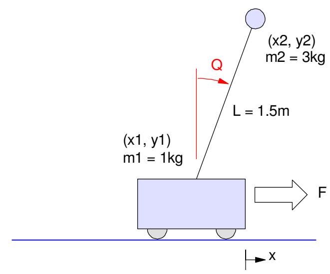
ECE 463/663 - Homework #12

LQG/LTR. Due Monday, May 1st

Please submit as a hard copy, email to jacob.glower@ndsu.edu, or submit on BlackBoard

LQG / LTR

For the cart and pendulum system of homework set #4:



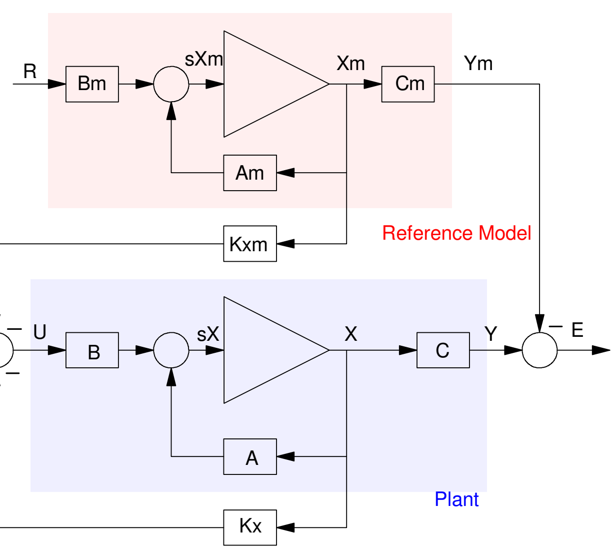
$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -29.4 & 0 & 0 \\ 0 & 26.133 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -0.667 \end{bmatrix} F$$

Design a control law so that the cart and pendulum system behaves like the following reference model:

$$y_m = \left(\frac{0.5}{s^2+s+0.5}\right) R$$

LQG/LTR without a Servo Compensator:

1) Give a block diagram for your controller plus servo compensator



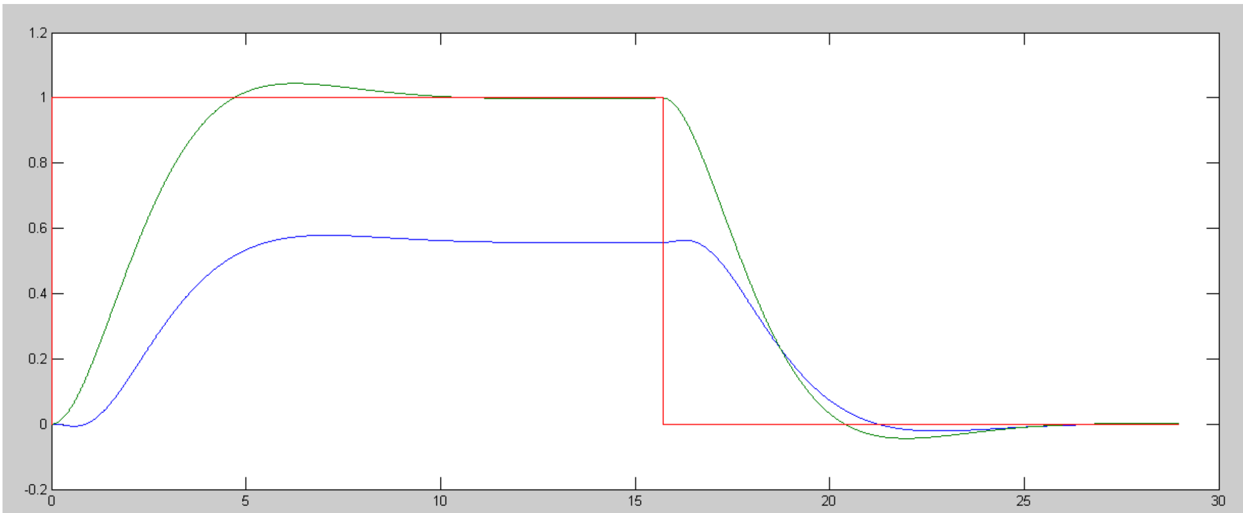
$$s\begin{bmatrix} X \\ X_m \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_m \end{bmatrix}\begin{bmatrix} X \\ X_m \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ B_m \end{bmatrix} R$$

$$E = \begin{bmatrix} C & -C_m \end{bmatrix}\begin{bmatrix} X \\ X_m \end{bmatrix}$$

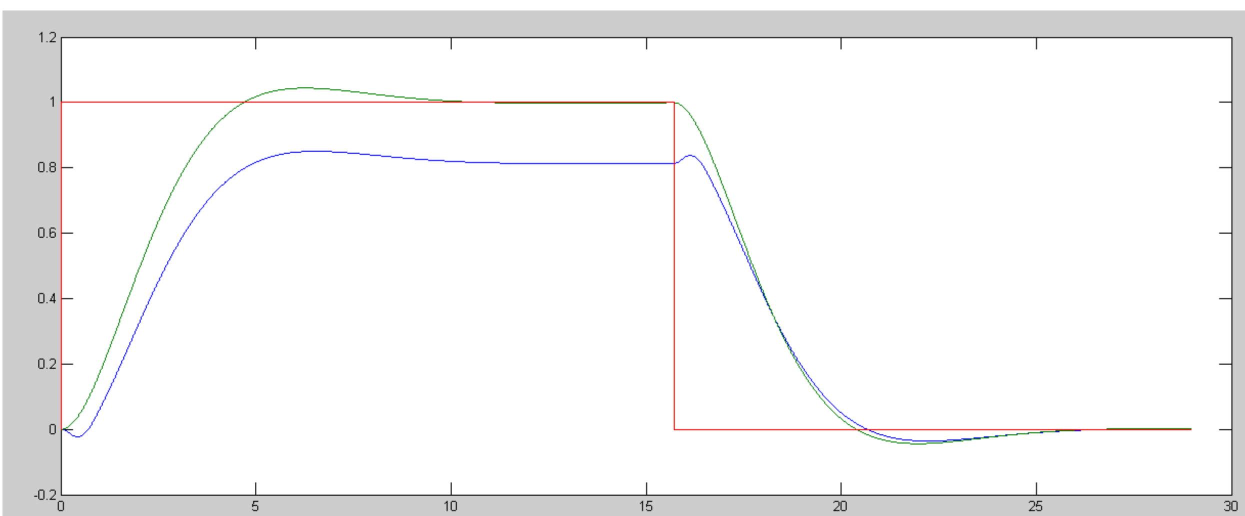
2) (20pt) Plot the step response of the model and the linearlized plant for yor control law for

- Q = 100 e^2
- Q = 1,000 e^2
- Q = 10,000 e^2

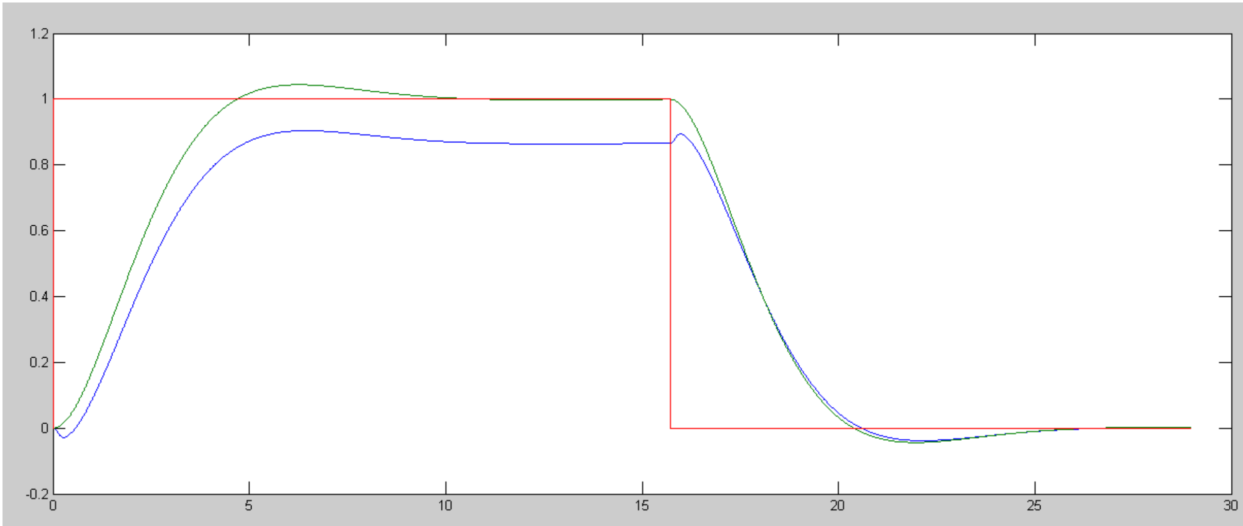
$$Q = \alpha[C - C_m]^T[C - C_m]$$



Q = 100 e2



Q = 1e4



Q = 1e6

As Q goes to infinity, the tracking gets better

The DC gain isn't one

Feedback Gains

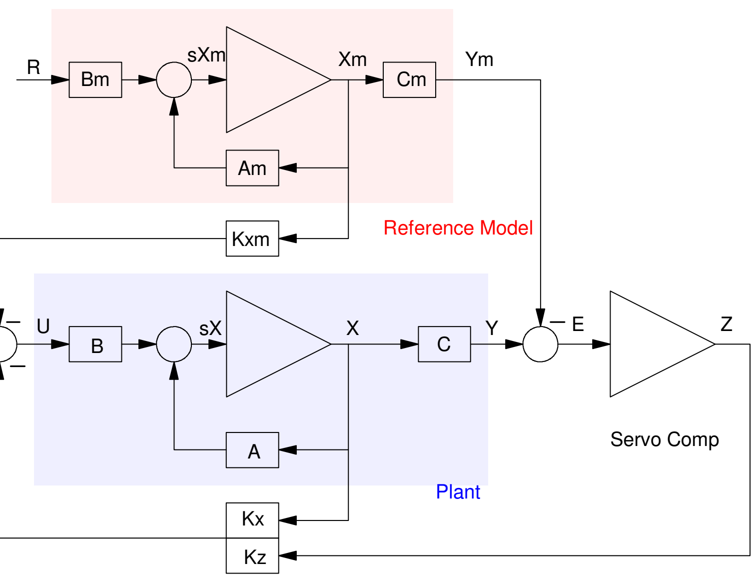
Q	x	q	x'	q'	xm	xm'
1e2	-10	-149.36	-18.404	-42.014	2.7862	3.0090
1e4	-100	-378.76	-106.72	-140.12	40.742	30.693
1e6	-1000	-2373.0	-899.78	-973.89	432.92	287.50

Code

```
A = [0,0,1,0;0,0,0,1;0,-20.4,0,0;0,26.133,0,0];
B = [0;0;0.5;-0.5];
C = [1,0,0,0];

Ref = 1;
dt = 0.01;
t = 0;

%Reference Model
Am = [0,1;-0.5,-1];
Bm = [0;1];
Cm = [0.5,0];
[n,m] = size(Am);

A6 = [ A, zeros(4,n) ;
       zeros(n,4), Am];
B6 = [B; zeros(n,1)];
B6r = [zeros(4,1); Bm];

C6 = [C, -Cm];
Q = C6' * C6;
R = 1;

K6 = lqr(A6, B6, Q*1e6, 1);

Kx = K6(1:4);
Km = K6(5:4+n);

X = zeros(4,1);
Xm = zeros(n,1);

n = 0;
y = [];
while(t < 29)
    Ref = 1*(sin(0.2*t) > 0);
    U = -Km*Xm - Kx*X;

    dX = CartDynamics(X, U);
    dXm = Am*Xm + Bm*Ref;

    X = X + dX * dt;
    Xm = Xm + dXm * dt;

    t = t + dt;
    n = mod(n+1, 5);
    if(n == 0)
       CartDisplay(X, [Cm*Xm;0;0;0], Ref);
       end
    y = [y ; X(1), Cm*Xm, Ref];
    end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```

LQG/LTR with a Servo Compensator:

3) Give a block diagram for your controller

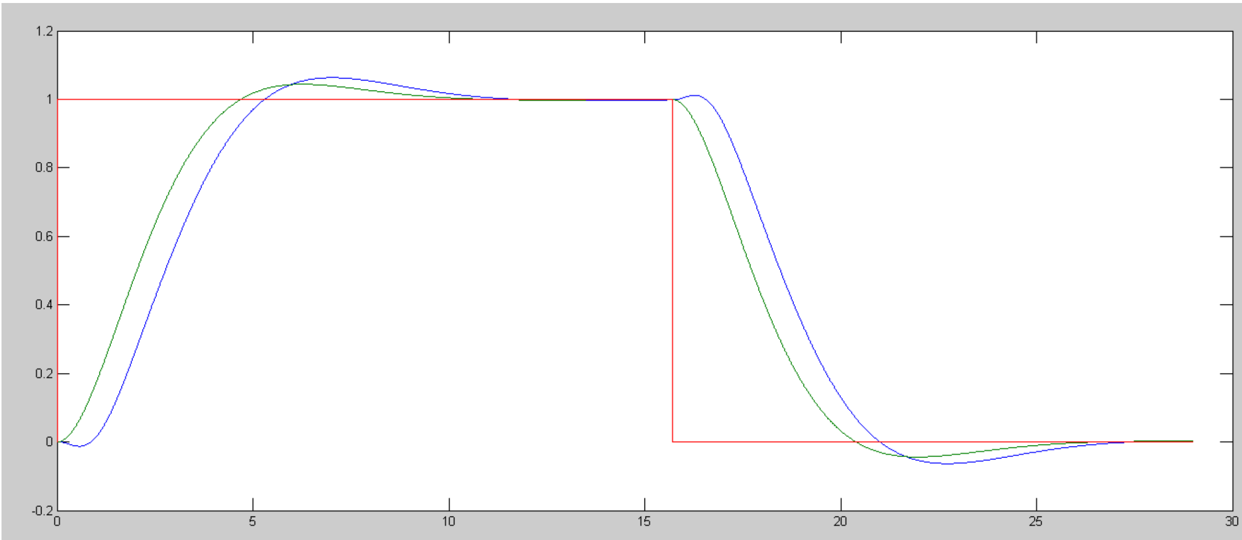


$$s\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & -C_m \\ 0 & 0 & A_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix} R$$

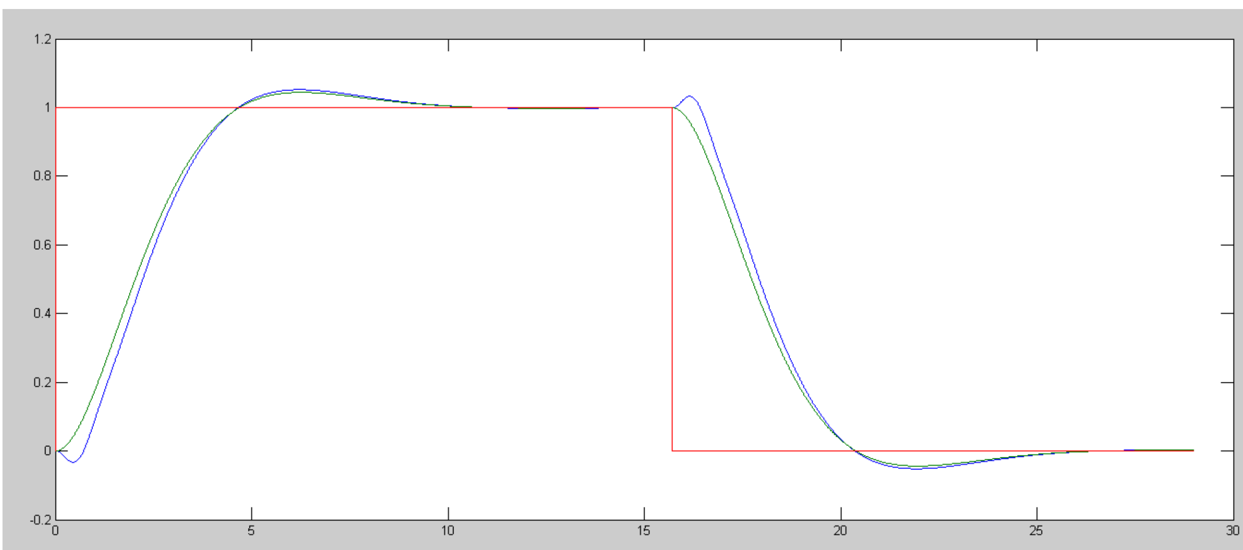
$$U = \begin{bmatrix} -K_x & -K_z & -K_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix}$$

4) (20pt) Plot the step response of the model and the linearlized plant for yor control law for

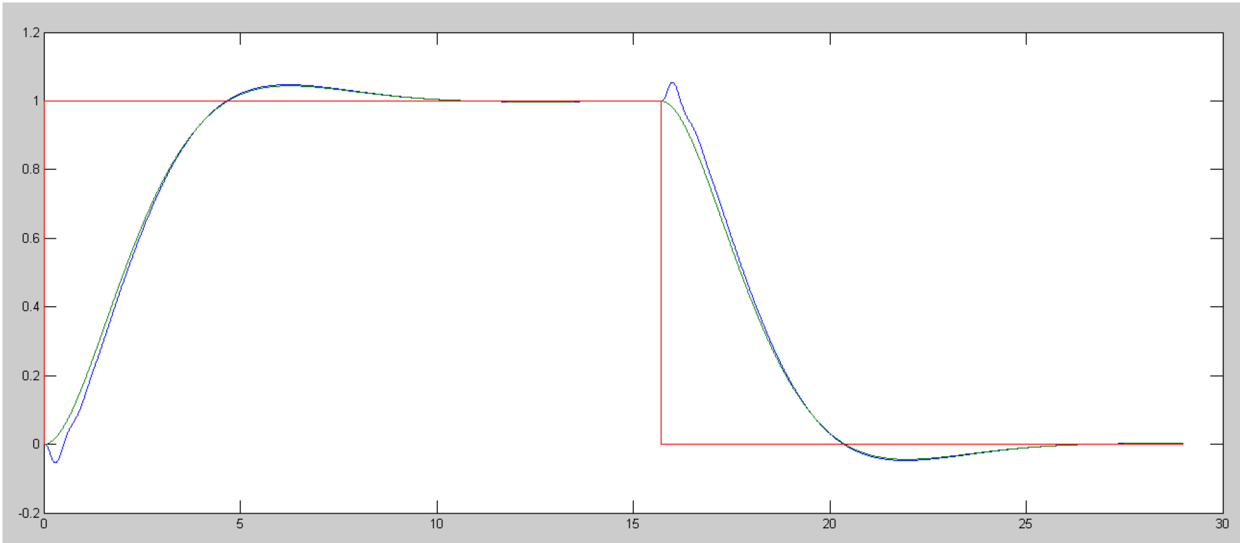
- Q = 100 e^2
- Q = 1,000 e^2
- Q = 10,000 e^2



Q = 100



Q = 1e4



Q = 1e6

With a servo compensator, the DC gain is one

As Q goes to infinity, the tracking gets better

Feedback Gains:

Q	x	q	x'	q'	z	xm	xm'
1e2	-24.221	-176.39	-29.333	-54.004	-10.000	8.8234	6.1618
1e4	-147.23	-375.10	-108.39	-138.58	-100.00	64.412	31.719
1e6	-1100.1	-1597.0	-605.12	-649.70	-1000.0	505.45	194.53

Code

```
A = [0,0,1,0;0,0,0,1;0,-20.4,0,0;0,26.133,0,0];
B = [0;0;0.5;-0.5];
C = [1,0,0,0];

Ref = 1;
dt = 0.01;
t = 0;

%Reference Model
Am = [0,1;-0.5,-1];
Bm = [0;1];
Cm = [0.5,0];
[n,m] = size(Am);

A7 = [ A, zeros(4,1), zeros(4,n) ;
       C, 0, -Cm;
       zeros(n,4), zeros(n,1), Am];
B7 = [B; 0; zeros(n,1)];
B7r = [zeros(4,1); 0; Bm];

C7 = [0*C, 1, 0*Cm];

Q = C7' * C7;
R = 1;

K7 = lqr(A7, B7, Q*1e6, 1);

Kx = K7(1:4);
Kz = K7(5);
Km = K7(6:5+n);

X = zeros(4,1);
Xm = zeros(n,1);

Z = 0;


n = 0;
y = [];
while(t < 29)
    Ref = 1*(sin(0.2*t) > 0);
    U = -Km*Xm - Kx*X - Kz*Z;

     dX = CartDynamics(X, U);
    dXm = Am*Xm + Bm*Ref;
    dZ = X(1) - Cm*Xm;

    X = X + dX * dt;
    Xm = Xm + dXm * dt;
    Z = Z + dZ*dt;

    t = t + dt;
    n = mod(n+1, 5);
    if(n == 0)
       CartDisplay(X, [Cm*Xm;0;0;0], Ref);
       end
    y = [y ; X(1), Cm*Xm, Ref];
    end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```