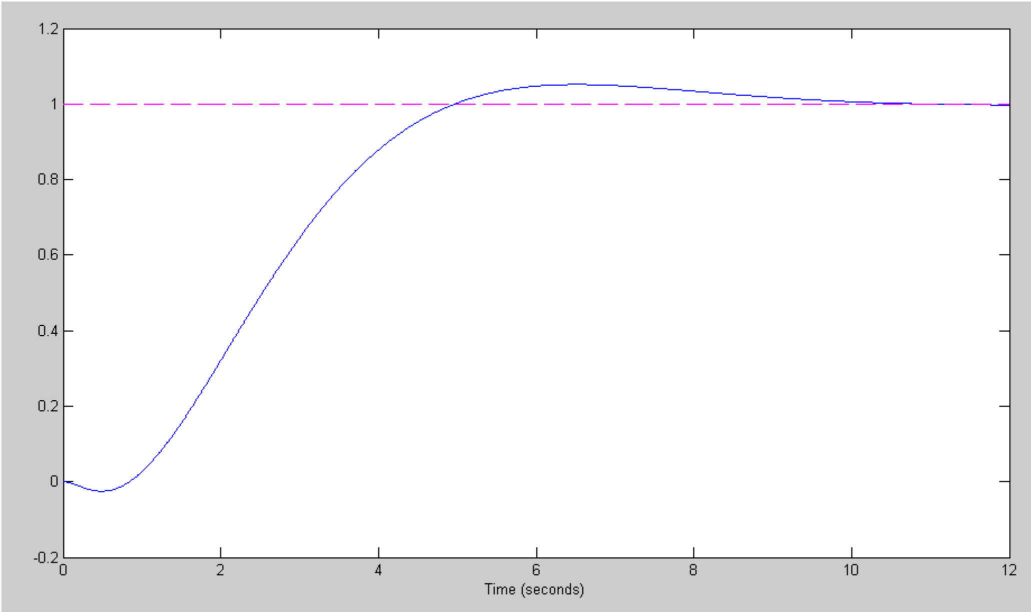
Assume you can only measure the cart position and beam angle.

```
>> A = [0,0,1,0;0,0,0,1;0,-29.4,0,0;0,26.133,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0  -29.4000         0         0
         0   26.1330         0         0

>> B = [0;0;1;-0.667]

         0
         0
    1.0000
   -0.6670

>> Kx = ppl(A, B, [-0.5+j*0.525,-0.5-j*0.525,-3,-4])

Kx =    -0.9669   -69.9034    -2.4036   -15.5976
```

```
>> C = [1,0,0,0];
>> DC = -C*inv(A - B*Kx)*B

DC =    -1.0342

>> Kr = 1/DC

Kr =    -0.9669
```

Plot the step response

```
>> t = [0:0.01:12]';
>> G = ss(A - B*Kx, B*Kr, C, 0);
>> y = step(G,t);
>> plot(t,y,t,0*y+1,'m--');
>> xlabel('Time (seconds)');
```



2) Design a full-order observer to estimate all four states so that the observer is 2-5 times faster than the plant. You may use either cart position or beam angle (or both) as measurements.

It isn't observable from cart angle, so use position

```
>> H = ppl(A', C', [-2,-3,-4,-5])'

   14.0000
  -21.8571
   90.6000
  -96.7224

>> eig(A - H*C)

   -2.0000
   -3.0000
   -5.0000
   -4.0000
```

3) Give the state-space model of the closde loop system using the actual states:

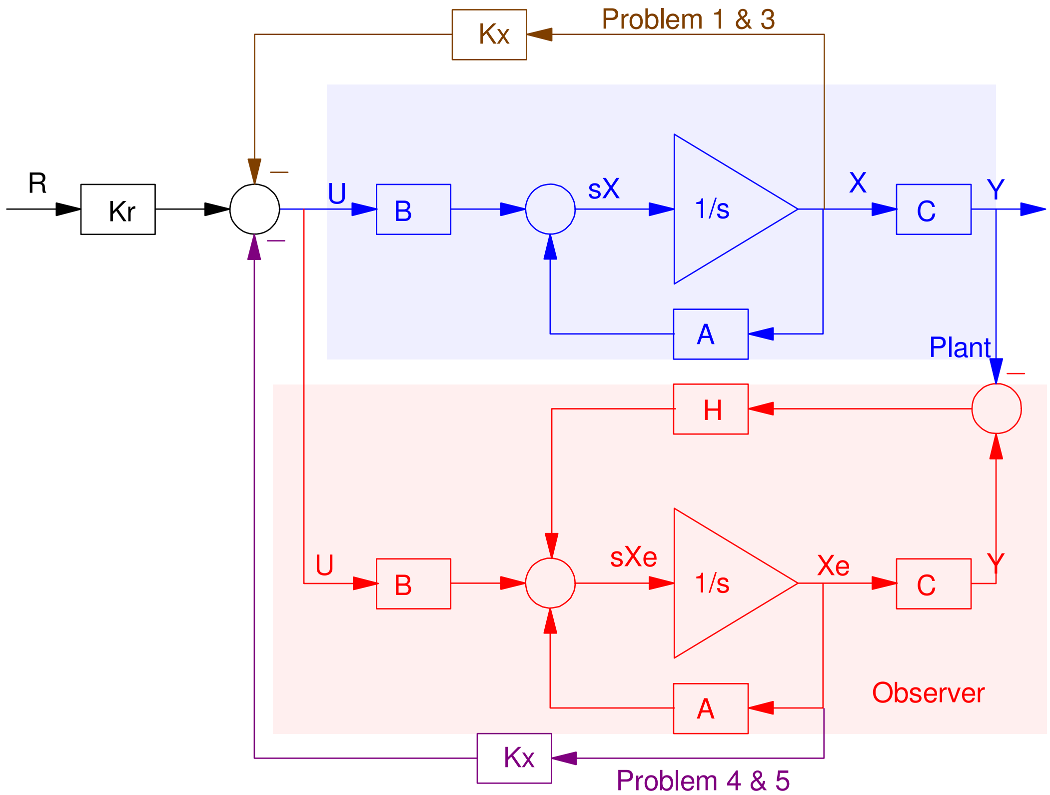
$$U = F = K_r R - K_x X$$

and plot the step response with initial conditions of

$X(0) = [0, 0, 0, 0]'$ $X_{observer}(0) = [0.1, 0.1, 0.1, 0.1]'$

(note: use the function step3)

$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A - BK_x & 0 \\ HC - BK_x & A - HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$



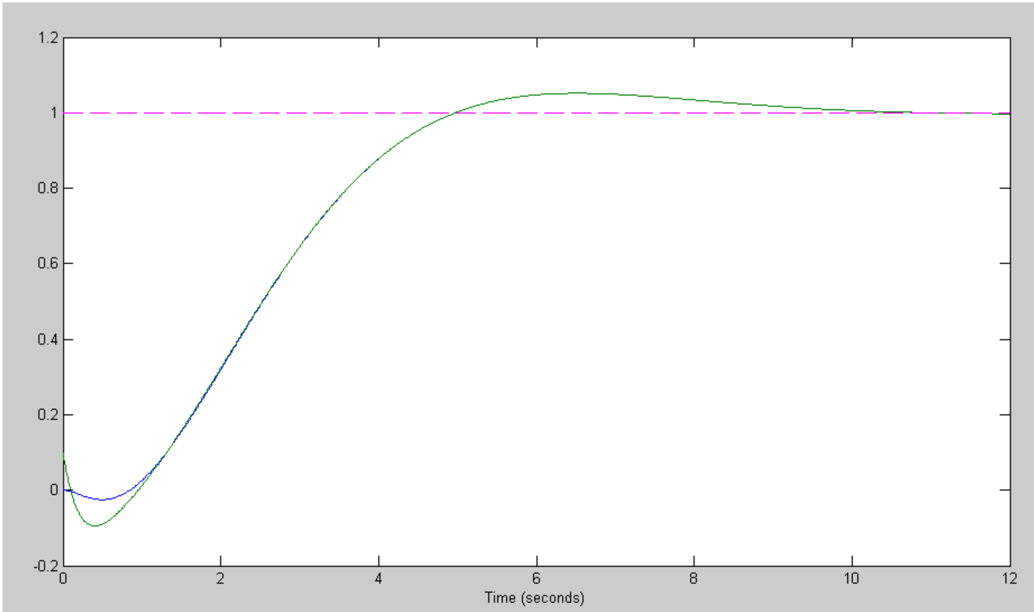

```
>> A8 = [A-B*Kx, zeros(4,4) ; H*C - B*Kx, A-H*C]

          0          0     1.0000          0          0          0          0          0
          0          0          0     1.0000          0          0          0          0
     0.9625    40.6225     2.3927    15.6125          0          0          0          0
    -0.6407   -20.4882    -1.5928   -10.3927          0          0          0          0
    14.0000          0          0          0   -14.0000          0     1.0000          0
   -21.8571          0          0          0    21.8571          0          0     1.0000
    91.5625    60.2225     2.3927    15.6125   -90.6000   -19.6000          0          0
   -97.3632   -40.0882    -1.5928   -10.3927    96.7224    19.6000          0          0
```

```
>> eig(A8)

  -5.0000
  -0.5000 + 0.5250i
  -0.5000 - 0.5250i
  -2.0000
  -4.0000
  -4.0000
  -3.0000
  -3.0000
```

note: It's stable (good)

Poles are at eig(A - BKx) and eig(A - HC) (also good)

```
>> B8r = [B*Kr ; B*Kr]

         0
         0
   -0.9625
    0.6407
         0
         0
   -0.9625
    0.6407

>> C8 = [1,0,0,0,0,0,0,0;0,0,0,0,1,0,0,0]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

>> D8 = [0;0]

     0
     0

>> X0 = [0;0;0;0;0.1;0.1;0.1;0.1]

         0
         0
         0
         0
    0.1000
    0.1000
    0.1000
    0.1000

>> t = [0:0.01:12]';
>> R = 0*t + 1;
>> y = step3(A8, B8r, C8, D8, t, X0, R);
>> plot(t,y,t,0*y+1,'m--');
>> xlabel('Time (seconds)');
```



Step response of the plant & observer
Feedback uses the plant's states

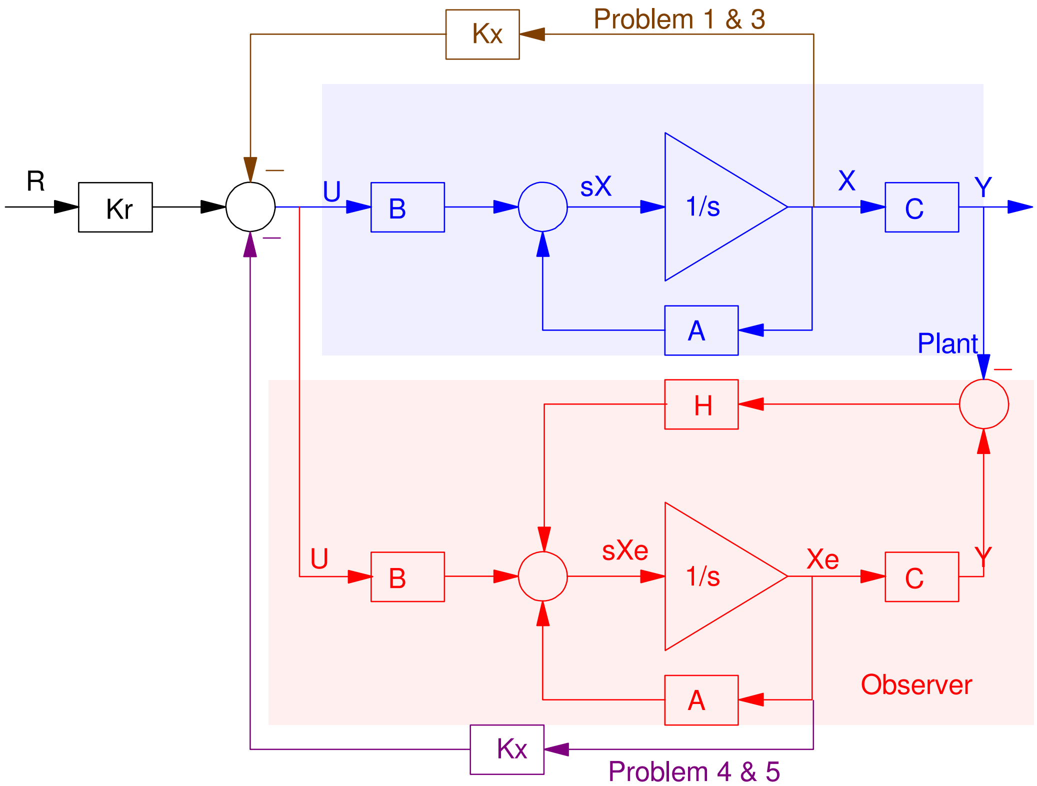
4) Give the state-space model of the closed loop system using the state estimates:

$$U = K_r R - K_x X_{observer}$$

and plot the step response with initial conditions of

X(0) = [0, 0, 0, 0]' $X_{observer}(0)$ = [0.1, 0.1, 0.1, 0.1]'

$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A & -BK_x \\ HC & A-BK_x-HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

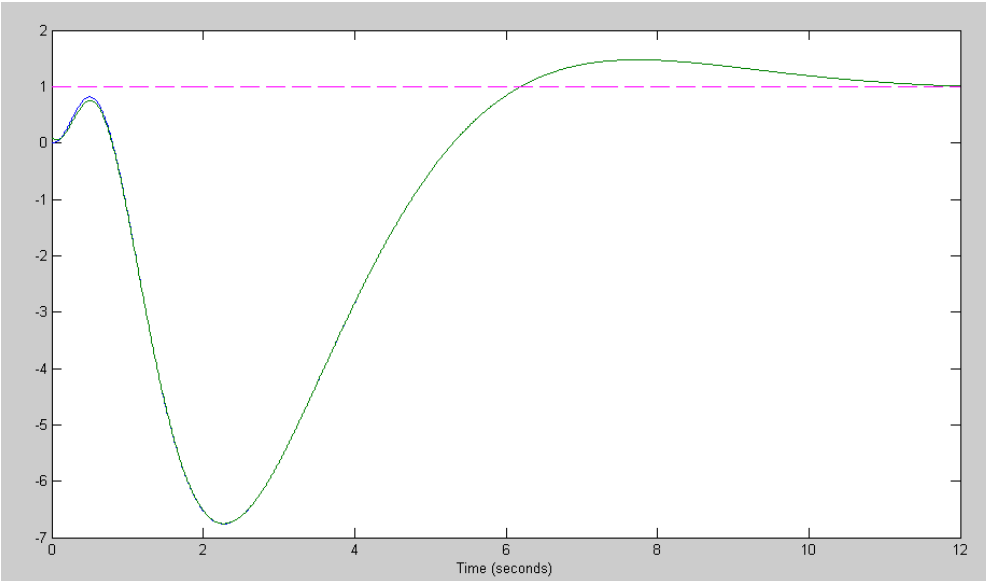



```
>> A8 = [A, -B*Kx ; H*C, A-B*Kx-H*C]

         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
         0  -19.6000         0         0    0.9625   60.2225    2.3927   15.6125
         0   19.6000         0         0   -0.6407  -40.0882   -1.5928  -10.3927
   14.0000         0         0         0  -14.0000         0    1.0000         0
  -21.8571         0         0         0   21.8571         0         0    1.0000
   90.6000         0         0         0  -89.6375   40.6225    2.3927   15.6125
  -96.7224         0         0         0   96.0817  -20.4882   -1.5928  -10.3927
```

```
>> eig(A8)

  -0.5000 + 0.5250i
  -0.5000 - 0.5250i
  -2.0000
  -5.0000
  -3.0000 + 0.0000i
  -3.0000 - 0.0000i
  -4.0000 + 0.0000i
  -4.0000 - 0.0000i
```

Stable with poles at the correct location - looks like A8 was put together correctly

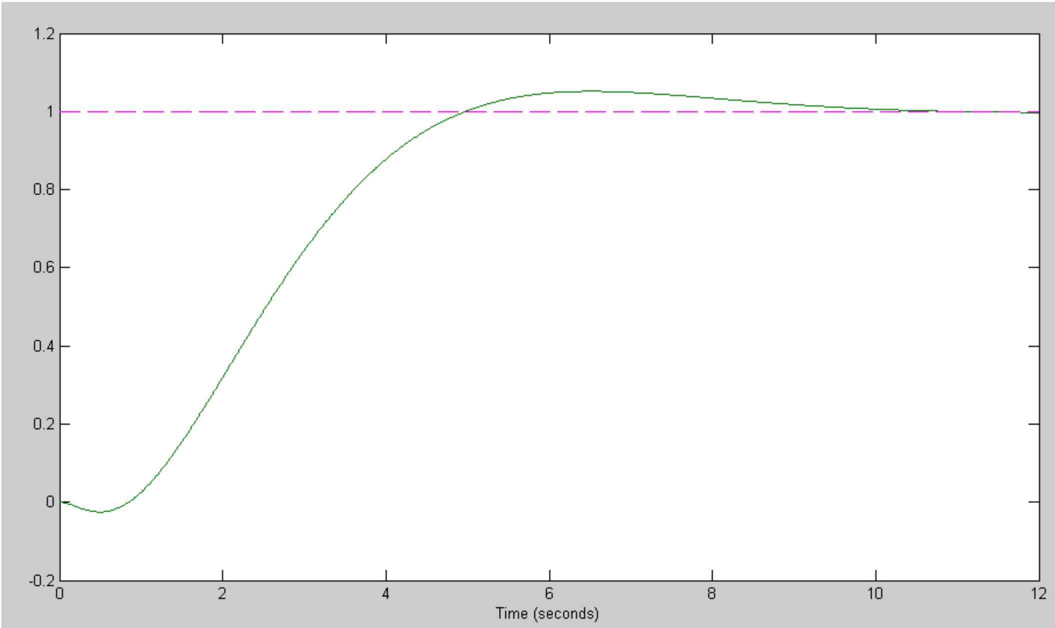
```
>> B8 = [B*Kr ; B*Kr]

         0
         0
   -0.9625
    0.6407
         0
         0
   -0.9625
    0.6407

>> C8 = [1,0,0,0,0,0,0,0;0,0,0,0,1,0,0,0]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

>> D8 = [0;0]

     0
     0

>> y = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y,t,0*y+1,'m--');
>> xlabel('Time (seconds)');
```



Step Response with Initial Error in the Observer States

This step response isn't very good. However, once the observer states converge, the step response is what you'd expect:

```
>> y = step3(A8, B8, C8, D8, t, X0*0, R);
>> plot(t,y,t,0*y+1,'m--');
>> xlabel('Time (seconds)');
```



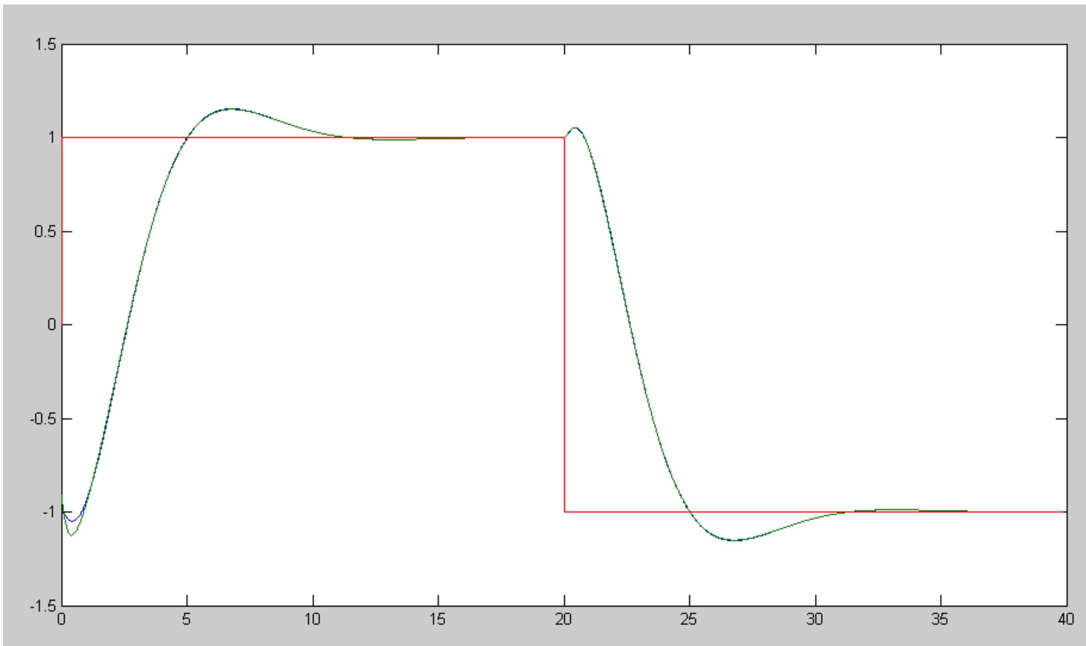
Step Response after the observer states converge

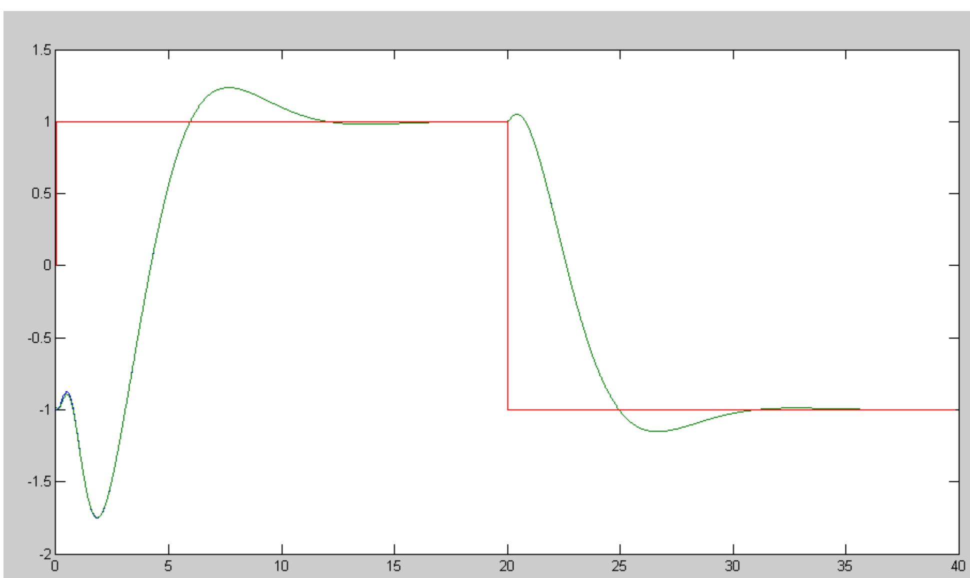
5) (20pt) Modify the cart and pendulum system to include

- your control law, and
- A full-order observer

Plot the step response of the nonlinear system + observer when

- Xe = $[0, 0, 0, 0]^T$
- Xe = $[0.1, 0.1, 0.1, 0.1]^T$





Code:

```
% Cart and Pendulum
% Lecture %20
% Separation Principle

X = [-1,0,0,0]';
Ref = 1;
dt = 0.01;
t = 0;
Kx = [-1.4431  -90.2887   -3.5873  -23.4070];
Kr = -1.4431;
% Full-Order Observer
Ae = [0,0,1,0;0,0,0,1;0,-29.4,0,0;0,26.14,0,0];
Be = [0;0;1;-0.667];
Ce = [1,0,0,0];
H = ppl(Ae', Ce', [-2,-3,-4,-5])';
Xe = X + 0.02;

n = 0;
y = [];
while(t < 39.9)
 Ref = sign(sin(t*pi/20));
 U = Kr*Ref - Kx*Xe;
 dX = CartDynamics(X, U);
 dXe = Ae*Xe + Be*U + H*(X(1) - Ce*Xe);

 X = X + dX * dt;
 Xe = Xe + dXe * dt;

 t = t + dt;
 n = mod(n+1, 5);
 if(n == 0)
    CartDisplay(X, Xe, Ref);
 end
 y = [y ; X(1), Xe(1), Ref];
end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```