ECE 463: Homework #8

Linear Observers. Due Monday, March 31st



Cart and Pendulum from homework #4 with a state estimator (green)

Use the dynamics for the cart and pendulum from homework set #4

$$s\begin{bmatrix} x\\ \theta\\ \dot{x}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -19.6 & 0 & 0\\ 0 & 19.6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x\\ \theta\\ \dot{x}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix} 0\\ 0\\ 0.6667\\ -0.4444 \end{bmatrix} F$$

1) Design a full-state feedback control law of the form

$$U = F = K_r R - K_x X$$

so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input.

Plot the step response of the linarized system in Matlab.

2) Design a full-order observer to estimate all four states so that the observer is 2-5 times faster than the plant. You may use either cart position or beam angle (or both) as measurements.

$$sX_e = AX_e + BU + H(Y - Y_e)$$

3) Give the state-space model of the closed-loop system using the actual states:

 $U = F = K_r R - K_x X$

and plot the step response with initial conditions of

$$X(0) = [0, 0, 0, 0]'$$
 $X_e(0) = [0, 0.1, 0, 0]'$

(note: use the function step3)

4) Give the state-space model of the closed loop system using the state estimates:

$$U = K_r R - K_x X_e$$

and plot the step response with initial conditions of

$$X(0) = [0, 0, 0, 0]'$$
 $X_{observer}(0) = [0, 0.1, 0, 0]'$

5) (20pt) Modify the cart and pendulum system to include

- your control law, and
- A full-order observer

Plot the step response of the nonlinear system + observer when

- $Xe = [0, 0, 0, 0]^T$
- $Xe = [0.1, 0.1, 0.1, 0.1]^T$

