ECE 463/663 - Homework #9

Calculus of Variations. Ricatti Equation. LQG Control. Due Monday, April 7th

Soap Film

1) Calculate the shape of a soap film connecting two rings around the X axis:
   - Y(0) = 8
   - Y(5) = 7

2) Calculate the shape of a soap film connecting two rings around the X axis:
   - Y(0) = 8
   - Y(3) = free

Hanging Chain

3) Calculate the shape of a hanging chain subject to the following constraints
   - Length of chain = 15 meters
   - Left Endpoint: (0,8)
   - Right Endpoint: (10,7)

Ricatti Equation

4) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^{10} (x^2 + 10\dot{x}^2)dt$$

x(0) = 8

x(10) = 7

5) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^{10} (x^2 + 10u^2)dt$$

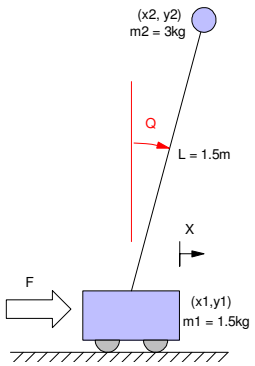
$$\dot{x} = -0.5x + u$$

x(0) = 8

x(10) = 7

LQG Control for a Cart & Pendulum

6) Cart & Pendulum (HW #4 & HW#6):



$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.667 \\ -0.444 \end{bmatrix} F$$

Design a full-state feedback control law of the form

$$F = U = K_r R - K_x X$$

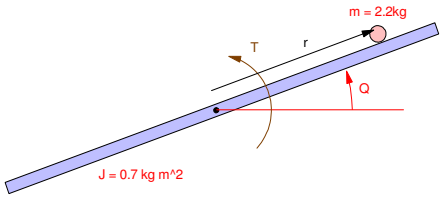
for the cart and pendulum system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

7) Ball and Beam (HW #4 & HW#6):



$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7/434 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.345 \end{bmatrix} T$$

Design a full-state feedback control law of the form

$$T = U = K_r R - K_x X$$

for the ball and beam system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?